Design of stable fuzzy controllers for an AGV

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Abstract

Fuzzy logic control is a relatively new technology and hence it needs rigorous comparative analyses with other well-established conventional control schemes. Further, fuzzy controller stability analysis is a major hindrance for its popularity among control engineers. This paper shows how stable fuzzy controllers may be synthesized for a typical AGV from the perspective of variable structure systems (VSS) theory. VSS or sliding model control (SMC) is an established robust non-linear control methodology. The AGV is characterized by highly non-linear, coupled and configuration dependent dynamics, with uncertainty in model parameters. Similarity in performance of the fuzzy controllers to the SMC controller is demonstrated through experimental results obtained for steer control of the AGV.

1. Introduction

Control of Autonomous Guided Vehicles (AGV) is a very challenging nonlinear control problem due to the complexity of the model, uncertainty of parameters and the significant coupling effects on the steer system by the drive system and vice versa. Other nonlinear effects like backlashes in the gear and road grade variations can also cause adverse effects.

Although, AGVs are characterized by nonlinear dynamics, the application of linear control methods has not been uncommon as is evident from the reported literature [7,8]. Here, the controller design is based on the assumption that the model dynamics are linear or on a linearised model of the AGV about a specified operating point/s. Most commonly used linear control techniques are, PI, PD and PID control methods.

Kagma et al [7] and Zalila et al [8] used a PD and a PI controller respectively in the steer control of a mobile robot. PID controller for steering control is reported in Kodagoda et al [4]. Although PI, PD, and PID controllers are commonly used in controlling nonlinear plants, their operation is limited to a narrower operating region. Further, such reported controllers perform poorly in the presence of significant parametric uncertainty.

Nonlinear methods of AGV control, where the non-linearities are catered for indirectly has also been reported in the literature. Hessburg et al[9] has demonstrated through real time implementation the efficacy of a fuzzy logic controller (FLC) for lateral (steering) control of an AGV. Lee et al [10] proposed a fuzzy-like gain scheduling PD controller for lateral control of an AGV. The control law is realized using a control structure similar to Sugeno's fuzzy model, however with non-overlapping membership functions.

Fuzzy logic has a lot of appeal in control. However, fuzzy logic control has a relatively short history. In addition, most of the fuzzy controllers proposed in the literature lack stability analysis and results. It is also important to note that there are also not much experimental results and analysis reported comparing established and conventional control schemes. In this paper we attempt to address the above issues through the systematic design, analysis and experimentation of fuzzy controllers. In the rest of the paper we describe the particular AGV and its dynamics used as the test bed. A stable SMC is designed for the dynamic system characterized by the AGV from Lyapunov theory. A stable fuzzy SMC is then derived through fuzzification of the sliding mode control law. A fuzzy PD state space controller is then synthesized by resolving the fuzzy SMC control law. This is followed by experimental results and the conclusions.

2. AGV and its dynamics

The vehicle utilized as a test bed is a *Carryall 1* golf car (see Figure 1). It is a front wheel steerable, rear wheel drive, electrically powered car and is suitably modified for autonomous control. A DC servomotor drives the steer system while a DC series motor powers the drive system.

The dynamic model of the AGV is [3],

$$\tau = M\ddot{\theta} + Q + d \tag{1}$$

$$\tau = \begin{bmatrix} \tau_d \\ \tau_s \end{bmatrix}, \qquad M = \begin{bmatrix} M_v r & Nr \\ N & M_\omega \end{bmatrix}$$

$$Q = \begin{bmatrix} \left(C_{d1}\omega^2 + C_{d2}v\omega + k_v v + F_{df}\right) r \\ C_{s1}\omega^2 + C_{s2}v\omega + \tau_{sf} \end{bmatrix}, \quad \ddot{\theta} = \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix}$$

M is the positive definite inertia matrix, τ is the vector of drive and steer torques, $\ddot{\theta}$ is the vector of linear and angular acceleration, Q is the vector of Coriolis, centrifugal and frictional forces, and $d \in R^{n\times 1}$ is the disturbance vector. v, γ and ω are speed, steer angle and rate of change of steer angle respectively. Other parameters are defined in Kodagoda et al [4]. It may be noted that the model is nonlinear, complex and configuration dependent.

Figure 1 Experimental AGV

3. Sliding Mode Controller (SMC)

Variable structure system or sliding mode control is an effective controller methodology for controlling of nonlinear systems in the presence of model uncertainties, parameter fluctuations and disturbances provided that their upper bounds are known.

