Temperature-based Stiffness Identification of Que-Ti's in a Historic Tibetan Timber Building

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ABSTRACT: Que-Ti, like the corbel brackets connecting beam and column in modern structures, is an important component in typical Tibetan historic timber buildings. It transfers shear, compression and bending moment by slippage and deformation of components as well as limited joint rotation. A rigorous analytical model of Que-Ti is needed for predicting the behaviour of a timber structure under load. However, few researches have been done with this model, particularly on the parameters describing the performances of this joint under load. The equivalent stiffness of a Que-Ti connection in its operating state is determined by using ambient temperature variations as a forcing function in the complete input(temperature)-output(local mechanical strains) relationship when it is incorporated in a finite element model of the structure. The identification is done iteratively via correlating the calculated strain responses with measured data.

1 INTRODUCTION

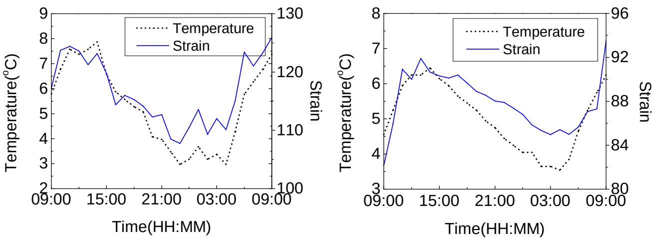
Que-Ti connection is commonly found in the beam-to-column joint in typical Tibetan historic timber buildings which are over thousands of years old. It can withstand bending moment, compression and shear loads. This connection is essential to describe the nonlinear behaviour of wooden structure under applied load. (Abdalla & Chen 1995; Kim & Chen 1998; Sekulovic et al. 2002). However, little can be found in the literature on the stiffness parameters of this connection. Efforts have been made on the structural identification from field-measured response data to the update analytical models of structures (Aktan et al. 1997; Brownjohn et al. 2003). Such research is based on the discrepancies between the predicted and measured responses of a constructed system with inaccurate modelling of some or all critical components in an *a priori* manner. Such errors are largely independent of any of the inherent shortcomings of simulation tools adopted, but rather, reflect a lack of knowledge of the target structure.

Identification methods using structural dynamic responses in time domain have been popular in the last two decades. Time histories of vibration response of the structure were adopted for smart structures identification (Cattarius & Inman, 1997). Structural stiffness parameters of a multi-storey framework were identified in a system identification approach (Koh et al. 2000). Recently, the dynamic response sensitivity-based model updating method has been developed and used to identify structural parameters using the measured dynamic responses (Lu & Law 2007; Lu & Liu 2011). Though it only needs a few measurement points, and yet it can provide highly accurate parameter identification taking advantage of the abundant time history data.

Parameter identification based on a reference set of measured data is usually subjected to environmental temperature in the two sets of measurements, and such effect is usually ignored in the subsequent model updating (Wei & Lv, 2015). The variation of intrinsic forces due to thermal and other mechanisms, can mask the effects from all other demands. Previous studies indicated the presence of large changes in the intrinsic forces over time but could not explain the exact mechanisms that give rise to these forces (Catbas & Aktan 2002).

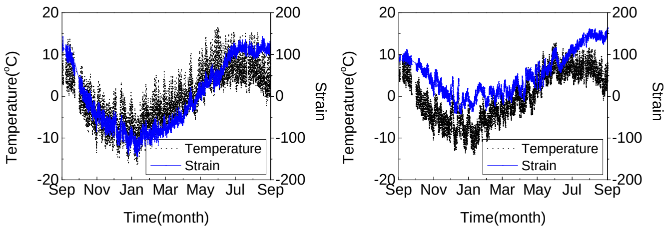
Strain responses from beam and column recorded over one day and one year on a Tibetan building are shown in Figures 1 and 2 respectively against the ambient temperature. The plots show clearly the structural strain responses follow closely the temperature variation implying that the temperature variation is a dominating factor affecting deformations in these components. The structural response of the building is dominated by the temperature response in the long-term monitoring of the structure.

