

Applications of Matrix Spaces in Quantum Information and Computational Complexity

by

Yinan Li

A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy

Supervisor: Dr. Runyao Duan Co-supervisor: Dr. Youming Qiao

at the

Centre for Quantum Software and Information Faculty of Engineering and Information Technology University of Technology Sydney

May 2018

Certificate of Original Authorship

I, Yinan Li, declare that this thesis, is submitted in fulfillment of the requirements for the award of Doctor of Philosophy, in the School of Software, Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise reference or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualification at any other academic institution.

Signature:

Production Note: Signature removed prior to publication.

Date: 31-05-2018

Applications of Matrix Spaces in Quantum Information and Computational Complexity

by

Yinan Li

Abstract

This thesis explores the applications of matrix spaces in quantum information and computational complexity, specifically in the areas of quantum state/channel discrimination, entanglement transformation and isomorphism testing. We achieve the following contributions:

- We derive a necessary condition which determines whether a set of orthogonal bipartite states can be distinguished by positive-partial-transpose (PPT) operations, in the many-copy scenario. We reduce the discrimination task to the following problem: Decide whether there exist a nonzero bipartite matrix with positive partial-transpose, such that all its eigenvectors with nonzero eigenvalues is orthogonal to a given bipartite vector space. As applications, we reprove and extend a variety of existing results, including the local indistinguishability of the bipartite maximally entangled state and its orthogonal complement [Yu, Duan and Ying, *IEEE transaction on Information Theory*, 2014] and the maximum dimension of non-positive-partialtranspose subspaces [Johnston, *Physical Review A*, 2013].
- We show that determining the parallel distinguishability of quantum channels is equivalent to determining whether the orthogonal complement of a given matrix space contains nonzero positive semidefinite matrices. Our characterization immediately implies a necessary condition to decide the parallel distinguishability. We further prove that our condition is also sufficient for two large families of quantum channels, which leads to an alternating proof for the parallel distinguishability of

unitary operations [Acín, *Physical Review Letters*, 2001]. We also present an illustrative example showing our necessary condition cannot determine the parallel distinguishability.

- We systematically study the tripartite-to-bipartite entanglement transformations by stochastic local operations and classical communication (SLOCC). Such a problem is equivalent to computing the maximal rank of a matrix space [Chitambar, Duan and Shi, *Physical Review A*, 2010]. We analyze the SLOCC convertibility in both the finite-copy and asymptotic regimes. In particular, we derive explicit formulas which compute asymptotic entanglement transformation rates for two families of tripartite states by resorting to certain results of the structure of matrix spaces, including the study of matrix semi-invariants. Significantly, we show that the problem of deciding the asymptotic SLOCC convertibility of tripartite pure states to the bipartite maximally entangled states and the non-commutative symbolic determinant identity testing problem is algorithmically equivalent, which builds a new connection between problems in algebraic complexity and problems in asymptotic SLOCC entanglement transformations.
- We devise algorithms which test isometry between alternating matrix spaces over finite field. Algorithms for such a problem in time polynomial in the underlying vector space size resolves the believed bottleneck case of the group isomorphism problem, i.e. testing isomorphism of *p*-groups of class 2 and exponent *p* in time polynomial in the group order. Our approach is to view it as a linear algebraic analog of the graph isomorphism problem. We devise an average-case efficient algorithm for the alternating matrix space isometry problem over a key range of parameters. in a random model of alternating matrix spaces in vein of the Erdős-Rényi model of random graphs. To devise our algorithm, we developed linear algebraic individualization and refinement techniques, which are crucial in the first average-case efficient algorithm for graph isomorphism, devised by Babai, Erdős, and Selkow in the 1970s [Babai, Erdős and Selkow, *SIAM Journal on Computing*, 1980]. We also adapt Luks' dynamic programming technique for graph isomorphism [Luks, *STOC*, 1999] to slightly improve the worst-case time complexity of the alternating matrix space isometry problem.

Acknowledgements

Very little of the work in this thesis would have been possible without the support of my supervisors, collaborators, colleagues, friends and family.

