Active Task Design in Adaptive Control of Redundant Robotic Systems

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Abstract

This paper seeks to use robots' kinematic redundancy to excite the system persistently, through actively designing a secondary task in the null space of a primary task. Resulted convergence of unknown parameters in adaptive control leads to better system stability and performance. A measure in Grassmannian, referred to as Subspace Discrepancy Measure (SDM), is proposed for evaluating the additional benefit from the secondary task in converging unknown parameters to their true values. This measure evaluates the angles among subspaces that the parameter estimations are converging to, given different secondary tasks. The subspaces are obtained from Principal Component Analysis (PCA) on a small amount of samples of parameter estimations. The SDM is used to determine the choice of the secondary task online through a trial-and-evaluation procedure actively. Numerical simulations demonstrated that the secondary task chosen by SDM enhances the parameter convergence.

1 Introduction

Having a model of robot dynamics with precise parameters is often prohibitive in many robot applications. For example, underwater robots are always subject to model uncertainties that are partially determined by environments or varying loads. While model-free control approaches often have issues of slow response, control strategies based on insufficient representation of dynamic models often lead to unsatisfactory stability or poor transient performance. Adaptive control theories pursue closed-loop stability and asymptotical convergence of the controlled robotic systems with uncertainties [Yucelen *et al.*, 2013]. It is usually built on known models with unknown parameters. Among other model choices, linear-in-parameters models, such as radial basis functions, equations of motions from Lagrange or Euler methods, and Gaussian Processes, have been widely used.

In general, the adaptive control algorithms do not guarantee that the estimations of unknown parameters converge to their true value unless a condition of persistency of excitation (PE) on the system states is satisfied. For linear time-invariant systems, as given in [Boyd and Sastry, 1986; Narendra and Annaswamy, 1987a], if the number of spectral lines contained in the spectrum of the reference inputs is no less than the number of unknown parameters, the system is PE and thus the parameter estimation error exponentially converges to zero. A parameter adaptive rule for robust adaptation without persistent excitation has been studied in [Narendra and Annaswamy, 1987b], which shows the controlled system is robust subject to a class of unmodelled dynamics. However when the robotic system is not PE, the true parameter values are not attractive. Therefore, the parameters have to be adapted to a new configuration when the reference input switches or parameter excitation changes. As a result, transient performance is usually unsatisfactory if the robotic system is not PE.

This paper seeks for the possibilities of using robot kinematic redundancy to make the system PE by actively designing/designating the secondary task in the null space of the primary task. The primary task for the robotic system is often given and has to be executed during the mission all the time, while the secondary task is often chosen for various purposes, such as maximizing the manipulability of the manipulator, maintaining the distance between the robot and obstacles, and imposing constraints of the robot base velocities, etc. In general, secondary tasks with more spectral lines will lead to higher probabilities in satisfying the PE condition. However, this may not be always true in practice due to the nonlinear transformation from the task space to the robot state space and different excitation frequencies of parameters. In addition, unnecessarily complicated secondary tasks may cause issues of system instabilities due to limited actuator and sensor bandwidth.

This paper pursues a representation of potential benefit from various secondary tasks in learning unknown parameters, based on a small amount of data about the estimations of unknown parameters collected from the adaptive control during secondary task execution. The idea is inspired by the phenomenon of partial convergence of unknown parameters in adaptive control, i.e., the parameter vector converges to a subspace when the system is not PE [Anderson, 1977; Boyd and Sastry, 1986].

In this paper, the subspaces that the parameters converge to are estimated through PCA [Moore, 1981]. The convergence of parameters are detected via the convergence of the task tracking error. Then, the SDM is used to measure the discrepancy between these subspaces of possibly different dimensions and is further used in designing secondary tasks for the purposes of parameter convergence. The design of a secondary task usually consists of two elements: (i) a task space and (ii) its reference inputs. This paper focuses on (ii) with (i) given and fixed. The SDM is a $L1$-norm Grassmannian distance between subspaces. Since the subspaces that the parameter estimations are converging to may have different dimensions, the SDM is extended to subspaces of different dimensions following [Ye and Lim, 2016], based on a Schubert variety in a Grassmannian.

Unlike those information-driven control methods (reviewed in the section on related work), the SDM approach for this study does not have a suitable model to predict the benefit of converging parameter estimations from a secondary task. Therefore, the task design approach presented in this paper is implemented through a trial-and-evaluation procedure actively. It is in particular useful for online applications, since the collection of the small amount of data on the evolution of parameter estimations is online, and so is the decision making on the task design.

This paper is organized as follows. Related work is summarized in Section 2. Problem formulation is given in Section 3. The adaptive control in task space with switched secondary tasks is presented in Section 4. The proposed SDM and the strategy of choosing reference inputs to the secondary task is given in Section 5. Simulation results and conclusions are summarized in Section 6 and 7, respectively.

2 Related Work

When the measurements or estimations on the acceleration of each task state or robot state are available and reliable, modelling robot dynamics falls into the category of supervised learning. The training data consists of samples of the acceleration measurement paired with the control inputs and robot state (or task state). The candidate model can be equations of motion for rigid bodies, polynomial models, neural networks, Gaussian Processes [Wei *et al.*, 2014], and other kernel methods. The parameters of these models can be learned from least square minimization, gradient descent methods, or variational approaches. An adaptive control approaches using Gaussian Processes as the model is studied in [Chowdhary *et al.*, 2015], where the acceleration is estimated using a Kalman filter-based fixed-point smoother. However, such acceleration estimation might be corrupted by noises and delays.

