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The influence of higher order incident modes on the performance of a hybrid reactive-dissipative splitter silencer

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A hybrid reactive-dissipative splitter silencer offers the potential to attenuate turbomachinery noise over a wide frequency range, including the problematic low to medium frequencies. This article uses a theoretical model to investigate the performance of a hybrid parallel baffle silencer for different complex incident sound fields. This includes an incident sound field with equal modal energy density, as well as the excitation of individual higher order modes. It is shown that provided horizontal and vertical partitions are used in the reactive element, the sound attenuation performance of the reactive chamber under complex incident sound fields is equivalent to that obtained using plane wave excitation over the frequency range of interest. Furthermore, it is demonstrated that the reactive elements work at frequencies above the first cut-on mode in the inlet duct, and so they are capable of extending sound attenuation into the low to medium frequency range. This delivers an efficient hybrid silencer design that is suitable for use in power generation applications, such as gas turbine exhaust systems.

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1. INTRODUCTION

The control of noise emitted by power generating equipment such as gas turbines presents many difficult challenges. These devices are normally relatively large so that noise control devices need to be designed for intake and exhaust ductwork with large transverse dimensions. This makes it difficult to attenuate noise over a wide frequency range because traditional methods such as combining reactive and dissipative elements do not work so well in the low to medium frequency range in larger duct systems. This is because the dissipative elements only work well in the medium to high frequency range,[1,2] and reactive elements placed around the outer surface of a duct do not perform well when higher order acoustic modes propagate in the duct. This creates a problem in the low to medium frequency range, where reactive elements are unable to provide sufficient levels of attenuation before the dissipative element takes over.

One possible solution is to combine the reactive elements into a hybrid splitter silencer, so that the reactive elements are used to partition rather than to surround the airway.[3] This has the potential to introduce a quasi-planar transverse sound pressure field in the low to medium frequency range, which will allow the reactive elements to continue to work at frequencies above the first cut-on frequency in the main duct. However, for this to work the transverse sound pressure field must provide suitable conditions for the reactive elements to continue to be effective at higher frequencies, and this will be heavily dependent on the characteristics of the incident sound field. For example, if the incident sound field contains energy propagating in higher order modes, then this is likely to cause the transverse sound field within the silencer to move further away from the quasi-planar conditions desirable to maintain reactive silencer performance. Accordingly, this article examines the influence of higher order incident modes on the performance of reactive elements over a low to medium frequency range extending above the first cut-on in the main duct.

The influence of the sound source on noise attenuating devices has received relatively little attention. This is because it is difficult to provide accurate estimates of the often complex modal content emitted by devices such as fans or turbines, and measuring modal amplitudes also presents a significant challenge. However, this does not mean that noise signatures of power generating devices cannot have a significant impact on the performance of silencer systems, especially those used in large ductwork. Accordingly, when designing silencers for real systems one should account for the possibility of higher order modal content in the sound source. This is particularly important for reactive silencers, as these are known to become ineffective under non-planar conditions.

To investigate this effect, it is convenient to start by using theoretical models, as these allow one to remove experimental uncertainty in order to focus on the physical processes involved. An estimate of the higher order modal content emitted by turbomachinery was reported by Mechel,[4] who introduced three different scenarios: equal modal energy density (EMED), equal modal amplitude (EMA), and equal modal power (EMP). Following this study, it has been generally accepted that EMED is the most appropriate representation of turbomachinery, although Kirby and Lawrie[5] did compare all three possibilities when calculating the transmission loss for a dissipative splitter silencer. Kirby and Lawrie show that for different incident modal characteristics the silencer transmission loss changes significantly, especially in the medium to high frequency range for dissipative elements. This demonstrates that it is important to understand the way in which a noise source and a noise control device interacts with one another, and that this should be accounted for when designing silencers for complex applications. In view of this, this article will investigate the effect of higher order incident modes on the performance of reactive elements used as part of a hybrid dissipative-reactive splitter design. This investigation will be based on the use of theoretical models and it will investigate the influence of an EMED incident sound field on the performance of reactive elements, and introduce ways in which the reactive elements can be designed to minimise the impact of higher order incident modes.

2. HYBRID REACTIVE-DISSIPATIVE SILENCER DESIGN

The hybrid reactive-dissipative silencers examined here combine separate dissipative and reactive elements. The geometry of the hybrid silencer is based on the parallel baffle splitter designs analysed by Kirby and Lawrie,[5] see Fig. 1. This design uses n_b rectangular baffles distributed across the cross-section of the duct with a reactive element placed at the rear of the splitter, see Fig. 2.