This paper aims to identify the equivalent stiffness of the Que-Ti connection in typical Tibetan historic building based on mechanical strain responses and temperature measurements. The Que-Ti is modelled with two rotational springs at the beam ends and one linear spring at the column top. The temperature variations would be treated as a measureable forcing function while the strain response is taken as the output. A complete input (measuring temperatures)-output (strains) relationship can then be formulated. The method is shown capable to identify the equivalent stiffnesses of the Que-Ti accurately with the simulation studies of a plane frame structure based on strain response sensitivity in the time domain with even 10% measurement noise.



(a) Beam (b) Column

Figure 1. Strain and temperature on members over 1 day



(a) Beam (b) Column

Figure 2. Strain and temperature on members over 1 year.

2 MODEL OF THE QUE-TI CONNECTION

One of the unique characteristics of typical Tibetan historic timber structure is the use of Que-Ti as connections transferring load between beam and column with an increase in bearing area at the end of the beam, and a decrease of the span of beam leading to an improved shear and bending resistance at the beam end. It seldom involves nail or pin in its construction (Fang et al., 2001).

The beam-column joint of historic timber architecture, as shown in Figure 3, is typically a planar structural component supporting column from the top and beams coming in from two horizontal directions with the beam discontinuous at the top of the column. The thickened parts of the connecting members close to the intersection form the Que-Ti. With consideration of this arrangement, three linear springs are used to simulate the behaviour of a Que-Ti in which two of them are rotational springs with stiffnesses K1 and K2 to simulate the behavior of the rotating restraint on the beam, and the other one with stiffness K3 has vertical compressive stiffness to simulate the compression behavior perpendicular to grain as shown in Figure 4. The three spring stiffness matrices are assumed linear and uncoupled.

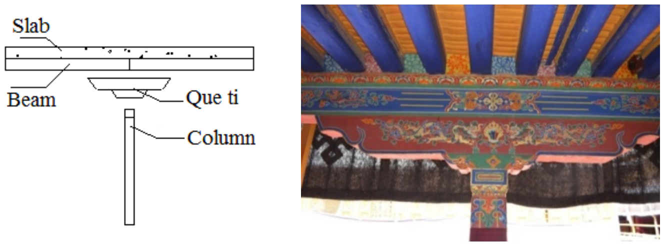


Figure 3. Composition of beam-column joints

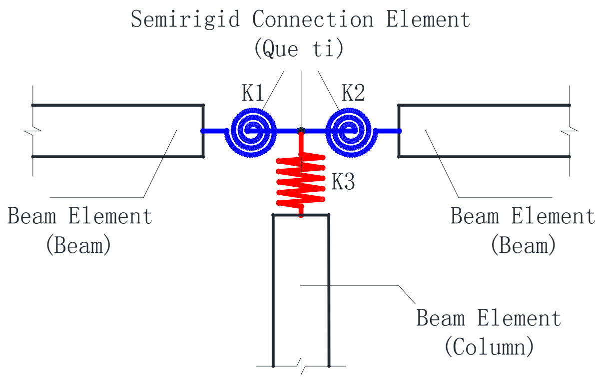


Figure 4. Simplified model of beam-column connection

3 SIMULATION AND PARAMETRIC STUDIES

The finite element mesh of a plane frame timber structure is shown in Figure 5. The structure is simply-supported at column bases and sliding-hinged at the outer end of beams. The cross-sectional area of all beam members is 0.25×0.5 m^2 and the cross-section of column varies from 0.25×0.25 m^2 to 0.4×0.4 m^2. Beam and column members are of 4.15 and 3.37 meters long respectively. The mass density and elastic modulus of timber are typically 0.418g/cm^3 and 6435MPa respectively. No external static load is applied on the frame other than the self-weight of the structure.

