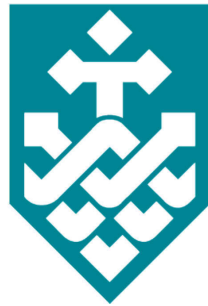


Semidefinite Optimization for Quantum Information



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A dissertation submitted for the degree of
Doctor of Philosophy

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July 2018

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CERTIFICATE OF ORIGINAL AUTHORSHIP

I hereby declare that I am the sole author of this thesis. I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as part of the collaborative doctoral degree and/or fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Xin Wang

This thesis is dedicated to my mother.

Acknowledgements

During my PhD study, I enjoyed a lot doing research and always felt excited to learn new things and discover interesting results. However, none of this would have been possible without the support and help of many people.

First and foremost, I must express my sincere gratitude to my supervisor Runyao Duan for his inspiring supervision and support. When I was a third year undergraduate student, I learned many exciting things about quantum information from him via emails and got interested in research. Without him, I probably would not have entered the academic world. I particularly thank him for the energy and passion he always brings to our meetings, for giving me the freedom in exploring different research directions, and for his encouragement on studying important problems regardless of whether they are old fashioned or popular.

I would especially like to thank my external supervisor Andreas Winter for his enlightening supervision. I want to thank him for many discussions on entanglement theory, strong converse, channel capacities, state redistribution, and Lovász number. He really knows everything. I wish I had visited Barcelona for at least one semester. Moreover, I am grateful to my QIP mentors Fernando Brandão, Debbie Leung and Thomas Vidick for their valuable advice on both research and career.

I sincerely thank all my co-authors: Gerardo Adesso, Mario Berta, John Calsamiglia, Runyao Duan, María García Díaz, Kun Fang, Ji Guan, Ludovico Lami, Yinan Li, Shusen Liu, Rosanna Nichols, Youming Qiao, Bartosz Regula, Matteo Rosati, Michalis Skotiniotis, Marco Tomamichel, Andreas Winter, Wei Xie, and Mingsheng Ying. It has been a pleasure to collaborate with these wonderful people. I want to further extend my gratitude to Gerardo Adesso, Mario Berta, Matthias Christandl, Xiongfeng Ma, and Man-Hong Yung for their hospitality and helpful discussions. I profited a lot from these visits.

I am grateful to the Center for Quantum Software and Information at UTS for providing an excellent environment for research, especially the travel opportunities provided for graduate students. I thank all the past and present members of UTS:QSI for creating such a pleasant research atmosphere. Special thanks goes to my friends Kun Fang and Yinan Li, with whom I discussed and collaborated frequently. I particularly thank Yinan for driving me to school during the past two years. I am also grateful to Marco Tomamichel for his supervision on some of my research projects, to Cheng Guo, Ching-Yi Lai for helpful suggestions, and to Michael Bremner, Hao-Chung Cheng,

Yuan Feng, Min-Hsiu Hsieh, Zhengfeng Ji, Ryan Mann, Youming Qiao, Peter Rohde, Kun Wang, and Nengkun Yu for interesting discussions.

I would furthermore like to thank Anurag Anshu, Charles H. Bennett, Eric Chitambar, Nilanjana Datta, Omar Fawzi, Li Gao, Mark W. Girard, Gilad Gour, Masahito Hayashi, Richard Josza, Felix Leditzky, Debbie Leung, Ke Li, Ziwen Liu, Laura Mančinska, Alexander Müller-Hermes, Martin Plenio, Peter W. Shor, John Watrous, Mark M. Wilde, Xiaodi Wu, Yunlong Xiao, Dong Yang, and Yuxiang Yang for interesting and stimulating discussions.

I am grateful in advance to my examiners Matthias Christandl and Barbara Kraus for reading my thesis and their feedback. The present version also benefited from the very helpful comments by Mark M. Wilde.

Finally, I thank all my great friends who have made my stay in Sydney full of fun and adventures. Many thanks to my family and friends for their encouragement and support, and I am most grateful to my dear Mom who has always been there to support me. Thank you all.