First and foremost, I would like to thank my supervisor, Runyao Duan. Without his guidance and insight, I would not have complete my PhD study. I am greatly indebted to him for leading me into the quantum world. Dating back to 2013 in Wuhan University, when I was still a third-year undergraduate student, I was so inspired by his talk about quantum computation. He encouraged me to investigate the parallel distinguishability of quantum channels, which becomes the departure point of my research career. Furthermore, I am grateful for Runyao's spiritual support for my life. He is not only my academic supervisor but also a spiritual mentor, sharing his experience and scope in both research and life.

Thanks to my co-supervisor, Youming Qiao, my research interests have been developed and extended. I would like to thank him for his inspirational discussions and conversations from which I really benefit a lot. I have learned about many interesting topics from him, especially for introducing me to the beauty of computational complexity theory. Meanwhile, I also want to extend my great indebtedness to my external supervisor, Andreas Winter, at Universitat Autónoma de Barcelona. Professor Winter has been my academic hero since I started my PhD, and I always believe that he is the kind of scientist I would like to become in the future.

I would also like to thank Ashely Montanaro and an anonymous examiner for their careful reading of my thesis and constructive comments and suggestions. I am very fortunate to have been part of the Centre for Quantum Software and Information (and the Centre for Quantum Computation and Intelligence Systems from 2014 to 2016). The centre has offered me an example of academic excellence for which to strive.

I want to thank my co-authors Runyao Duan, Cheng Guo, Chi-Kwong Li, Youming Qiao, Xin Wang for giving me the opportunity to work with them. I wish to acknowledge Mingsheng Ying, Yuan Feng, Zhengfeng Ji, Michael Bremner, Ching-Yi Lai, Nengkun Yu, Chris Ferrie, Kun Fang, Wei Xie, Hao-Chung Cheng, Ryan Mann, Mario Berta, Andreas Winter, Li Gao, Marius Junge, David Gross, Andrew Childs, Ronald de Wolf, Xie Chen, Thomas Vidick, Fernando Brandao, Chris Umans, Micheal Walter, Xiaoming Sun and Man-Hong Yung for their help and discussions in the past four years. I also want to thank Andrew Childs, Xie Chen and Man-Hong Yung for their hospitality during my visit in University of Maryland, California Institute of Technology and South University of Science and Technology of China, and offering me the opportunities to introduce my work in their groups.

I have been blessed with great friends who have made the years spent in Sydney full of fun and adventures. My heartfelt thanks also goes to my family. My love and gratitude for my parents and my grandparents can hardly be expressed in words. Thanks for the endless love and support over all these years. It has been a great privilege to grow in such a loving family and they are and will always be my greatest strength and comfort. Last but by no means least, I would like to thank my beloved fiancè, Zhuoling Tian. Thanks for being part of my life and light it up. We missed in Wuhan University, and we also missed in Europe and USA. But we finally meet down under in Australia, both doing our PhD. I really feel lucky to have someone who is so supportive and understandable, sharing happiness and sorrow together, and at the same time being the greatest comrade in academia, encouraging and inspiring each other and exploring the mysteries of science and arts hand in hand. The most romantic thing that conjures up in my mind is waking up with her smile aside.

Contents

D	Declaration of Authorship						
A	Abstract						
A	Acknowledgements vii						
\mathbf{Li}	st of	Figures	xi				
\mathbf{Li}	st of	Publications	ciii				
1	Intr	oduction	1				
	$\begin{array}{c} 1.1 \\ 1.2 \end{array}$	Quantum Mechanics in a NutshellMatrix Spaces in a Nutshell	5 7				
2	PP	Γ-distinguishability of Orthogonal Bipartite States	9				
	2.1	The Local Distinguishability of Orthogonal Quantum States	10				
	2.2	Notations and Preliminaries	13				
	2.3	PPT-indistinguishability of Orthogonal Bipartite States	15				
		2.3.1 A Sufficient Condition for PPT-indistinguishability in the Many-	15				
		2.3.2 Constructions of PPT-indistinguishable Orthogonal Bipartite States	10				
		in the Many-copy Scenario	17				
		2.3.3 Minimum Dimension of Strongly PPT-unextendible Spaces in $\mathcal{H}_m \otimes \mathcal{H}_n$	18				
	2.4	Summary and Discussion	22				
3	Dist	inguishing Quantum Channels with Parallel Schemes	23				
	3.1	Introduction: Quantum Channel Discrimination	24				
	3.2	Notations and Preliminaries	26				
	3.3	Parallel Distinguishability of Quantum Channels	28				
		3.3.1 Characterizing the Parallel Distinguishability	28				
		3.3.2 Determining the Parallel Distinguishability for Two Families of Quan-					
		tum Channels	30				
		3.3.3 A Counterexample for the Sufficiency of Corollary 3.6	31				
3.4 Summary and Discussion							