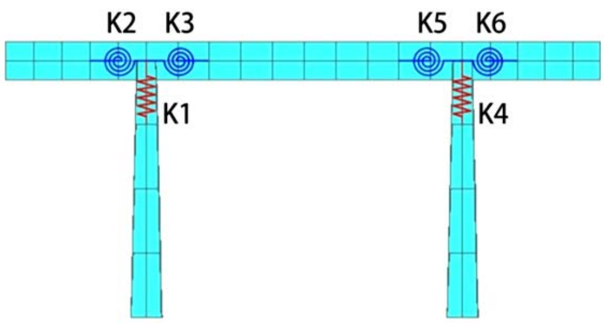


Figure 5. A simple two dimensional frame structure

3.1 *Methodology*

For a general finite element model of a linear elastic time-invariant system with *m* elements, the strain caused by the thermal variation $[\boldsymbol{\varepsilon}]$ is given by

$$[\boldsymbol{\varepsilon}] = [\mathbf{B}][\mathbf{K}]^{-1}[\mathbf{F}] \quad (1)$$

Since the temperature variation is treated as forcing function in this study, Eq. (2) can be further rewritten as

$$\begin{bmatrix} \varepsilon_i^1 \\ \varepsilon_i^2 \\ \vdots \\ \varepsilon_i^{m\text{-}1} \\ \varepsilon_i^m \end{bmatrix} = [\mathbf{B}][\mathbf{K}]^{-1}\alpha\mathrm{EA} \begin{bmatrix} \Delta T_i^1 \\ \Delta T_i^2 \\ \vdots \\ \Delta T_i^{m\text{-}1} \\ \Delta T_i^m \end{bmatrix} \quad (i=1,2,\ldots,n) \quad (2)$$

where $[\mathbf{B}]$ and $[\mathbf{K}]$ are the system strain-displacement relation matrix and stiffness matrix respectively. T is the temperature and α is the thermal expansion coefficient vector. E is the modulus of elasticity and A is the cross-sectional area of member. m is the total number of finite element in the structure and *n* is the total time steps.

The difference of responses from measurements and calculation is

$$[\Delta\mathbf{R}] = [\boldsymbol{\varepsilon}]_c - [\boldsymbol{\varepsilon}]_m$$

$$= [\mathbf{B}][\mathbf{K}]^{-1}\alpha EA \begin{bmatrix} \Delta T_i^1 \\ \Delta T_i^2 \\ \vdots \\ \Delta T_i^{m\text{-}1} \\ \Delta T_i^m \end{bmatrix} - \begin{bmatrix} \varepsilon_i^1 \\ \varepsilon_i^2 \\ \vdots \\ \varepsilon_i^{m\text{-}1} \\ \varepsilon_i^m \end{bmatrix}_m \quad (i=1,2,\ldots,n) \quad (3)$$

Differentiating both sides of Equation (2) with respect to the stiffness parameter of the system, the strain sensitivity matrix can be written as

$$S_i = \frac{[\partial\varepsilon]}{[\partial p_i]} = [\mathbf{B}][\mathbf{K}]^{-1}\frac{\partial[\mathbf{K}]}{\partial p_i}[\mathbf{K}]^{-1}\alpha EA \begin{bmatrix} \Delta T_i^1 \\ \Delta T_i^2 \\ \vdots \\ \Delta T_i^{m\text{-}1} \\ \Delta T_i^m \end{bmatrix} \quad (4)$$

where $\{p_i, i = 1,2,\cdots m\}$ are unknown stiffness parameters.

The length of the sensitivity vector is the same as the number of measured data points, and the sensitivity vector corresponding to a fractional change of stiffness in the *i*th element can be rewritten as S_i. The sensitivity vectors for all structural elements can be computed, and the sensitivity matrix is assembled as

$$\mathrm{S} = [\mathrm{S}_1 \ \mathrm{S}_2 \ \cdots \ \mathrm{S}_m] \quad (5)$$

The identification equation for the stiffness parameters of a structure can be expressed as

$$\mathbf{S}\Delta\mathbf{P} = \Delta\mathbf{R} \quad (6)$$

where ΔP is the unknown incremental stiffness parameters. Eq. (6) can be solved with an iterative Gauss-Newton method, and Tikhonov regularization is used for optimizing the following objective function in the *k*th iteration as

$$P_k = P_{k\text{-}1} + [\mathbf{S}_k^T\mathbf{S}_k]^{-1}\mathbf{S}_k^T\Delta\mathbf{R} \quad (7)$$

$$J(\Delta P_k, \lambda_k) = \left\|\mathbf{S}_{k\text{-}1}\Delta P_k - \Delta\mathbf{R}\right\| + \lambda_k\|\Delta P_k\|^2 \quad (8)$$

where λ_k is the regularization parameter in the *k*th iteration obtained with the *L*-curve method (Hansen, 1992), $\mathbf{S}_{k-1}$ is the sensitivity matrix with which the structural model is updated.