The AGV dynamics can be expressed as a linear combination of a suitably chosen parameter vector $\Omega \in R^{r\times 1}$, using equation (1):

$$\tau = W\Omega + d \tag{2}$$

$W \in R^{n\times r}$ is a matrix of functions. Let us choose the following control torque, τ :

$$\tau = \hat{M}u + \hat{Q} + \tau^{smc} \tag{3}$$

u is chosen as, $u = \ddot{\theta}_d + \Lambda(\dot{\theta}_d - \dot{\theta})$, $\Lambda \subset R^{n\times n}$ is a diagonal gain matrix with elements $\lambda_i \ (i = 1,\ldots,n)$, $\dot{\theta}_d$ is desired vector of linear and angular velocities, and $\ddot{\theta}_d$ is its derivative. $\hat{M}$ and $\hat{Q}$ are the estimates of M and Q. The component $(\hat{M}u + \hat{Q})$ represents the computed torque component, which attempts to linearize and decouple the system (1). The term τ^{smc} is used to remove the effects of inexact de-coupling as a result of model mismatch and bounded disturbances. As detailed in [1] the closed loop equation can now be written as:

$$(\ddot{\theta} - \ddot{\theta}_d) + \Lambda(\dot{\theta} - \dot{\theta}_d) = M^{-1}\left[\tau^{smc} - (W\psi + d)\right] \tag{4}$$

ψ represents the parameter mismatch vector representing the mismatch between the actual parameter vector (Ω)and its estimate ($\hat{\Omega}$). Now, let us define the i^{th} sliding surface as,

$$s_i = \left(\dot{\theta}_i - \dot{\theta}_{d_i}\right) + \lambda_i\left(\theta_i - \theta_{d_i}\right) \tag{5}$$

A condition for the intersection of switching planes, $s = (s_1, s_2) = 0$, to be attractive can be derived by defining a quasi Lyapunov function *V(t)* as,

$$V(t) = \frac{1}{2}s^T M s \tag{6}$$

M is the AGV's positive definite inertia matrix. Differentiating (6) we have,

$$\dot{V}(t) = s^T M\dot{s} + \frac{1}{2}s^T \dot{M}s \tag{7}$$

Now the i^{th} ($i = 1,2.$) component of τ^{smc} can be chosen as given in equation (8) to ensure $\dot{V}(t) \le 0 \ \forall \ (t > 0)$ so as to guarantee the asymptotic stability of s_i and hence the convergence of tracking errors to zero [2].

$$\tau_i^{smc} = -\mathrm{sgn}(s_i)\left[\sum_{j=1}^{r}\bar{W}_{ij}\bar{\psi}_j + \bar{d}_i + \delta_i\right] - s_i\sum_{i=1}^{n}\frac{\bar{\dot{M}}_{ij}}{2}, \quad i = 1,2 \tag{8}$$

where,

$$\left|d_i\right| < \bar{d}_i \ , \ \left|M_{ij}\right| < \bar{\dot{M}}_{ij} \ , \ \delta_i > 0 \ \ (i = j = 1,2.),$$

$$\left|W_{ij}\right| < \bar{W}_{ij} \ \ (i = j = 1,2), \quad \left|\psi_j\right| < \bar{\psi}_j \ \ (j = 1,\ldots,r)$$

4. Fuzzy Sliding Mode Controller

Fuzzy control offers simple but robust control solutions for nonlinear systems [1]. Further fuzzy logic provides a convenient framework to incorporate the heuristic knowledge about the plant. Also, unlike SMC, fuzzy control approach permits greater flexibility in fine-tuning the controller to achieve arbitrary complex control surfaces. Next we show how an approximate fuzzy SMC may be derived from the SMC control law (8). Now, equation (8) can be expressed as:

$$\tau_i^{smc} = -\mathrm{sgn}(s_i)\left[K_{i1} + |s_i| K_{i2}\right] \quad i = 1,2 \tag{9}$$

$$K_{i1} = \sum_{j=1}^{r}\left[\bar{W}_{ij}\psi_j + \bar{d}_i + \delta_i\right],\; K_{i2} = \sum_{i=1}^{2}\frac{\bar{M}_{ij}}{2}, \quad i = 1,2$$

K_{i1} and K_{i2} are positive constants whose magnitudes depend on the model mismatch and the bounded disturbance d. We choose $K_{i1,}$ and K_{i2} large enough so as to eliminate the need for the computed torque component of the control law (3). From equation (9) it can be seen that farther the system response is from the sliding surface, larger the magnitude of the torque needed to drive the system on to it. However, the sign of the control torque is opposite to that of s_i. One possible fuzzification of the SMC law (9) with a boundary layer using m rules is through the following rule structure [6].