At the front of each baffle is a rigid fairing designed to smooth the flow. This is followed by a dissipative element made from rock wool, with length L_D. This is exposed to the airway using a perforated screen made from steel. The reactive element follows, and this is a quarter wave resonator with an opening just behind the dissipative element. The opening also uses the same perforated screen as that used by the dissipative section. In order to explore the action of the reactive element, the internal design is varied following designs similar to that used by Williams et al.[3] Accordingly, two different reactive chamber designs are examined here: (i) the empty chamber design A, which is shown in Fig. 2(a); and (ii) the same empty chamber, but with a vertical partition that extends the entire length of the reactive element, to give chamber design B, see Fig. 2(b). This then delivers three different silencer geometries, which are listed in Table 1. The first silencer, silencer 1, uses the chamber design A, and has three baffles, so that $n_b = 3$; the chamber for this silencer is designed so that the target centre frequency is 50 Hz. Two further designs are examined, silencers 2 and 3, and both silencers use chamber design B,

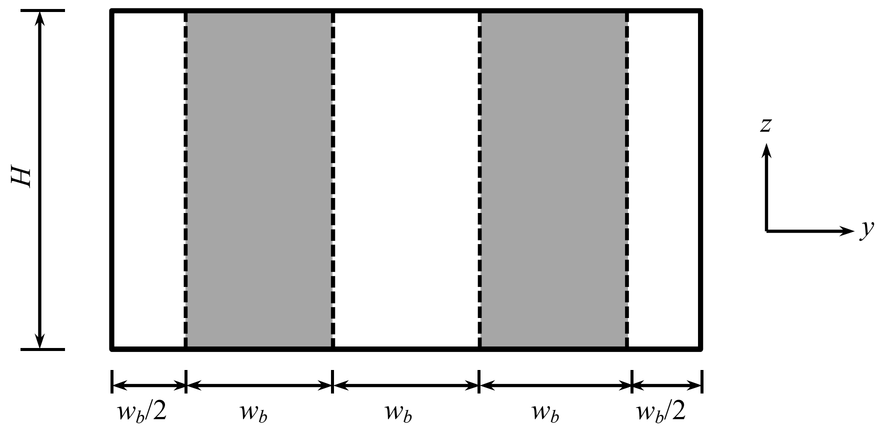

Figure 1. Front view. Silencer cross-section with n_b = 2 baffles.

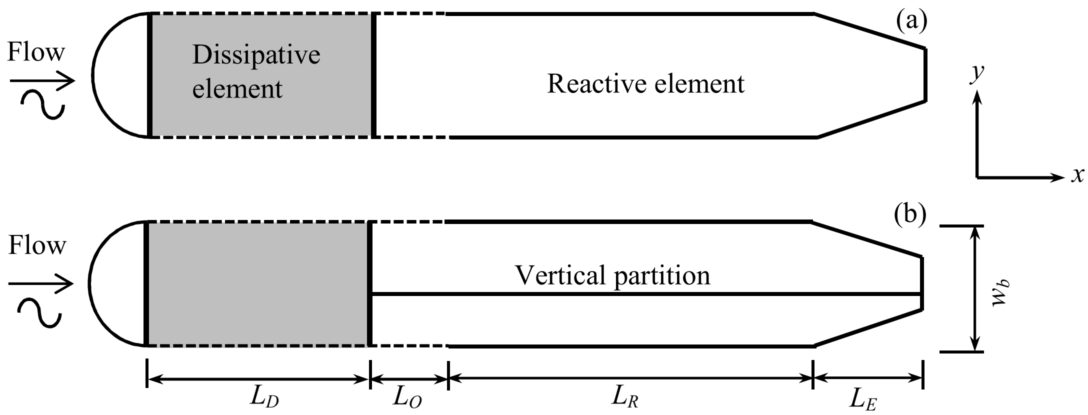

Figure 2. Plan view. Hybrid dissipative-reactive silencer design. (a) Design A; (b) Design B.

Table 1. Silencer parameters.