The structural stiffness matrix is updated after ΔP_k is obtained. Then the structural responses and the sensitivity matrix can be re-calculated based on the updated stiffness matrix, and the vector ΔP_k for the next iteration is calculated until the convergence is achieved with the following criterion as

$$\frac{\|P_{k+1} - P_k\|}{P_k} < Tol \quad (9)$$

The value of *Tol* is such selected to suit the difficulty with convergence of the identified results there there is noise effect.

3.2 *Accuracy of parameter identification*

The environmental temperature of the structure is assumed changed periodically with a maximum 5°C variation as shown in Figure 6. The data is recorded at one hour interval. The strain responses of the structure are calculated as the "measured" responses for the parameter identification, and only two strain responses at the beam and column as shown in Figure 7 are required in this study.

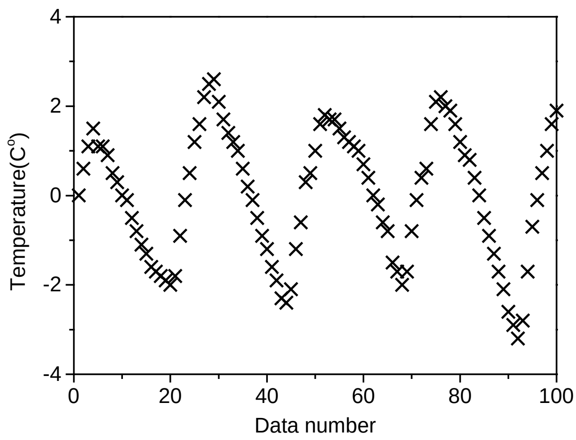


Figure 6. Environmental temperature changes

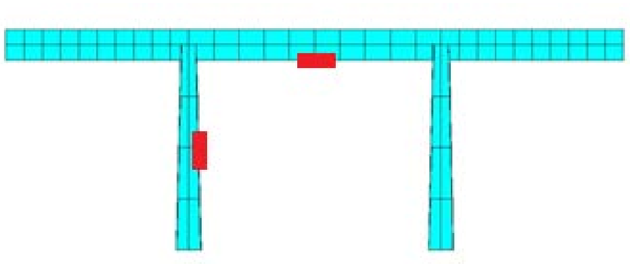


Figure 7. Sensor arrangement

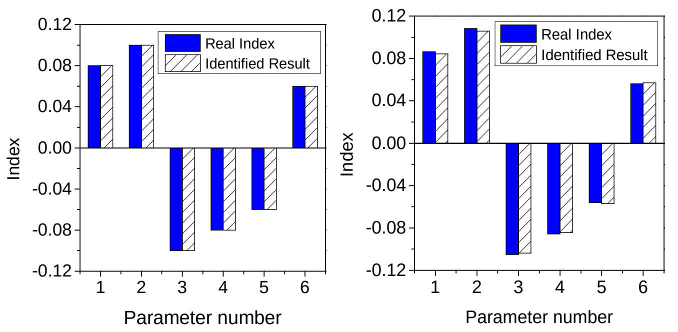
The operating stiffness of the Que-Ti are assumed as K_1=1.08×10^{5}kN/m, K_2=7700kN·m/r, K_3=6300kN·m/r, K_4=0.92×10^{5}kN/m, K_5=7420kN·m/r and K_6=6580kN·m/r, and their initial values at start of the iterative computation is set as: K_1=K_4=1×10^{5}kN/m, K_2=K_3=K_5=K_6=7000kN·m/r making reference to test results from Leichti et al., (2000). The incremental fraction of the parameters identified are 0.08, 0.1, -0.1, -0.08,-0.06 and 0.06 respectively for the six stiffnesses. This fraction is related to the real parameters in the form of:

$$(1 + fraction) \times K_{initial} = K_{real} \qquad (10)$$

3.3 *Identification with or without noise effect*

The effectiveness of the method adopted is studied with the "measured" responses without noise effect, and the computation convergence criterion *Tol* is set as 10^{-9} (Ali et al. 2013). The identified stiffness fractions are shown in Figure 8(a). The results show that the stiffness of springs of the Que-Ti connection can be identified accurately.