$$\text{IF } \bar{s}_i \text{ is } A_i^k \text{ THEN } \tau_i^{fsmc} \text{ is } B_i^k \quad , \quad i = 1,2,\; k = 1,\cdots,m \tag{10}$$

$$\bar{s}_i = \dot{e}_i + \lambda_i \cdot e_i,\; \bar{s}_i = -s_i,\; e_i = (\theta_{di} - \theta_i),\; i = 1,2$$

An appropriate choice for the linguistic terms A_i^k and $B_i^k,\; k = 1,..,m$ of the linguistic variables $\bar{s}_i$ and τ_i^{fsmc} are:

$$A_i^k = B_i^k = \{NVL, NL, NM, NS, Z, PS, PM, PL, PVL\}$$

The nine terms are, *PVL* (Positive Very Large), *PL* (Positive Large), *PM* (Positive Medium), *PS* (Positive Small), *Z* (Zero), *NS* (Negative Small), *NM* (Negative Medium), *NL* (Negative Large) and *NVL* (Negative Very Large). We choose triangular membership functions, singleton fuzzification, Mamdani inferencing system and center of gravity (COG) defuzzification. Hence the inferred crisp torque output of the fuzzy SMC is:

$$\tau_i^{fsmc\_crisp} = \frac{\sum_{j=1}^{r} b_j^i \int_\Gamma \mu_{B_i^j}(\tau_i^{fsmc})\, d\tau_i^{fsmc}}{\sum_{j=1}^{r} \int_\Gamma \mu_{B_i^j}(\tau_i^{fsmc})\, d\tau_i^{fsmc}} \tag{11}$$

b^i_j is the center of the implied fuzzy set corresponding to the rule j, r is the total number of rules and $\int_\Gamma \mu_{B_i^j}(\tau_i^{fsmc})\, d\tau_i^{fsmc}$ denotes the area under $\mu_{B_i^j}(\tau_i^{fsmc})$.

Fuzzy SMC (11) approximation of the smoothed SMC controller (9) can be arbitrarily controlled by the number of linguistic terms and hence the rules. However, what is really important to ensure stability and convergence of the fuzzy SMC is to guarantee the negative semi-definiteness of the Lyapunov function derivative, i.e. $\dot{V}(t) \le 0 \;\forall\; (t > 0)$. This translates to the condition that the crisp fuzzy SMC output is larger or equal in magnitude to the SMC torque for all $\bar{s}$, i.e.

$$\left|\tau_i^{smc}(\bar{s}_i)\right| \le \left|\tau_i^{fsmc\_crisp}(\bar{s}_i)\right| \quad \forall\; \bar{s}_i \tag{12}$$

5. Fuzzy PD Controller

The heuristic control knowledge is usually available in terms of state variables and their changes. To facilitate inclusion of such knowledge and to add more degrees of freedom for tuning, it is useful to design state based fuzzy controllers. One of such controllers is a fuzzy PD controller. The following illustrates how such a law may be derived from the fuzzy sliding mode controller. Switching plain, $\bar{s}_i$ can be normalized to $\hat{s}_i$ as:

$$\hat{s}_i = \hat{\dot{e}}_i + \hat{e}_i \tag{13}$$

where, $\hat{s}_i = \frac{-s_i}{N_{si}},\; \hat{\dot{e}}_1 = \frac{\dot{e}_i}{N_{si}},\; \hat{e}_i = \frac{\lambda_i e_i}{N_{si}}$, N_{si} is a positive scalar.

From (13) it follows that $\hat{s}_i\; (i = 1,2)$ is a linear combination of the state variables ($\hat{e}_i$ and $\hat{\dot{e}}_i$). Thus, after normalizing we may replace $\bar{s}_i$ of the fuzzy SMC rules (10) by corresponding sets of pairs of the state variables to yield a rule base in state space giving the same or higher crisp control output as the crisp FSMC [1]. The resultant state space rule base is given in Table 1, assuming all inputs ($\hat{s}_i, \hat{e}_i, \hat{\dot{e}}_i$) and outputs ($\tau_i^{fsmc}, \tau_i^{fpd}$) are partitioned into nine linguistic terms in their respective universe of discourses. Here again we use singleton fuzzification, Mamdani inferencing and center of gravity (COG) defuzzification technique to obtain the crisp output of the fuzzy PD controller $\tau_i^{fpd-crisp}(\hat{s}_i)$. Again, to ensure stability and convergence it is sufficient that the following condition (14) is satisfied at all points in the state space to guarantee the Lyapunov stability criteria.

$$\left|\tau_i^{smc\_crisp}(\hat{s}_i)\right| \le \left|\tau_i^{fpd\_crisp}(\hat{s}_i)\right| \quad \forall\; \hat{s}_i \tag{14}$$

Whilst maintaining the above (14), parameters of the fuzzy PD controller can be tuned in order to satisfy the performance requirements.

