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**Performance Analysis of Fractional Frequency
Reuse in Random Cellular Networks**

by

Sinh Cong Lam

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Certificate of Authorship/Originality

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Sinh Cong Lam

ABSTRACT

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In a Long Term Evolution (LTE) cellular network, Fractional Frequency Reuse (FFR) is a promising technique that improves the performance of mobile users which experience low Signal-to-Interference-plus-Noise Ratios (SINRs). Recently, the random cellular network model, in which the Base Stations (BSs) are distributed according to a Poisson Point Process (PPP), is utilised widely to analyse the network performance. Therefore, this thesis aims to model and analyse performance of two well-known FFR schemes called Strict Frequency Reuse (FR) and Soft FR, in the random cellular network. Monte Carlo simulation is used throughout the thesis to verify the analytical results.

The first part of this thesis follows 3rd Generation Partnership Project (3GPP) recommendations to model the Strict FR and Soft FR in downlink and uplink single-tier random cellular networks. The two-phase operation model is presented for both Cell-Center User (CCU) and Cell-Edge User (CEU). Furthermore, the thesis follows the resource allocation technique and properties of PPP to evaluate Intercell Interference (ICI) of the user. The closed-form expressions of the performance metrics in terms of classification probability and average coverage probability of the CCU and CEU are derived.

Thereafter, the performance of FFR is analysed in multi-tier cellular networks which are comprised of different types of cells such as macrocells, picocells and femtocells. The focus of this part is to examine the effects of the number of users and number of Resource Blocks (RBs) on the network performance. A new network

model, in which the SINR on data channels are used for user classification purpose, is proposed. The analytical results indicate that the proposed model can reduce the power consumption of the BS while improving the network data rate. This chapter introduces an approach to analyse the optimal value of SINR threshold and bias factor. The analytical results indicate that the proposed model can increase the network data rate by 16.08% and 18.63% in the case of Strict FR and Soft FR respectively while reducing power consumption of the BS on the data channel.

Finally, the thesis develops an FFR random cellular network model with an FR factor of 1 using either Joint Scheduling or Joint Transmission with Selection Combining. The performance metrics in terms of average coverage probability are derived for Rayleigh fading environment.

Generally, this thesis makes contributions to uplink and downlink of LTE networks in terms of performance analysis.

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List of Publications

The contents of this dissertation are based on the following papers that have been published, accepted, or submitted to peer-reviewed journals and conferences.

Journal Papers

1. **Lam, S.C.** & Sandrasegaran, K, Ha. N, Tuan. N “Performance Analysis of Fractional Frequency Reuse in Multi-Tier Random Cellular Networks”, submitted Springer Wireless Networks (**IF 2016 = 1.584**).
2. **Lam, S.C.**, Sandrasegaran, K. & Ghosal, P. “A Model based Poisson Point Process for Downlink Cellular Networks using Joint Scheduling”, submitted Springer Wireless Personal Communication (Aug 2017) (**IF 2016 = 0.95**).
3. **Lam, S.C.** & Sandrasegaran, K “Performance Analysis of Fractional Frequency Reuse in Uplink Random Cellular Networks”, Elsevier Physical Communication (Sept 2017) (**IF 2016 = 1.58**)
doi: <https://doi.org/10.1016/j.phycom.2017.09.008>.
4. **Lam, S.C.**, Sandrasegaran, K. & Ghosal, P. “Performance Analysis of Frequency Reuse for PPP Networks in Composite RayleighLognormal Fading Channel”, Wireless Personal Communication (2017) (**IF 2016 = 0.95**).
doi:10.1007/s11277-017-4215-2
5. **Lam, S.C.** & Sandrasegaran, K 2016, “Analytical Coverage Probability of a Typical User In Heterogeneous Cellular Networks”, Journal of Networks (**ERA 2010=A**), Vol 11, No 2 (2016), 56-61, Feb 2016, doi:10.4304/jnw.11.2.56-61.

