UNIVERSITY OF TECHNOLOGY SYDNEY
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Decomposition of Quantum Markov Chains and Its Applications

by

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Certificate of Authorship/Originality

I certify that the work in this thesis has not been previously submitted for a degree nor has it been submitted as a part of the requirements for other degrees except as fully acknowledged within the text.

I also certify that this thesis has been written by me. Any help that I have received in my research and in the preparation of the thesis itself has been fully acknowledged. In addition, I certify that all information sources and literature used are quoted in the thesis.

ABSTRACT

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Markov chains have been widely employed as a fundamental model in the studies of probabilistic and stochastic communicating and concurrent systems. It is well-understood that decomposition techniques play a crucial role in reachability analysis and model-checking of Markov chains. (Discrete-time) Quantum Markov chains have been introduced as a model of quantum communicating systems [66] and also a semantic model of quantum programs [67]. The BSCC (Bottom Strongly Connected Component) and stationary coherence decompositions of quantum Markov chains were introduced in [62, 68, 5]. This thesis presents a new decomposition technique, namely periodic decomposition, for quantum Markov chains. This decomposition further helps us find sufficient and necessary conditions for limiting states of quantum Markov chains.

To confirm the power of these decomposition techniques, we apply them to characterizing the one-shot zero-error capacity of quantum channels, finding the structure of quantum decoherence-free subsystems against quantum noises and super-activating quantum memory with entanglement via modeling the underlying quantum systems by quantum Markov chains.

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List of Publications

Journal Papers

J-1. Guan, J., Feng, Y. and Ying, M., 2018. Decomposition of quantum Markov chains and its applications. Journal of Computer and System Sciences, 95, pp.55-68.

J-2. Su, Z., Guan, J. and Li, L., 2018. Efficient quantum repeater with respect to both entanglement-concentration rate and complexity of local operations and classical communication. Physical Review A, 97(1), p.012325.

J-3. Liu, S., Zhou, L., Guan, J., He, Y., Duan, R. and Ying, M., 2017. $Q|SI\rangle$: a quantum programming environment. SCIENTIA SINICA Informationis 47(10), 1300-1315(2017);

Preprint

P-1. Guan, J., Feng, Y. and Ying, M., 2017. Super-activating Quantum Memory with Entanglement. arXiv preprint arXiv:1708.00700.

P-2. Guan, J., Feng, Y. and Ying, M., 2018. The structure of decoherence-free subsystems, arXiv preprint arXiv:1802.04904, 2018.

P-3. Liu, S., Wang, X., Zhou, L., Guan, J., Li, Y., He, Y., Duan, R. and Ying, M., 2017. $Q|SI\rangle$: a quantum programming environment. arXiv preprint arXiv:1710.09500.

Contents

Abbreviation

MC - Markov chain

qMC - quantum Markov chain

gcd - the greatest common divisor

BSCC - bottom strongly connected component

CPTP - completely positive and trace-preserving

RHS - right-hand side

LHS - left-hand side