Silencer	w_d (m)	n_b	w_b (m)	L_D (m)	f_r (Hz)	Reactive Element	Horizontal partitions
1	0.8	3	0.13	0.2	50	A	0
2	0.8	3	0.13	0.2	50	B	0
3	0.4	1	0.20	0.1	125	B	0
4	0.4	1	0.20	0.1	125	B	2

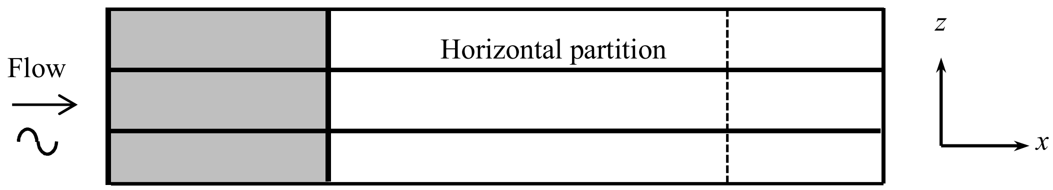

Figure 3. Side view. Horizontal baffle design.

but here the target frequency is 50 Hz for silencer 2, and 250 Hz for silencer 3, see Table 1. Finally, a further silencer is examined, silencer 4, which is identical to silencer 3 apart from the addition of two horizontal partitions, and these have been added for reasons that will become apparent later on. Note that these horizontal partitions run the entire length of the baffle, including both the dissipative and reactive elements, see Fig. 3.

The dissipative element has a length of L_D = 200 mm for each silencer, and a cross sectional percentage open area of Δ = 50% is also maintained for each design, which requires the number of baffles across the duct to be changed in order to maintain a constant open area.

3. THEORY

The development of theoretical models for large splitter silencers has been covered extensively in the literature. See for example the analytic model of Lawrie and Kirby[6], the finite element models reported by Kirby et al.[7,8], and the boundary element models reported by Zhou et al.[2] andWang.[9] Therefore, the theoretical model used to analyse silencer performance is not reported in detail here, and the reader is referred instead to the finite element based approach reported by Kirby et al.,[7] and in particular the hybrid approach reported by Kirby[7,10] which is of most relevance to this current work. To summarise: this approach is based on a full finite element discretisation for the main silencer section, which is then matched onto a modal solution for the uniform duct sections that lie on the inlet and outlet sides of the silencer. This approach avoids the need to mesh the inlet and outlet ducts, whilst at the same time retaining flexibility for the silencer section so that non-uniform designs can be analysed. Crucially, this approach also enables higher order modes to be easily included in the inlet and outlet ducts, which is not the case if one uses a full finite element discretisation of the entire system.

The focus of this article is the analysis of the effects that higher order incident modes have on silencer performance, and especially reactive silencer performance. This will involve generating predictions for a planar sound source, a sound source with equal modal energy density (EMED), as well as exciting individual modes. Accordingly, the assumed ansatz for the pressure in the inlet and outlet ducts is

$$p(x,y,z;t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \mathrm{A}_{m,n}\psi_{m,n}(y,z)e^{i\omega t}e^{-ik_0\lambda_{m,n}x}, \quad (1)$$

Here, p is the acoustic pressure, $\mathrm{A}_{m,n}$ is the modal amplitude, $\psi_{m,n}(y,z)$ is the eignefunction for the duct, t is time, $k_0 = \omega/c_0$ where ω is the radian frequency and c_0 is the speed of sound in the airway, $\lambda_{m,n}$ is the eigenvalue for the duct, and finally $i = \sqrt{-1}$. It is straightforward to obtain an analytic expression for $\psi_{m,n}(y,z)$ and $\lambda_{m,n}$ for an empty rectangular duct, and for the eigenvalue this gives

$$\lambda_{m,n} = \sqrt{1 - \left(\frac{m\pi}{k_0 w_d}\right)^2 - \left(\frac{n\pi}{k_0 H}\right)^2}. \qquad (2)$$

In the analysis that follows, various combinations of m and n are applied in order to investigate how the silencer responds to different methods of excitation. In the first case only modes of the kind (m, 0) are excited, so that the eigenfunction is constant along the z axis and the pressure varies in the y direction only. This enables symmetry to be used to reduce the theoretical model from three to two dimensions.[6] This is the problem analysed for silencers 1 and 2. The second case will consider all possible modes propagating at a given frequency, so that $m \geq 0$ and $n \geq 0$. This demands a full three dimensional analysis for the silencer and to enable this to be tractable within a sensible solution time at higher frequencies, it is necessary to analyse a smaller silencer, so that the duct width is reduced from 800 mm to 400 mm.