The polluted response is simulated by adding a normal random component to the calculated response expressed as a fraction of the standard deviation of the responses. 10% noise level is assumed and the convergence criterion is set as 10^{-5}. The identified results shown in Figure 8(b) are noted stable even with 10% measurement noise.



(a) without noise effect (b) with 10% noise effect

Figure 8. Identified Parameter with and without noise effect

3.4 *Parametric studies*

The range of temperature varation, the length of data and sensor placement are selected in this parametric study. The computation convergence criterion *Tol* is set as 10^{-9} for all studies in this section.

3.4.1 *Range of temperature variation*

Three different temperature records with 5^{o}C, 15^{o}C and 25^{o}C maximum range of variation shown in figure 9 are used for the identification. The identified results with 10% noise in the strain responses are shown in figure 10. The identified results are noted better with larger range of temperature variation, and the errors of identification are list in Table 1.

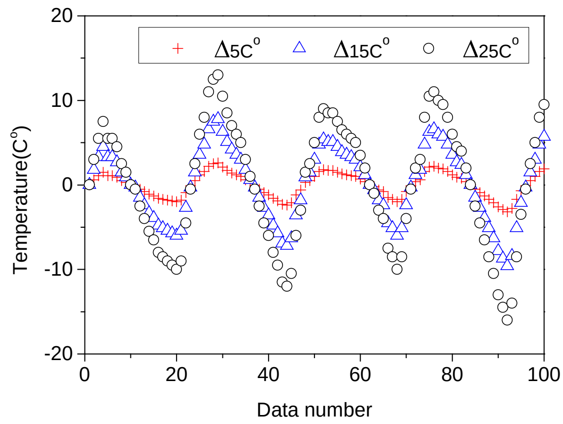


Figure 9. Different temperature records

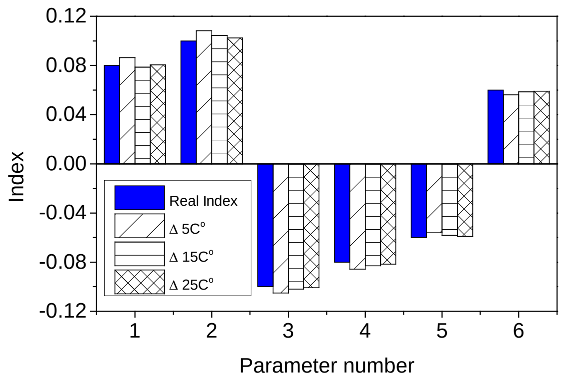


Figure 10 Identified results from using different temperature records

Table 1. Errors (%) of identification from using different temperature records

Parameters	Temperature difference		
	5^{o}C	15^{o}C	25^{o}C
	%	%	%
K_1	8.00	1.55	0.72
K_2	8.20	4.44	2.40
K_3	5.20	1.98	0.72
K_4	7.15	3.60	2.04
K_5	6.38	2.98	1.62
K_6	6.55	2.50	1.53

3.4.2 *Data length*

Four different length of data from the same temperature record are studied as shown in figure 11. They contain 300, 200 100 and 50 data points each. The results show that 300 and 200 data points give better accuracy as noted in figure 12, and the errors of identification are listed in Table 2. A longer length of data can give more accurate result but it also takes a longer time to calculate.

