Semidefinite Optimization for Quantum Information

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CERTIFICATE OF ORIGINAL AUTHORSHIP

I hereby declare that I am the sole author of this thesis. I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as part of the collaborative doctoral degree and/or fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Xin Wang

This thesis is dedicated to my mother.

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Abstract

This thesis aims to improve our understanding of the structure of quantum entanglement and the limits of information processing with quantum systems. It presents new results relevant to three threads of quantum information: the theory of quantum entanglement, the communication capabilities of quantum channels, and the quantum zero-error information theory.

In the first part, we investigate the fundamental features of quantum entanglement and develop quantitative approaches to better exploit the power of entanglement. First, we introduce a computable and additive entanglement measure to quantify the amount of entanglement, which also plays an important role as the improved semidefinite programming (SDP) upper bound of distillable entanglement. Second, we show that the Rains bound is neither additive nor equal to the asymptotic relative entropy of entanglement. Third, we establish SDP lower bounds for the entanglement cost and demonstrate the irreversibility of asymptotic entanglement manipulation under positive-partial-transpose-preserving quantum operations, resolving a major open problem in quantum information theory.

In the second part, we develop a framework of semidefinite programs to evaluate the classical and quantum communication capabilities of quantum channels in both the non-asymptotic and asymptotic regimes. In particular, we establish the first general SDP strong converse bound on the classical capacity of an arbitrary quantum channel and give in particular the best known upper bound on the classical capacity of the amplitude damping channel. We further establish a finite resource analysis of classical communication over quantum erasure channels, including the first second-order expansion of classical capacity beyond entanglement-breaking channels. For quantum communication, we establish the best SDP-computable strong converse bound and refine it as the so-called max-Rains information.

In the third part, we investigate the quantum zero-error information theory. In contrast to the conventional Shannon theory, there is a very different-looking information theory when errors are required to be precisely zero, where the communication problem reduces to the analysis of the so-called confusability graph (non-commutative graph) of a classical channel (quantum channel). We develop an activated communication model and explore its novel properties. Notably, we separate the quantum Lovász number and the entanglement-assisted zero-error capacity, resolving an intriguing open problem in the area of zero-error information.

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