The overall duct size used for silencers 1-4 is relatively small for gas turbine exhaust systems, however this still allows for a number of higher order modes to cut-on within a frequency range where three dimensional solutions can easily be achieved. Moreover, it is key here to generate an accurate numerical model in order to ensure reliable predictions, and this is why the size of the problem is moderated here; however, it is expected that the observations and conclusions identified will scale for large silencers, so that higher order modes simply cut-on at lower frequencies than that seen in this article. Note that for silencers 1 and 2, the higher order cut-on frequencies in the inlet and outlet duct, $f_{m,n}$, are at $f_{1,0}$ = 215Hz, $f_{2,0}$ = 430Hz, $f_{3,0}$ = 644Hz and $f_{4,0}$ = 859Hz for modes (1,0), (2,0), (3,0) and (4,0) respectively at 20°C. The plane wave region of the duct is therefore restricted to frequencies below 214Hz. For silencers 3 and 4, the first 7 higher order mode cut-on frequencies are analysed and these cut-on at $f_{0,1}$ = 287Hz, $f_{1,0}$ = 430Hz, $f_{1,1}$ = 516Hz, $f_{0,2}$ = 573Hz, $f_{1,2}$ = 716Hz, $f_{0,3}$ = 859Hz and $f_{2,0}$ = 859Hz.

In order to solve the problem, it is necessary first to assign the properties of the incident sound field. This is accomplished by changing the relative values of the incident modal amplitudes. Thus, if the incident sound field is a plane wave, and the modal amplitudes in the incident duct are $\mathrm{F}_{m,n}$, then $\mathrm{A}_{0,0} = 1$, and $\mathrm{A}_{m,n} = 0$ for $m > 0$ and $n > 0$. For EMED excitation then[4]

$$\left|\frac{\mathrm{F}_{i,j}}{p_0}\right|^2 = \frac{\mathrm{I}_{0,0}}{\mathrm{I}_{i,j}\sum_{m=0}^{\mathrm{M}}\sum_{n=0}^{N}\lambda_{m,n}}, \qquad (3)$$

where, M and N are the number of modes found on solution of the eigenproblem in region Ω_{A}, and $\mathrm{I}_{i,j} = \int \left|\psi_{i,j}(y,z)\right|^2 dy\mathrm{d}z$. The silencer transmission loss, TL, is then defined as the ratio of the inlet to the outlet sound power, and so in general this gives:

$$\mathrm{TL} = -10\log_{10}\frac{\sum_{m=0}^{\mathrm{M}}\sum_{n=0}^{N}\mathrm{Re}(\lambda_{m,n})\mathrm{I}_{m,n}\left|\mathrm{D}_{m,n}\right|^2}{\sum_{m=0}^{\mathrm{M}}\sum_{n=0}^{N}\mathrm{Re}(\lambda_{m,n})\mathrm{I}_{m,n}\left|\mathrm{A}_{m,n}\right|^2}, \qquad (4)$$

where, $\mathrm{D}_{m,n}$ are the modal amplitudes in the outlet duct obtained following solution of the problem.[7,8]

To solve the problem a finite element approach is used to obtain the eigensolution for the uniform duct and a full finite element discretisation is used for the central silencer section. For one dimensional regions three noded quadratic line elements are used, for two dimensional regions eight noded quadrilateral elements are used, and for three dimensions 20 node hexahedral elements are used. The predictions were checked for convergence and an element density of 50 elements per metre was used in each mesh to ensure an accurate solution.

4. RESULTS AND DISCUSSION

In this section, TL predictions are reported for the four different silencer combinations outlined in Table 1. A prediction for silencer 1, when excited by a plane wave, is shown in Fig. 4. Two basic characteristics can be determined for the transmission loss of silencer 1 under plane wave conditions. First, there exists a broad-band component which provides attenuation across all frequencies and rises to a maximum in the 1kHz octave band, although this is seen to provide poor levels of attenuation at low frequencies. Second, a series of narrow bandwidth peaks adds to the dissipative TL, and this is generated by the reactive elements. Note that a relatively small dissipative element is chosen in these numerical experiments in order to lower the TL of the dissipative elements so that one may focus on the behaviour of the reactive elements. The maximum TL for the resonator is located at the design frequency of 50Hz in Fig. 4, however a series of peaks continue above this frequency and these appear at harmonics of the design frequency. Each peak is seen to deliver a contribution to transmission loss that is comparable in amplitude to that seen for the fundamental peak. This demonstrates that the reactive element is effective at reducing noise over a range of frequencies, and this extends beyond the first cut-on mode in the inlet duct. For example, the fundamental peak is located below $f_{1,0}$, however the resonant peaks continue up to a frequency of $f_{6,0}$, and above this frequency the peaks continue to appear although the attenuation they provide becomes negligible. This demonstrates that with a plane wave sound source the hybrid silencer is capable of providing attenuation over a relatively wide frequency range that extends outside of the plane wave region. This is an important attribute of the hybrid silencer for large ducts, and this behaviour occurs because the reactive element is used to partition up the duct and this delivers quasi-planar conditions over a wider frequency range than is normally encountered in reactive silencer placed on the perimeter of the duct. Thus, if one wishes to increase the frequency range over which the reactive elements deliver sound attenuation, then this can be achieved by adding more baffles so that the duct cross-section is partitioned into ever smaller segments. Following an extensive numerical investigation, an approximate upper frequency limit for the reactive element, beyond which it is no longer considered to be active, occurs at a frequency equal to $f_{2n_b,0}$.