Abstract

This thesis aims to improve our understanding of the structure of quantum entanglement and the limits of information processing with quantum systems. It presents new results relevant to three threads of quantum information: the theory of quantum entanglement, the communication capabilities of quantum channels, and the quantum zero-error information theory.

In the first part, we investigate the fundamental features of quantum entanglement and develop quantitative approaches to better exploit the power of entanglement. First, we introduce a computable and additive entanglement measure to quantify the amount of entanglement, which also plays an important role as the improved semidefinite programming (SDP) upper bound of distillable entanglement. Second, we show that the Rains bound is neither additive nor equal to the asymptotic relative entropy of entanglement. Third, we establish SDP lower bounds for the entanglement cost and demonstrate the irreversibility of asymptotic entanglement manipulation under positive-partial-transpose-preserving quantum operations, resolving a major open problem in quantum information theory.

In the second part, we develop a framework of semidefinite programs to evaluate the classical and quantum communication capabilities of quantum channels in both the non-asymptotic and asymptotic regimes. In particular, we establish the first general SDP strong converse bound on the classical capacity of an arbitrary quantum channel and give in particular the best known upper bound on the classical capacity of the amplitude damping channel. We further establish a finite resource analysis of classical communication over quantum erasure channels, including the first second-order expansion of classical capacity beyond entanglement-breaking channels. For quantum communication, we establish the best SDP-computable strong converse bound and refine it as the so-called max-Rains information.

In the third part, we investigate the quantum zero-error information theory. In contrast to the conventional Shannon theory, there is a very different-looking information theory when errors are required to be precisely zero, where the communication problem reduces to the analysis of the so-called confusability graph (non-commutative graph) of a classical channel (quantum channel). We develop an activated communication model and explore its novel properties. Notably, we separate the quantum Lovász number and the entanglement-assisted zero-error capacity, resolving an intriguing open problem in the area of zero-error information.

Contents

1	Introduction	1
1.1	Overview	2
2	Preliminaries	8
2.1	Basics of linear algebra	8
2.2	The formalism of quantum information	8
2.2.1	Quantum states	9
2.2.2	Quantum channels and measurements	9
2.2.3	Bipartite quantum states	12
2.3	Bipartite quantum operations	13
2.3.1	Local operations and classical communication	13
2.3.2	Non-local operations	15
2.3.3	Channel composition	17
2.4	Semidefinite optimization	19
2.4.1	Basics of semidefinite programming	19
2.4.2	Duality of semidefinite programming	20
2.4.3	Applications of semidefinite programming in quantum information	21
2.5	Symmetries	22
2.6	Distance measures	23
2.6.1	Distance between states	23
2.6.2	Distance between channels	25
2.7	Entropies	26
2.7.1	Entropy of a single system	26
2.7.2	Relative entropies	27
2.7.3	Smoothed entropies	29
I	Entanglement Theory	31
3	Entanglement distillation and quantification	32
3.1	Introduction	32
3.1.1	Background	32
3.1.2	Outline	35

3.2	Distillation under PPT operations	35
3.3	Improved SDP upper bound on distillable entanglement	37
3.3.1	max-Rains relative entropy	43
3.4	Deterministic Distillable Entanglement	44
3.4.1	One-copy deterministic distillable entanglement	45
3.4.2	Asymptotic deterministic distillable entanglement	46
3.5	Nonadditivity of Rains bound	49
3.5.1	Rains bound on distillable entanglement	50
3.5.2	Nonadditivity of Rains bound	52
3.6	Discussion	54
3.6.1	Summary	54
3.6.2	Outlook	55
4	Irreversibility of Asymptotic Entanglement Manipulation	56
4.1	Introduction	56
4.1.1	Background	56
4.1.2	Outline	57
4.2	Lower bounds for entanglement cost	58
4.2.1	Entanglement cost	58
4.2.2	Lower bounds for entanglement cost	59
4.3	Irreversibility of PPT entanglement manipulation	63
4.3.1	PPT entanglement cost of ρ_v	64
4.3.2	PPT distillable entanglement of ρ_v	65
4.3.3	General irreversibility under PPT operations	67
4.4	Discussion	68
4.4.1	Summary	68
4.4.2	Outlook	68
II	Quantum Shannon Theory	70
5	Classical communication via quantum channels	71
5.1	Introduction	71
5.1.1	Background	72
5.1.2	Outline	74
5.2	One-shot communication capability	75
5.2.1	Task of information processing	75
5.2.2	Semidefinite programs for optimal success probability	77
5.2.3	Semidefinite programs for coding rates	81
5.2.4	Matthews-Wehner converse via activated NS codes	84
5.3	Non-asymptotic communication capability	88
5.3.1	New meta-converse for classical communication	88
5.3.2	Second-order analysis for quantum erasure channel	91
5.4	Asymptotic communication via quantum channels	93
5.4.1	SDP strong converse bounds for the classical capacity	93

