

UNIVERSITY OF TECHNOLOGY SYDNEY
Centre for Quantum Software and Information

**Decomposition of Quantum Markov Chains and
Its Applications**

by

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Certificate of Authorship/Originality

I certify that the work in this thesis has not been previously submitted for a degree nor has it been submitted as a part of the requirements for other degrees except as fully acknowledged within the text.

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ABSTRACT

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Markov chains have been widely employed as a fundamental model in the studies of probabilistic and stochastic communicating and concurrent systems. It is well-understood that decomposition techniques play a crucial role in reachability analysis and model-checking of Markov chains. (Discrete-time) Quantum Markov chains have been introduced as a model of quantum communicating systems [66] and also a semantic model of quantum programs [67]. The BSCC (Bottom Strongly Connected Component) and stationary coherence decompositions of quantum Markov chains were introduced in [62, 68, 5]. This thesis presents a new decomposition technique, namely periodic decomposition, for quantum Markov chains. This decomposition further helps us find sufficient and necessary conditions for limiting states of quantum Markov chains.

To confirm the power of these decomposition techniques, we apply them to characterizing the one-shot zero-error capacity of quantum channels, finding the structure of quantum decoherence-free subsystems against quantum noises and super-activating quantum memory with entanglement via modeling the underlying quantum systems by quantum Markov chains.

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- J-2. Su, Z., Guan, J. and Li, L., 2018. Efficient quantum repeater with respect to both entanglement-concentration rate and complexity of local operations and classical communication. *Physical Review A*, 97(1), p.012325.
- J-3. Liu, S., Zhou, L., Guan, J., He, Y., Duan, R. and Ying, M., 2017. $Q|SI\rangle$: a quantum programming environment. *SCIENTIA SINICA Informationis* 47(10), 1300-1315(2017);

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- P-1. Guan, J., Feng, Y. and Ying, M., 2017. Super-activating Quantum Memory with Entanglement. arXiv preprint arXiv:1708.00700.
- P-2. Guan, J., Feng, Y. and Ying, M., 2018. The structure of decoherence-free subsystems, arXiv preprint arXiv:1802.04904, 2018.
- P-3. Liu, S., Wang, X., Zhou, L., Guan, J., Li, Y., He, Y., Duan, R. and Ying, M., 2017. $Q|SI\rangle$: a quantum programming environment. arXiv preprint arXiv:1710.09500.

Contents

Certificate	ii
Abstract	iii
Acknowledgments	iv
List of Publications	v
Abbreviation	ix
I Introduction	1
II Preliminaries	7
1 Review of Classical Markov Chains	9
1.1 Markov Chains	9
1.2 Classification of States and Chains	11
1.3 Stationary Distribution and Limit Theorem	12
2 Review of Quantum Markov Chains	14
2.1 Definition	14
2.2 Essential Tools	15
III Decomposition of Quantum Markov Chains	18
3 BSCC Decomposition and Reachability	20
3.1 Reachability	20

3.2	Classification of Quantum States	21
3.3	BSCC Decomposition	23
4	Stationary Coherence Decomposition and Fixed Points	24
4.1	Stationary Coherence Decomposition	24
4.2	Fixed Points	26
5	Periodic Decomposition and Limiting States	30
5.1	Irreducibility and Periodicity	30
5.2	Periodic Decomposition	39
5.3	Limiting States	41
5.4	Conclusion	42
IV	Applications	44
6	One-shot Zero-error Capacity of Quantum Channels	46
6.1	One-shot Zero-error Capacity	46
6.2	A New Characterization	51
6.3	Conclusion	52
7	The Structure of Decoherence-free Subsystems	54
7.1	Continuous Coherence	55
7.2	Structure Theorem	58
7.3	Matrix Product States	65
7.4	Conclusion	67
8	Super-activating Quantum Memory with Entanglement	68
8.1	Quantum Memory	69

8.2 Super-activation for Storing Classical Information	70
8.3 Super-activation for Storing Quantum Information	73
8.4 Conclusion	77
V Concluding Remarks	78
VI Appendix	80
Bibliography	83

Abbreviation

MC - Markov chain

qMC - quantum Markov chain

gcd - the greatest common divisor

BSCC - bottom strongly connected component

CPTP - completely positive and trace-preserving

RHS - right-hand side

LHS - left-hand side