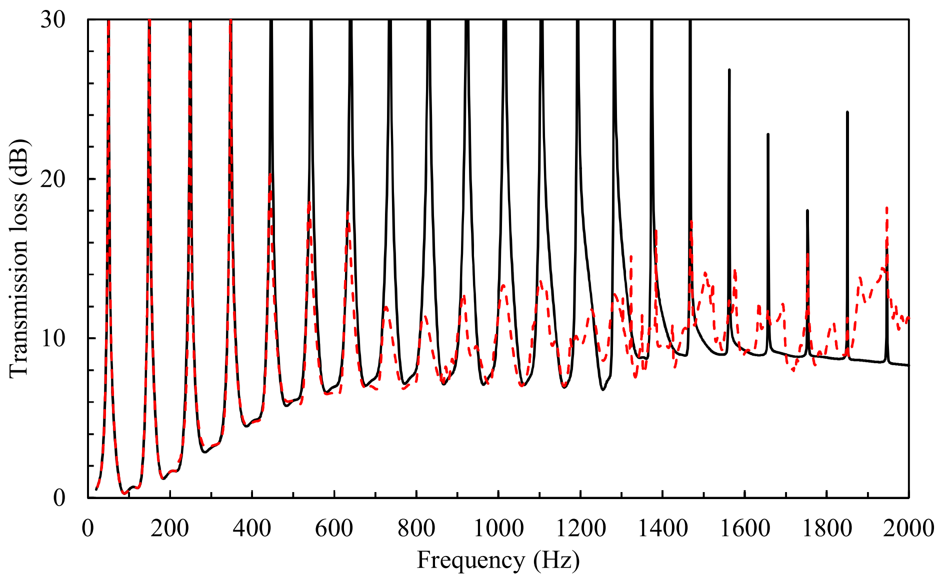

Figure 4. Transmission loss of silencer 1. ——— ***, plane wave excitation;*** - - - ***;EMED excitation.***

Figure 4 demonstrates that under a plane wave incident sound field the reactive element will continue to work at higher frequencies. However, performance of reactive elements is known to be heavily dependent on the incident sound field, and so Fig. 4 also includes EMED excitation. It is immediately clear that when the incident energy is distributed across different modes the action of the reactive element changes. The peaks in TL begin to lose their characteristic shape at higher frequencies, and above $f_{2,0}$ the additional TL provided by the reactive element begins to disappear. However, the reactive elements continue to work above $f_{1,0}$ and continue to provide additional attenuation over a relatively wide frequency range. This demonstrates that even under multi-modal EMED excitation the performance of the reactive element is retained. Thus, the presence of non-planar modes does affect the ability of the reactive elements to work at higher frequencies, however the hybrid silencer continues to be effective well above the first cut-on mode.

Further explanation of the behaviour seen in Fig. 4 can be achieved by exciting individual modes in the silencer. For example, in Fig. 5 TL predictions for silencer 1 are compared with the modes (0,0), (1,0), (2,0), and (3,0) excited individually. It is seen that once mode (1,0) cuts on at $f_{1,0}$, it generally overlays mode (0,0) at frequencies up to and including $f_{2,0}$. As a consequence, sound power in modes (0,0) or (1,0) experience similar levels of attenuation between $f_{1,0}$ and $f_{2,0}$ and so the transmission loss remains unaltered. This is consistent with the overlay in TL seen between $f_{1,0}$ and $f_{2,0}$ for EMED and plane wave excitation in Fig. 4. However, if excitation is limited to the (2,0) mode, then the TL shows resonances with a narrower bandwidth than mode (0,0), and at different peak frequencies with an amplitude that decreases with frequency. Moreover, those resonances above $f_{2,0}$ are shifted towards lower frequencies and with much lower amplitudes than those below $f_{2,0}$. Finally, excitation using mode (3,0) shows no resonances and so any power in this mode will not be effectively attenuated by the reactive element. The reason for mode (3,0) showing minimal attenuation is because the sound pressure at the entrance to the reactive chamber on either side of a baffle is out of phase.