4	Trij	oartite	-to-Bipartite SLOCC Entanglement Transformation	35
	4.1	Introd	uction	36
	4.2	Notati	ions and Preliminaries	43
	4.3	Multi-	Copy Transformation	48
	4.4	Asym	ptotic Transformation	51
		4.4.1	Asymptotic Maximal Rank of Matrix Spaces without Shrunk Subspace	52
		4.4.2	Asymptotic Maximal Rank of Maximal Compression Matrix Spaces	53
	4.5	Summ	ary and Discussion	64
5	Tes	ting Is	ometry between Alternating Matrix Spaces	67
	5.1	Introd	uction	68
		5.1.1	Relation with the Group Isomorphism Problem	69
		5.1.2	Relation with the Graph Isomorphism Problem	71
		5.1.3	Current Status and ALgorithmic Results	72
	5.2	Towar	ds the Main Algorithm	75
		5.2.1	A Variant of the Naive Refinement Algorithm	75
		5.2.2	Individualization and Refinement in the ALTMATSPISO Setting	76
		5.2.3	Algorithm Outline	80
	5.3	Prelin	ninaries	82
		5.3.1	Matrix Tuples and Matrix Spaces	84
		5.3.2	Random Alternating Matrix Spaces	89
	5.4	Proof	of the Main Algorithm	90
		5.4.1	Properties of Alternating Spaces and Alternating Tuples	91
		5.4.2	Estimate the Probability of $\mathbb{G} \in F'(n, m, q, r)$	93
		5.4.3	Algorithm Analysis	96
	5.5	Dynar	nic Programming	100
	5.6	Summ	ary and Discussion	108
6	Cor	nclusio	n	111

List of Figures

1.1	A systematic comparison between GRAPHISO and ALTMATSPISO	4
1.2	A 3-tensor.	8
3.1	Parallel scheme to distinguish an unknown quantum channel $\mathcal{O} \in \{\mathcal{E}, \mathcal{F}\}$	
	with N uses on the input state $ \Phi\rangle\langle\Phi _{RQ}$, where \mathcal{I}_R representing the identity	<u>م</u> ۲
	channel is applied on the auxiliary system R	25
5.1	For a given graph G . Individualize the top (red) and lower left (blue) ver-	
	tices. We obtain the induced bipartite graphs and label the rest of vertices	
	based on their adjacency relations with the individualized vertices	76
5.2	The 3-tensor \mathbb{G} , and flipping \mathbb{B}' to get \mathbb{B}	81
5.3	Slicing \mathbb{B}	81
$5.2 \\ 5.3$	The 3-tensor \mathbb{G} , and flipping \mathbb{B}' to get \mathbb{B}	

List of Publications

- Runyao Duan, Cheng Guo, Chi-Kwong Li, and Yinan Li. Parallel distinguishability of quantum operations. In 2016 IEEE International Symposium on Information Theory (ISIT), pages 2259 - 2263, July 2016.
- [2] Yinan Li, Xin Wang, and Runyao Duan. Indistinguishability of bipartite states by positive-partial-transpose operations in the many-copy scenario. *Phys. Rev. A*, 95:052346, May 2017.
- [3] Yinan Li and Youming Qiao. On rank-critical matrix spaces. Differential Geometry and its Applications, 55:68 - 77, 2017.
- [4] Yinan Li and Youming Qiao. Linear algebraic analogues of the graph isomorphism problem and the Erdős-Rènyi model. In 2017 IEEE 58th Annual Symposium on Foundations of Computer Science (FOCS), pages 463-474, Oct 2017.
- [5] Yinan Li, Youming Qiao, Xin Wang, and Runyao Duan. Tripartite-to-bipartite entanglement transformation by stochastic local operations and classical communication and the structure of matrix spaces. *Communications in Mathematical Physics*, 358(2):791814, Mar 2018.