Conference Papers

1. **S. C. Lam**, K. Sandrasegaran, "Performance Analysis of Joint Scheduling in Random Cellular Networks," 2017 17th International Symposium on Communications and Information Technologies (ISCIT), Cairns, 2017
2. **S. C. Lam**, K. Sandrasegaran, "Optimal Strict Frequency Reuse in Cellular Networks-based Stochastic Geometry Model," 2017 17th International Symposium on Communications and Information Technologies (ISCIT), Cairns, 2017
3. **S. C. Lam**, K. Sandrasegaran and T. N. Quoc, "Strict frequency reuse algorithm in random cellular networks," 2016 International Conference on Advanced Technologies for Communications (ATC), Hanoi, 2016, pp. 447-452. DOI: 10.1109/ATC.2016.7764824
4. **S. C. Lam**, K. Sandrasegaran and T. N. Quoc, "Performance of soft frequency reuse in random cellular networks in Rayleigh-Lognormal fading channels," 2016 22nd Asia-Pacific Conference on Communications (APCC), Yogyakarta, 2016, pp. 481-487. DOI: 10.1109/APCC.2016.7581454
5. **Lam, S.C.**, Heidary, R., and Sandrasegaran, K., "A closed-form expression for coverage probability of random cellular network in composite Rayleigh-Lognormal fading channels. 2015 International Telecommunication Networks and Applications Conference (ITNAC), Sydney, Australia. DOI: 10.1109/ATNAC.2015.7366806
6. **Lam, S.C.**, Subramanian, R., Ghosal, P., Barua, S., and Sandrasegaran, K. "Performance of well-known frequency reuse algorithms in LTE downlink 3GPP LTE systems. 9th International Conference on Signal Processing and Communication Systems (ICSPCS), Cairns, Australia 2015. DOI: 10.1109/ICSPCS.2015.7391766

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Abbreviation

3GPP	3rd Generation Partnership Project
BS	Base Station
CC	Cell Center
CCU	Cell-Center User
CDF	Cumulative Density Function
CE	Cell Edge
CEU	Cell-Edge User
CoMP	Coordinated Multi-Point
e-UTRA	Evolved Universal Terrestrial Radio Access
FFR	Fractional Frequency Reuse
FR	Frequency Reuse
ICI	Intercell Interference
ICIC	Intercell Interference Coordination
LTE	Long Term Evolution
MRC	Maximum Ratio Combining
MIMO	Multiple-Input and Multiple-Output
PDF	Probability Density Function
PGF	Probability Generating Function
PPP	Poisson Point Process
RB	Resource Block
RV	Random Variable
SNR	Signal-to-Noise Ratio

SINR	Signal-to-Interference-plus-Noise Ratio
UE	User Equipment

List of Symbols

α	Path Loss Exponent
α_j	Path Loss Exponent of Tier- j
Δ	Frequency Reuse Factor
Δ_j	Frequency Reuse Factor of Tier- j
ϵ	Power Control Exponent
$\epsilon_k^{(c)}$	Allocation ratio in CC Area of Tier- k
$\epsilon_k^{(e)}$	Allocation ratio in CE Area of Tier- k
$\epsilon_k^{(z)}$	Allocation ratio in z Area of Tier- k
\hat{T}	Coverage Threshold
\hat{T}_j	Coverage Threshold of Tier- j
λ	Density of BSs
$\lambda^{(c)}$	Density of BSs transmitting at CC power
$\lambda_j^{(c)}$	Density of BSs transmitting at a CC power in Tier- j
$\lambda^{(e)}$	Density of BSs transmitting at a CE power
$\lambda_j^{(e)}$	Density of BSs transmitting at a CC power in Tier- j
$\lambda^{(u)}$	Density of users
λ_j	Density of BSs of Tier- j
$\mathcal{P}_c^{(c)}$	Average coverage probability of a CCU
$\mathcal{P}_{FR}^{(c)}$	Average coverage probability of a CCU under FFR
$\mathcal{P}_{Soft}^{(c)}$	Average coverage probability of a CCU under Soft FR
$\mathcal{P}_{Str}^{(c)}$	Average coverage probability of a CCU under Strict FR