One way to address the loss in sound attenuation experienced by some of the higher order modes in the previous example is to isolate the two opposite resonator openings from each other, so that sound cannot communicate between airways. This is addressed in silencer design B, shown in Fig. 2(b). This design maintains the same geometry as silencer 1, but applies a vertical partition in the expansion

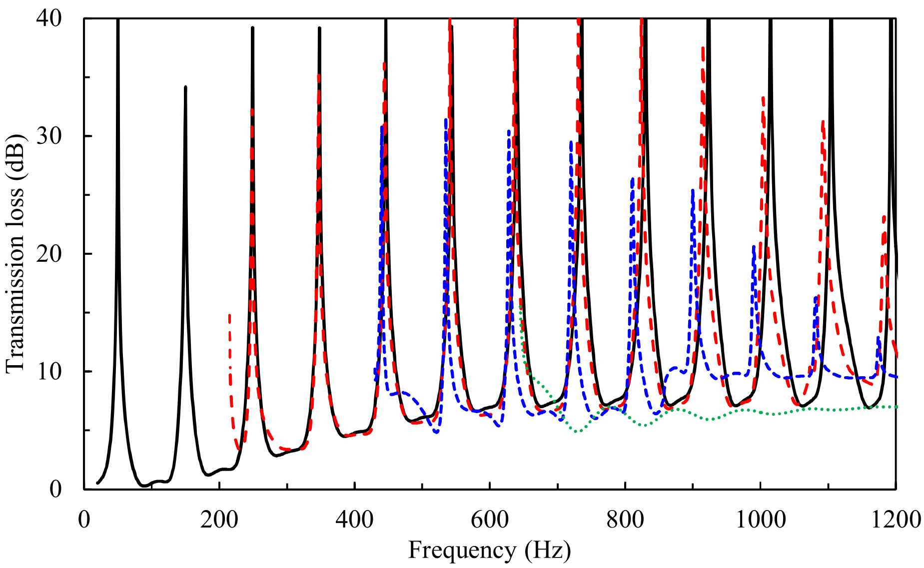

Figure 5. Silencer 1 when excited by individual modes.* ———*, mode (0,0);* – – – –*, mode (1,0);* – – – – *, mode (2,0);* ⋯⋯⋯ *, mode (3,0).

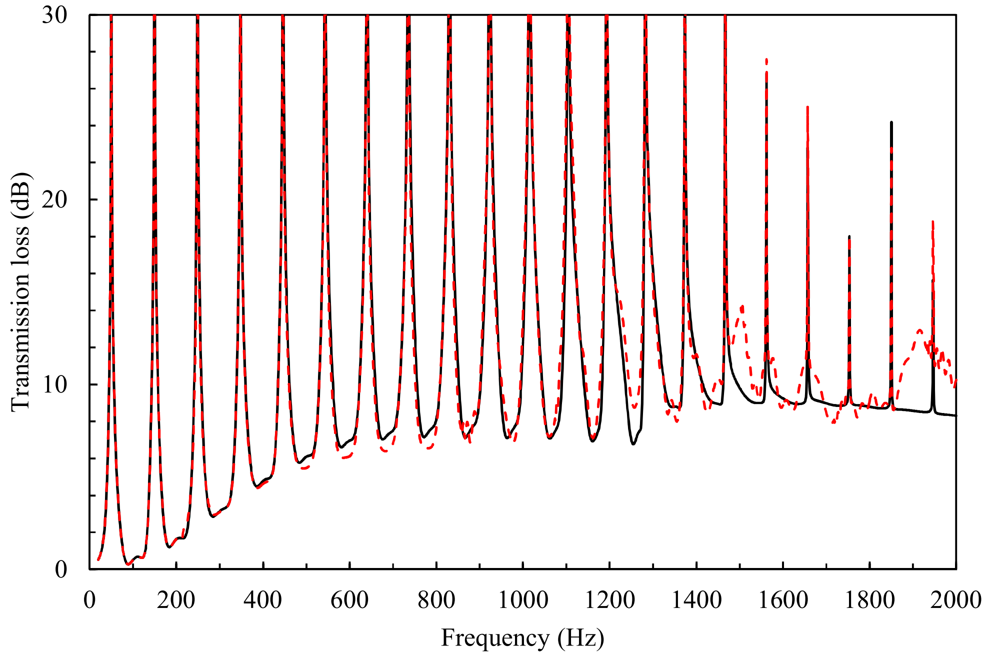

Figure 6. Transmission loss for silencers 1 and 2. ——— , silencer 1 plane wave; – – – , silencer 2 EMED.