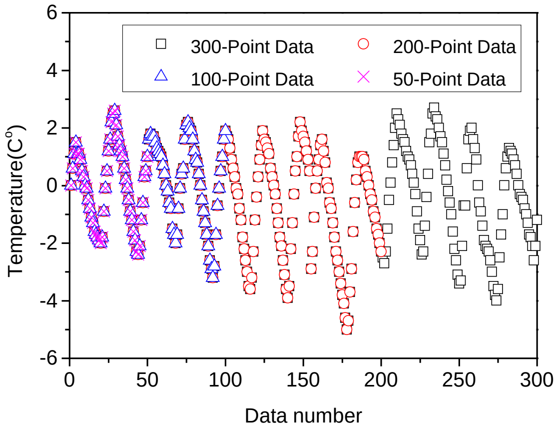


Figure 11. Temperature records with different lengths

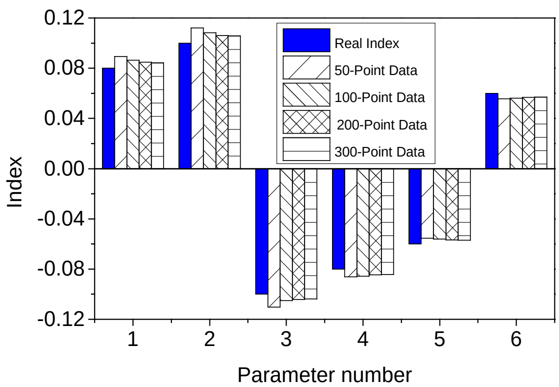


Figure 12 Identified results from using different length of data

Table 2 Errors (%) of identification from using different length of data

Parameters	Length of Data			
	50	100	200	300
	%	%	%	%
K_1	11.75	8.00	5.88	5.50
K_2	12.20	8.20	6.10	5.83
K_3	10.20	5.20	4.30	3.82
K_4	7.52	7.15	5.65	5.41
K_5	7.71	6.38	5.55	5.07
K_6	7.21	6.55	5.38	5.05

3.4.3 *Sensor placements*

Four scenarios of sensor placements are shown in Figure 13. The identified results from using 100-data point and the temperature record with maximum 5°C variation in Figure 8 are shown in figure 14. The errors of identification are listed in Table 3. It is noted that the results with sensors on all members are the best while those from Scenarios II to IV with only two sensors on two members are poorer and they are similar.

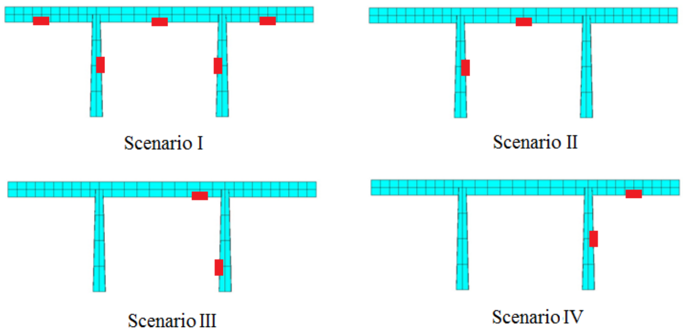


Figure 13 Four scenarios of sensor arrangements

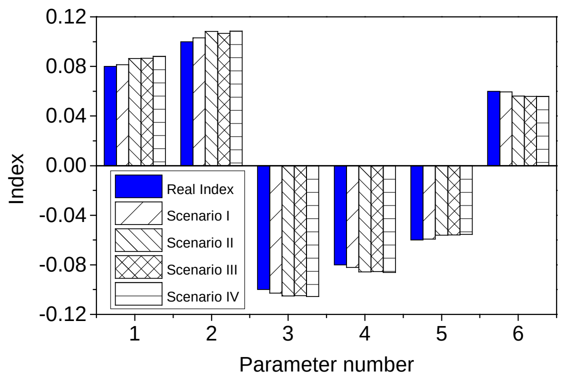


Figure 14 Identified results from using different sensor arrangements

Table 3. Errors (%) of identification with different sensor placement

Parameters	Scenario			
	I	II	III	IV
	%	%	%	%
K_1	1.81	8.00	8.29	10.16
K_2	3.15	8.20	6.60	8.39
K_3	2.85	5.20	4.93	5.51
K_4	2.50	7.15	6.89	7.80
K_5	1.21	6.38	7.05	7.73
K_6	0.97	6.55	6.98	7.03

4 CONCLUSIONS

The equivalent stiffness of the semi-rigid Que-Ti joint in typical Tibetan historic building in its operating state is identified based on the mechanical strain responses resulting from the environmental temperature variation via a sensitivity approach. The Que-Ti at the joint of a Tibetan timber frame is modelled with three linear springs. Temperature variations are treated as a measureable forcing function to obtain the complete input (temperature variation)-output (local mechanical strains) relationship. The method adopted is shown capable to update all equivalent stiffness of the Que-Ti joints accurately in the simulation study of a plane frame structure based on strain response sensitivity in the time domain with even 10% measurement noise.

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