

# Parameter Estimation and Application for Static Nonlinear and Dynamic Linear Systems



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## Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements.

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## Abstract

General issues associated with parameter estimation have been extensively studied. During the past several decades, a vast number of methods have been developed for solving different parameter estimation issues in different areas. Thanks to numerous newly-introduced areas, the parameter estimation and its related techniques still play important roles and need to be expanded to solve new challenges. Since the research topics of parameter estimation are extremely wide, this dissertation is concerned with two topics within parameter estimation related to calibration of MEMS accelerometer and modelling of oxygen uptake.

It is challenging to obtain the unknown parameters of tri-axial accelerometer accurately based on auto-calibration as the cost function is nonlinear and non-convex. Furthermore, it is more challenging to solve this nonlinear and non-convex cost function online to overcome the variation of parameter caused by the external change of environment. To overcome these challenges, an iterative parameter estimation method with the experimental design to solve the accelerometer model is proposed. Furthermore, two algorithms are explored based damped recursive estimation and expectation maximum algorithm to online estimate the unknown parameter in the model. This topic can be summarised as a parameter estimation problem of a special nonlinear static system. Over the past decades, the modelling of oxygen uptake response to exercise has always been a challenging topic due to measurement noise, insufficient stimulations of the system and individual differences of human beings. To overcome these difficulties, a nonparametric estimation method is investigated for the modelling of oxygen uptake response and ensure its accuracy and reliability. The second topic can be summarised as a parameter estimation problem of a noisy dynamic linear system

with limited stimulation. These two topics are highly prized for academic significance but also remained open due to their challenges in mathematics.

First, for parameter estimation problem of the offline auto-calibration of accelerometer, a 6-orientation G-optimal experimental scheme is proposed for a special second-degree model based on the statistical experimental design. Then, a new linearisation approach is developed to apply the proposed G-optimal experimental scheme. Then, a convergence-guaranteed recursive parameter estimation algorithm is developed that can be easily implemented in a portable wearable device. The region of convergence of the proposed algorithm is proved. Numerous simulations and experiments are carried out to validate the efficiency of the proposed method.

Second, for parameter estimation problem of the online auto-calibration of accelerometer, a linearisation method of the 6-parameter tri-axial accelerometer model is explored. Then, a modified damped recursive least square (MDRLS) estimation method is proposed to estimate the unknown parameters in real time. Meanwhile, the MDRLS can iteratively remove the bias caused by the linearisation during the online estimation process. The convergence speed and estimation effectiveness of the proposed method are discussed based on both simulations and experiments. The results show that the proposed method can achieve similar accuracy with significantly fewer measurements. In the end, the region of convergence of the proposed online estimation method is analysed and discussed based on Monte Carlo simulation. Simulations and experiments also demonstrate the performance of the proposed method.

Third, in real life, the misalignments exist between axes for some tri-axial accelerometers. Therefore, the 9-parameter model can achieve higher accuracy for those accelerometers. However, this model will reduce accuracy for those accelerometers without misalignments. To online estimate the unknown parameter with automatic model selection of tri-axial accelerometer, a sparse least square (SPARLS) estimation is explored. To apply this SPARLS, a linearisation method of the 9-parameter model is proposed. Based on the linearised method, the SPARLS is modified to solve the unknown parameter in real time while penalising the insufficient parameters. Therefore, the model of tri-axial accelerometer can be adjusted automatically in real time to remove the insignificant parameters caused by noise. Then, the conditions for the con-



vergence of this iterative approach are identified and investigated based on simulations for different situations. A self-designed device is also used to validate the performance of the proposed algorithm.

Forth, for modelling oxygen uptake response to exercise system, a nonparametric estimation method for impulse response is developed to identify any order systems. To estimate the impulse response based on a simple step input signal, a novel kernel-based estimation method is investigated. The proposed method can efficiently reduce the order of impulse response model by incorporating a  $\mathcal{L}_1$  norm penalty term. Furthermore, the overall penalty terms can be converted into a special elastic net to simplify the calculation procedure. Then, to consider the prior information, kernels are investigated by extensive simulations, and the stable spline (SS) kernel is recommended as the best candidate. It is demonstrated by experiments and simulations that the proposed method is efficient for the modelling of impulse response of oxygen uptake to dynamic exercise, which often confronts a highly noised measurement under the stimulation of a simple input signal. Finally, an averaged impulse response model is established, which can quantitatively describe the oxygen uptake on-kinetics for treadmill exercise.



## Publications

The contents of this thesis are based on the following papers that have been published, accepted, or submitted to peer-reviewed journals and conferences.

### Journal Papers:

1. Lin Ye, Ying Guo, and Steven W. Su, “An Efficient Autocalibration Method for Triaxial Accelerometer,” *IEEE Transactions on Instrumentation and Measurement*, vol. 66, no. 9, pp. 2380-2390, June 2017.
2. Lin Ye, Ahmadreza Argha, Branko G Celler, Hung T. Nguyen and Steven W, Su, “Online Auto-calibration of Triaxial Accelerometer with time-variant model structures,” *Sensors and Actuators A: Physical*, vol. 266, pp. 294-307, October 2017.
3. Lin Ye, Ahmadreza Argha, Branko G Celler, Hung T. Nguyen and Steven W, Su, “Dynamic Characteristics of Oxygen Uptake,” *BioMedical Engineering OnLine*, vol. 17, no. 1, pp. 44-62, April 2018
4. Lin Ye, et al, “A Fast-Converge Real-time Auto-Calibration Algorithm for Tri-axial Accelerometer,” under second round of review at *IEEE Transactions on Instrumentation and Measurement*, 2017.
5. Hairong Yu, Lin Ye, et al “Nonparametric Dynamical Model of Cardiorespiratory Responses at the Onset and Offset of Treadmill Exercises,” under second round of review at *Medical & Biological Engineering & Computing*, 2017.

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### Conference Papers:

1. Lin Ye, Ahmadreza Argha, Branko G Celler, Yi Zhang, Hung T. Nguyen and Steven W, Su, “Nonparametric modelling of VO 2 response to exercise,” in *Proc. 2017 39th International Conference of IEEE Engineering in Medicine and Biology Society*, pp. 1525-1528, July 2017.
2. Lin Ye, et al, “An online recursive autocalibration of triaxial accelerometer,” in *Proc. 2016 38th International Conference of IEEE Engineering in Medicine and Biology Society*, pp. 2038-2041, July 2016.
3. Lin Ye, et al, “Inertial Sensor based Post Fall Analysis for False Alarming Reduction,” in *Proc. 2016 Telehealth and Assistive Technology*, pp. 864-011, October 2016.
4. Lin Ye, and Steven W. Su “Experimental design for the calibration of tri-axial Magnetometers,” in *Proc. 2015 9th International Conference on Sensing Technology* , pp. 864-011, October 2016.
5. Lin Ye, and Steven W. Su “Optimum Experimental Design applied to MEMS accelerometer calibration for 9-parameter auto-calibration model,” in *Proc. 2015 37th International Conference of IEEE Engineering in Medicine and Biology Society*, July 2015.
6. Admadreza Argha, Lin Ye, Steven W. Su, and Branko G. Celler “Real-time modelling of heart rate response during exercise using a novel constrained parameter estimation method,” in *Proc. 2016 38th International Conference of IEEE Engineering in Medicine and Biology Society*, July 2016.
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