chamber in order to isolate the two resonator openings. This silencer is labelled Silencer 2 in Table 1, and the TL obtained for plane wave and EMED excitation for the silencer is shown in Fig. 6. It is evident in Fig. 6 that the TL of silencers 1 and 2 is the same for PW excitation because the transverse sound pressure field is symmetrical for all frequencies. However, when EMED is used the partitioning of the reactive chamber restores performance and the quarter wave resonator under EMED excitation now acts as though the incident sound field was planar.

A three dimensional analysis is now introduced, so that the influence of non-planar modes in the vertical direction, with $n > 0$ as well as $m > 0$, is investigated. A duct height of 600 mm is used, although it is necessary to lower the duct width to 400 mm in order maintain acceptable requirements for computational power, see Table 1. Furthermore, only a single baffle is modelled here, which also enables the lowering of computational demands. Accordingly, higher order modes cut-on in the duct at frequencies of 285Hz for mode (0,1), and 429Hz for mode (1,0).

The reactive element for silencers 3 and 4 is tuned to a fundamental frequency of 125Hz. A vertical partition is also used in the resonator chamber (design B) so that the reduction in TL caused by duct modes of the kind (m,0) is negated. In addition, silencer 3 has no horizontal partitions so that the reactive chamber has a height of 600mm, whereas silencer 4 has two horizontal partitions, which form three equal chambers of height 200 mm, see Fig. 3.

The transmission loss for silencer 3 and 4 is presented in Figs. 7 and 8. If each silencer is excited by a plane wave then the TL shows no noticeable difference in TL. This is to be expected as the partitioning plates have a negligible thickness ≤1% of the resonator height, and the transverse pressure distribution at the inlet will be constant over the cross-section with no variation along either the y or z axis. The pressure will therefore be symmetric about each baffle in the y direction, and constant along the z axis at the resonator opening. Application of an EMED source to silencer 3 is seen to reduce the amplitude of each peak in Fig. 7, so that most of the attenuation gained by the use of the reactive element is lost once the (0,1) duct mode cuts on at 285Hz. Modes with order $n > 0$ are therefore shown to be as problematic as those modes with $m > 0$. The cause of this decreased performance comes from the angle of propagation associated with each mode within the resonator. Each mode travels along the resonator at a

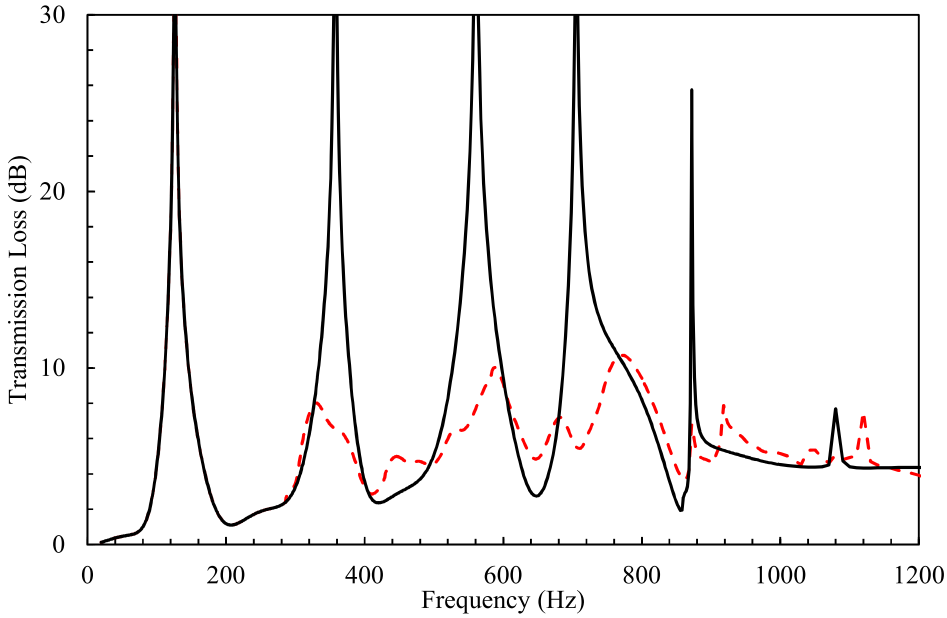

Figure 7. Transmission loss for silencer 3.* ——, *plane wave,* - - - -; *EMED.