$\mathcal{P}_c^{(e)}$	Average coverage probability of a CEU
$\mathcal{P}_{FR}^{(e)}$	Average coverage probability of a CEU under FFR
$\mathcal{P}_{Soft}^{(e)}$	Average coverage probability of a CEU under Soft FR
$\mathcal{P}_{Str}^{(e)}$	Average coverage probability of a CEU under Strict FR
$\mathcal{P}_c^{(z)}$	Average coverage probability of user z
\mathcal{P}_c	Average coverage probability of a typical user
\mathcal{P}_{FR}	Average coverage probability of a typical user under FFR
\mathcal{P}_{Soft}	Average coverage probability of a typical user under Soft FR
\mathcal{P}_{Str}	Average coverage probability of a typical user under Strict FR
ϕ	Ratio between Transmit Power on a CEU and CCU
ϕ_j	Ratio between Transmit Power on a CEU and CCU of Tier- j
σ	Gaussian noise
θ	Set of BSs in the networks (<i>Chapter 5</i>)
$\theta^{(c)}$	Set of interfering BSs (<i>Chapter 5</i>)
$\theta_{Soft}^{(c)}$	Set of users (<i>chapter 2</i>), BSs (<i>other chapters</i>) transmitting at a CC power under Soft FR
$\theta_{Str}^{(c)}$	Set of users (<i>chapter 2</i>), BSs (<i>other chapters</i>) transmitting at a CC power under Strict FR
$\theta_{Soft}^{(e)}$	Set of users (<i>chapter 2</i>), BSs (<i>other chapters</i>) transmitting at a CE power under Soft FR
$\theta_{Str}^{(e)}$	Set of users (<i>chapter 2</i>), BSs (<i>other chapters</i>) transmitting at a CE power under Strict FR
$A^{(c)}$	CCU classification probability
$A_{Soft}^{(c)}$	CCU classification probability under Soft FR
$A_{Str}^{(c)}$	CCU classification probability under Strict FR
$A^{(e)}$	CEU classification probability
$A_{Soft}^{(e)}$	CEU classification probability under Soft FR
$A_{Strict}^{(e)}$	CEU classification probability under Strict FR
$C_{FR,k}^{(c)}$	Average CCU data rate in Tier- k

$C_{FR,k}^{(e)}$	Average CEU data rate in Tier- k
d_{jz}	Distance between a user j and serving BS of user z
g	Channel Power Gain from user to it's serving BS
g_{jz}	Channel Power Gain from user z to BS j
$I_{Sof}^{(z)}$	InterCell Interference of user z at under Soft FR
$I_{Str}^{(z)}$	InterCell Interference of user z at under Strict FR
K	Number of Tiers (<i>chapter 4</i>), coordinated BS (<i>chapter 5</i>)
$M_k^{(c)}$	Number of CCUs during communication phase in CE Area of Tier- k
$M_k^{(e)}$	Number of CEUs during communication phase in CE Area of Tier- k
$M_k^{(nc)}$	Number of new CCUs during communication phase in CC Area of Tier- k
$M_k^{(ne)}$	Number of new CEUs during communication phase in CE Area of Tier- k
$M_k^{(nz)}$	Number of new z users during communication phase in z Area of Tier- k
$M_k^{(oc)}$	Number of CCUs during establishment phase in CC Area of Tier- k
$M_k^{(oe)}$	Number of CEUs during establishment phase in CE Area of Tier- k
$M_k^{(oz)}$	Number of z users during establishment phase in z Area of Tier- k
$M_k^{(z)}$	Number of z users during communication phase in CE Area of Tier- k s
N_{GL}	Degree of a Laguerre polynomial
N_G	Degree of a Legendre polynomial
P	Transmit Power of a user (<i>chapter 2</i>), a BS (<i>other chapters</i>)
$P^{(z)}$	Transmit Power of user z
P_j	Transmit Power of BS in Tier- j
r	Distance between a user and BS
R_k	Average cell data rate in Tier- k
r_{jz}	Distance between a user z and BS j
$SINR$	SINR during a communication phase
$SINR^{(o)}$	SINR during a establishment phase
T	SINR Threshold

T_j	SINR Threshold of Tier- j
t_n and w_n	n -th node and weight of Gauss - Laguerre quadrature
x_n and c_n	n -th abscissas and weight of Gauss-Legendre quadrature
z	CC or CE