$\hat{\tau}_i^{fpd}$		$\hat{\dot{e}}_i$								
		NVL	NL	NM	NS	Z	PS	PM	PL	PVL
$\hat{e}_i$	NVL	NVL	NVL	NVL	NVL	NVL	NL	NM	NS	Z
	NL	NVL	NVL	NVL	NVL	NL	NM	NS	Z	PS
	NM	NVL	NVL	NVL	NL	NM	NS	Z	PS	PM
	NS	NVL	NVL	NL	NM	NS	Z	PS	PM	PL
	Z	NVL	NL	NM	NS	Z	PS	PM	PL	PVL
	PS	NL	NM	NS	Z	PS	PM	PL	PVL	PVL
	PM	NM	NS	Z	PS	PM	PL	PVL	PVL	PVL
	PL	NS	Z	PS	PM	PL	PVL	PVL	PVL	PVL
	PVL	Z	PS	PM	PL	PVL	PVL	PVL	PVL	PVL

Table 1 Equivalent state space rule base

6. Experimental Results

Experiments were carried out to assess the performance of SMC, fuzzy SMC and fuzzy PD controllers. Each controller was given a trapezoidal steer angle profile starting from a zero angle.

Figure 2 shows the SMC performance with a boundary layer. Width of the boundary layer was chosen to reduce the control chatter. Smaller boundary layers give better accuracy at the expense of high control torque chatter.

Figure 3 shows the tracking performance of the fuzzy SMC, which was synthesized using the SMC controller. Fuzzy SMC tracking performance is very similar to that of SMC (Figure 2) in terms of tracking accuracy and control output. Further, it was observed that tighter spacing of membership functions around the origin yields better tracking accuracy, however, with increased control chatter. This is very much similar to the behavior of the equivalent SMC controller with changes in the boundary layer.

The tracking performance of the synthesized fuzzy PD controller based on the fuzzy SMC is shown in Figure 4. It is seen that the performance of the fuzzy PD controller is very similar to the fuzzy SMC and also to the SMC. Again, it was observed that a higher density of membership functions around the origins yields better accuracy and more control chatter. Nature of membership functions around the origin determines the shape and thickness of the effective boundary layer of the equivalent SMC and hence the observed behavior.

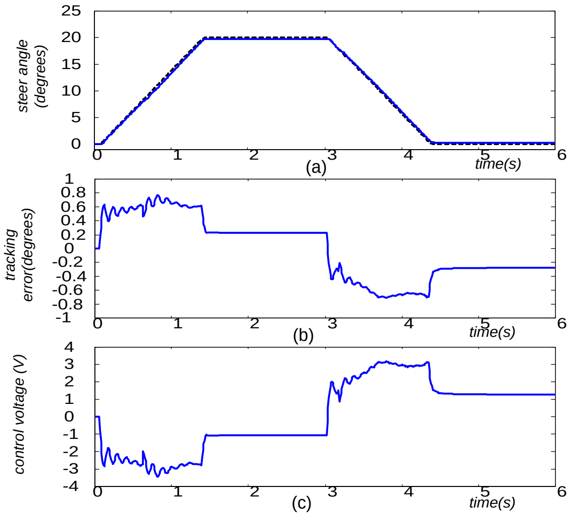

Figure 2 Sliding mode controller, (a) steer angle, (b) tracking error, (c) control voltage

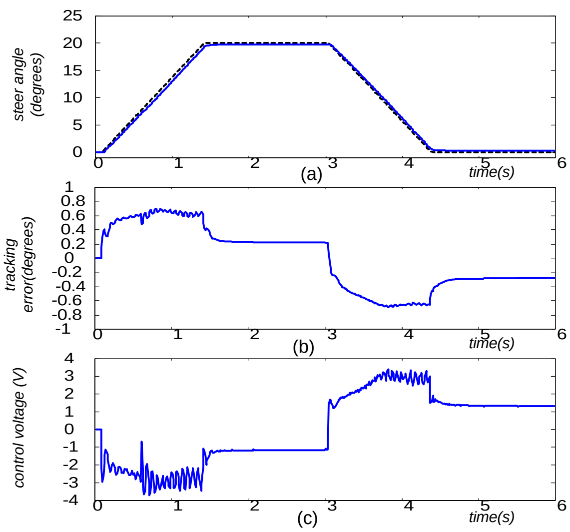

Figure 3 Fuzzy sliding mode controller, (a) steer angle, (b) tracking error, (c) control voltage