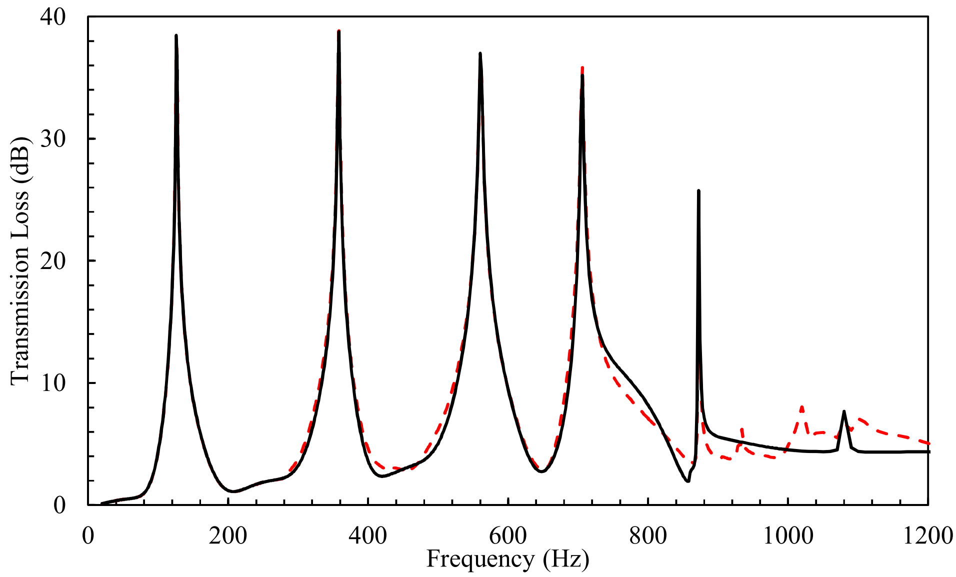

Figure 8. Transmission loss for silencers 3 and 4.* ——, *silencer 3 with plane wave excitation;

- - - -, *silencer 4 with EMED excitation.*

different angle with respect to the duct axis[4] and so each mode travels a different distance within the resonator and the effective length of the resonator changes.

The reduction in TL seen in Fig. 7 can be rectified by partitioning the resonator into vertical chambers so that only quasi plane wave type distributions are permitted within the silencer. This is demonstrated for silencer 4 in Fig. 8, which has two equally spaced horizontal partitions. Each partitioned chamber has a height of 200 mm and so the equivalent cut-on frequency of the (0,1) mode in a partitioned chamber increases from 286Hz to 858Hz, which coincides with the reactive element's upper frequency limit.

Higher order modes continued to be excited at the silencer inlet but planar like sound propagation is now maintained inside the resonators. Accordingly, the TL for the EMED excitation of silencer 4 in Fig. 8 is similar to that obtained for plane wave excitation at frequencies up to $f_{2n_b,0}$. Thus, the addition of the horizontal partitions restores the acoustic performance of the reactive silencers in a similar way to the vertical partitioning of the two dimensional silencer in Fig. 6. However, it should be noted that further increasing the number of partitions will not confer any further improvements in performance because the upper frequency of operation will then coincide with the cut-on frequency in the chamber itself, which removes resonant behaviour.

5. CONCLUSION

A hybrid reactive-dissipative splitter silencer is examined here using a theoretical model, which uses the finite element method to analyse the influence of the sound source on silencer performance. This is carried out for different types of excitation, including plane wave and equal modal energy density (EMED) excitation. The analysis shows that the reactive part of a hybrid silencer works well under plane wave conditions. Moreover, the reactive element provides attenuation well above the first cut-on mode in the inlet duct, and this provides an important extension of reactive silencer performance. However, when energy incident on the silencer propagates in higher order modes this may have a negative impact on silencer performance. To overcome this effect, it is necessary to further partition up the silencer using vertical and horizontal partitions for the resonant chamber. These partitions are seen to restore quasi-planar sound pressure distributions within the reactive chamber, and this restores the action of the reactive chamber to that seen for an incident plane wave. This is an important result, as it demonstrates that even under complex incident sound fields, the reactive chambers in a hybrid dissipative-reactive splitter silencer will continue to work at frequencies well above the first modal cut-on in the inlet duct.

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