UNIVERSITY OF TECHNOLOGY SYDNEY

Applications of Matrix Spaces in Quantum Information and Computational Complexity

by

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Certificate of Original Authorship

I, Yinan Li, declare that this thesis, is submitted in fulfillment of the requirements for the award of Doctor of Philosophy, in the School of Software, Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise reference or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Abstract

This thesis explores the applications of matrix spaces in quantum information and computational complexity, specifically in the areas of quantum state/channel discrimination, entanglement transformation and isomorphism testing. We achieve the following contributions:

- We derive a necessary condition which determines whether a set of orthogonal bipartite states can be distinguished by positive-partial-transpose (PPT) operations, in the many-copy scenario. We reduce the discrimination task to the following problem: Decide whether there exist a nonzero bipartite matrix with positive partial-transpose, such that all its eigenvectors with nonzero eigenvalues is orthogonal to a given bipartite vector space. As applications, we reprove and extend a variety of existing results, including the local indistinguishability of the bipartite maximally entangled state and its orthogonal complement [Yu, Duan and Ying, *IEEE transaction on Information Theory*, 2014] and the maximum dimension of non-positive-partial-transpose subspaces [Johnston, *Physical Review A*, 2013].

- We show that determining the parallel distinguishability of quantum channels is equivalent to determining whether the orthogonal complement of a given matrix space contains nonzero positive semidefinite matrices. Our characterization immediately implies a necessary condition to decide the parallel distinguishability. We further prove that our condition is also sufficient for two large families of quantum channels, which leads to an alternating proof for the parallel distinguishability of unitary operations [Acín, *Physical Review Letters*, 2001]. We also present an illustrative example showing our necessary condition cannot determine the parallel distinguishability.

- We systematically study the tripartite-to-bipartite entanglement transformations by stochastic local operations and classical communication (SLOCC). Such a problem is equivalent to computing the maximal rank of a matrix space [Chitambar, Duan and Shi, *Physical Review A*, 2010]. We analyze the SLOCC convertibility in both the finite-copy and asymptotic regimes. In particular, we derive explicit formulas which compute asymptotic entanglement transformation rates for two families of tripartite states by resorting to certain results of the structure of matrix spaces, including the study of matrix semi-invariants. Significantly, we show that the problem of deciding the asymptotic SLOCC convertibility of tripartite pure states to the bipartite maximally entangled states and the non-commutative symbolic determinant identity testing problem is algorithmically equivalent, which builds a new connection between problems in algebraic complexity and problems in asymptotic SLOCC entanglement transformations.

- We devise algorithms which test isometry between alternating matrix spaces over finite field. Algorithms for such a problem in time polynomial in the underlying vector space size resolves the believed bottleneck case of the group isomorphism problem, i.e. testing isomorphism of p-groups of class 2 and exponent p in time polynomial in the group order. Our approach is to view it as a linear algebraic analog of the graph isomorphism problem. We devise an average-case efficient algorithm for the alternating matrix space isometry problem over a key range of parameters. in a random model of alternating matrix spaces in vein of the Erdős-Rényi model of random graphs. To devise our algorithm, we developed linear algebraic individualization and refinement techniques, which are crucial in the first average-case efficient algorithm for graph isomorphism, devised by Babai, Erdős, and Selkow in the 1970s [Babai, Erdős and Selkow, *SIAM Journal on Computing*, 1980]. We also adapt Luks' dynamic programming technique for graph isomorphism [Luks, *STOC*, 1999] to slightly improve the worst-case time complexity of the alternating matrix space isometry problem.

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