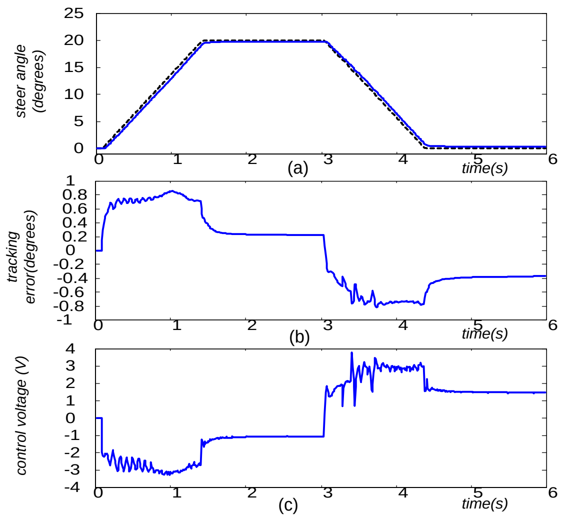

Figure 4 Performance of fuzzy PD controller (a) steer angle, (b) tracking error, (c) control voltage

Integral action

In the designed fuzzy PD control scheme, the steady state error is mainly dependant on the input normalization gain and the distribution of membership functions of the linguistic variable, “angle error”. Selecting higher normalization gains as well as selecting densely packed membership functions near the origin can minimize the steady state error. However, both methods will increase the control chatter. One of the ways to overcome that problem is to introduce an integral part as a separate input. With two inputs and each input consists of 7 membership functions, the rule base size is (7x7). After introducing the integral of error as a separate input, the rule base size becomes (7x7x7), which will make it more complex and computationally expensive. Without increasing the number of inputs we can incorporate integral action by redefining the normalized switching surface as follows:

$$\hat{s}_i = \hat{e}_i + \left(\dot{\hat{e}}_i + \int \hat{e}_i \, dt\right) \tag{13}$$

Now, we may introduce a new state variable equal to $\left(\dot{\hat{e}}_i + \int \hat{e}_i \, dt\right)$. Following the derivation detailed in sections 3, 4, and 5, we may obtain an equivalent fuzzy P(I+D) controller. The structure of the rule base for this choice of inputs will correspond to Table 1, although the scales and the gain parameters would be different. Figure 5 shows the improvement in tracking performance with the new fuzzy P(I+D) controller.

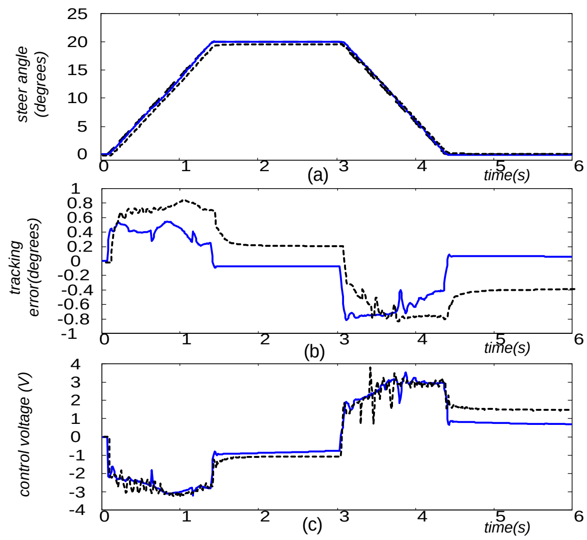

Figure 5 Performance of fuzzy PD controller (*dotted curve*) and fuzzy P(I+D) controller (*solid curve*)

7. Conclusion

For a class of nonlinear systems characterized by AGV dynamics, stable fuzzy SMC, PD/PID control laws can be synthesized using sliding mode control theory. The synthesized fuzzy controllers yield performance similar to the equivalent sliding mode controllers. There is not much to be gained from using a fuzzy SMC against an SMC with fixed torque bounds, especially, if the inputs are less than three. However, the state space equivalent of the SMC is advantageous in that it gives more degrees of freedom in performance tuning and also provides for the inclusion of heuristic control knowledge, which is usually available in terms of simple state variables. Another important consequence of the fuzzy control law development presented is that it shows how additional state variables, such as a variable's derivative and integral, can be incorporated into the controller by suitably modifying the switching surface. This permits the treatment of linear combinations of state variables (e.g.

D+I), as inputs to the fuzzy controller thus simplifying the rule base, computational complexity, and memory requirements, especially when the state inputs exceed two.

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