5.4.2	Amplitude damping channel	97
5.4.3	A special class of quantum channels	100
5.4.4	New converse via channel divergence	103
5.5	Discussion	110
5.5.1	Summary	110
5.5.2	Outlook	111
6	Quantum communication via quantum channels	112
6.1	Introduction	112
6.1.1	Background	112
6.1.2	Outline	114
6.2	One-shot communication capability	114
6.2.1	Task of information processing	114
6.2.2	SDP converse bounds for quantum communication	117
6.2.3	Example: amplitude damping channel	118
6.3	Asymptotic communication capability	120
6.3.1	Quantum capacity	120
6.3.2	An SDP strong converse bound on quantum capacity	120
6.3.3	Comparison with other converse bounds	122
6.4	Discussion	128
6.4.1	Summary	128
6.4.2	Outlook	128
III	Quantum Zero-error Information	130
7	Advancing quantum zero-error information theory	131
7.1	Introduction	131
7.1.1	Background	131
7.1.2	Outline	132
7.2	Zero-error capacity of a quantum channel	132
7.2.1	Graphs and their quantum generalizations	132
7.2.2	Zero-error capacity of a quantum channel	134
7.2.3	An upper bound on the independence number	135
7.3	Separating C_{0E} and quantum Lovász number	136
7.3.1	Zero-error communication quantities	137
7.3.2	Establishing the gap	139
7.4	Activated zero-error communication	146
7.4.1	Activated one-shot zero-error capacity	146
7.4.2	Classical-quantum channel	151
7.4.3	Asymptotic zero-error capacity	153
7.4.4	Separating $C_{0,NS}$ and semidefinite packing number	154
7.5	Discussion	155
7.5.1	Summary	155
7.5.2	Outlook	155

List of Tables

2.1	Overview of notational conventions	9
2.2	Constraints of bipartite operations	17
2.3	Different kinds of bipartite operations and codes	18
3.1	Partial zoo of entanglement measures	51
5.1	Classical communication capabilities of basic channels	111
6.1	Comparison of converse bounds on quantum capacity	123
6.2	Quantum communication capabilities of basic channels	129
7.1	Classical graphs and their quantum analogs	133
7.2	Zero-error capacities of different classes of channels	156

List of Figures

1.1	Structure of this thesis	3
2.1	Local operations and classical communication	14
2.2	Local operation with shared entanglement	16
2.3	No-signalling operations	16
2.4	Hierarchy of quantum bipartite operations	17
2.5	Channel composition	18
3.1	Entanglement distillation and formation	33
3.2	Comparison between E_W and E_N	41
3.3	Estimation of deterministic entanglement distillation	48
3.4	Nonadditivity of Rains bound	53
4.1	Illustration of entanglement irreversibility	57
4.2	Zoo of entanglement measures	69
5.1	Classical communication over channels	71
5.2	Strong vs. weak converse	74
5.3	General code scheme	75
5.4	Channel coding rate over quantum erasure channel	92
5.5	Bounds on the classical capacity of amplitude damping channel	98
5.6	Capacities of amplitude damping channel	100
6.1	Quantum communication over channels	112
6.2	General codes	115
6.3	Quantum coding with two uses of amplitude damping channel	119
6.4	Quantum coding with three uses of amplitude damping channel	119
6.5	Comparison between Q_Γ and partial transposition bound	127
7.1	Activated classical communication.	146