In the multi-task adaptive control through null space saturation [Khatib, 1987], the dynamically-consistent inversion of the mass matrix breaks the property of linearity in parameters that adaptive/learning control relies on. An adaptive control approach via inverse dynamics is proposed in [Tee and Yan, 2011], by introducing a regressor in order to approximate the dynamically consistent generalized inverse of the mass matrix in the task space. However, the parameter convergence is not discussed.

The idea of choosing actions/tasks based on difference/divergence in belief space of unknown parameters has been well explored in information-driven sensor path planning and control. The benefit of sensor measurements in estimating unknown variables is represented by information functions, which is then optimized [Lu *et al.*, 2014a; 2014b].

Small data, different from big data, is usually better organized and packaged, presenting information in subspaces or locally. However, small data needs to be incorporated with fundamental knowledge of robot dynamics, kinematics, and control, which has been explored in system identifications [Ljung, 1998]. Recently, semi-parametric models have been studied in [Nguyen-Tuong and Peters, 2010; Wu and Movellan, 2012], which combine parametric and nonparametric models together to approximate humanoid robot dynamics. The parametric model plays a major role in its dynamics at a robot state where no data is available. On the other hand, the nonparametric model, at a robot state where data is available, is able to compensate for errors from the parametric model.

3 Problem Formulation

The dynamic model of manipulator robotics with n Degrees of Freedom (DoFs) is given as

$$\boldsymbol{M}(\boldsymbol{\nu},\boldsymbol{\eta})\dot{\boldsymbol{\nu}}+\boldsymbol{C}(\boldsymbol{\nu},\boldsymbol{\eta})\boldsymbol{\nu}+\boldsymbol{D}(\boldsymbol{\nu},\boldsymbol{\eta})\boldsymbol{\nu}+\boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}, \quad (1)$$

where $\boldsymbol{\eta}$ and $\boldsymbol{\nu}$ are generalized coordinates and (quasi) velocities, respectively, $\boldsymbol{M} \in \mathbb{R}^{n\times n}$ denotes the inertial matrix including the added mass from hydrodynamic effects, $\boldsymbol{C} \in \mathbb{R}^{n\times n}$ denotes the Coriolis matrix and centripetal terms including the added mass, $\boldsymbol{D} \in \mathbb{R}^{n\times n}$ denotes hydrodynamic damping matrix, and $\boldsymbol{g} \in \mathbb{R}^n$ is the gravitational and buoyant generalized forces. The parameters in these matrices are unknown. The generalized force input is denoted as $\boldsymbol{\tau}$.

Let $n_p \in \mathbb{Z}_+$ denote the dimensions of the primary task state $\boldsymbol{x}_p$ and $n_s = n - n_p$ denote the dimensions of the secondary task state $\boldsymbol{x}_s$. Also, let $\boldsymbol{x}_p^d \in \mathcal{C}^1$ ($\boldsymbol{x}_s^d \in \mathcal{C}^1$) denote the primary (secondary) task reference inputs. The primary and secondary tasks are related to $\boldsymbol{\nu}$ by

$$\dot{\boldsymbol{x}}_p = \boldsymbol{J}_p(\boldsymbol{\eta})\boldsymbol{\nu},\ \dot{\boldsymbol{x}}_s = \boldsymbol{J}_s(\boldsymbol{\eta})\boldsymbol{\nu}, \tag{2}$$

where $\boldsymbol{J}_p(\boldsymbol{\eta}) \in \mathbb{R}^{n_p\times n}$ and $\boldsymbol{J}_s(\boldsymbol{\eta}) \in \mathbb{R}^{n_s\times n}$ are the jacobians. The robot generalized coordinates are observable, and the mechanical structure of the robot is often known accurately. Therefore, $\boldsymbol{J}_p$ and $\boldsymbol{J}_s$ can be obtained if the task state can be expressed in the robot generalized coordinates.

Adaptive control seeks for the control inputs and parameter updates to the robot system under various uncertainties. In general, the parameter estimations in adaptive control algorithms do not guarantee that the error of parameter estimations converges to zero unless the PE condition is met. However, without prior knowledge of the robot system, it might be difficult to access the transfer function and thus to determine if the system is PE.

Problem. *Find a measure for evaluating the benefit of $\boldsymbol{x}_s^d \in \mathcal{X}$ in converging parameter estimations and then find $\boldsymbol{x}_s^d$ through trial-and-evaluation procedures. The set $\mathcal{X}$ is assumed countable and predefined.*

4 Adaptive Control in Task Space

The robot dynamics in task space for designing adaptive control is given first. Because the SDM approach designs/changes the secondary task online, the resulted robotic system with the switched secondary task is particularly analyzed and is proven stable.

4.1 Dynamics in task space

Aggregating the primary and secondary task together to a new task, we have the following mapping

$$\dot{\boldsymbol{x}} = \begin{bmatrix}\dot{\boldsymbol{x}}_p\\ \dot{\boldsymbol{x}}_s\end{bmatrix} = \begin{bmatrix}\boldsymbol{J}_p\\ \boldsymbol{J}_s\end{bmatrix}\boldsymbol{\nu} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{\nu}. \tag{3}$$

In order to guarantee stability and boundedness over robot state, tasks are chosen such that $\boldsymbol{J}$ is full rank. Designing the secondary task for maintaining manipulability is not discussed in this paper.

Differentiating equation (3) with respect to time t leads to

$$\ddot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{\eta})\dot{\boldsymbol{\nu}} + \dot{\boldsymbol{J}}(\boldsymbol{\eta})\boldsymbol{\nu}. \tag{4}$$

Substituting equation (4) into equation (1) yields

$$\begin{aligned}&\boldsymbol{M}(\boldsymbol{\nu},\boldsymbol{\eta})\boldsymbol{J}^{\dagger}(\boldsymbol{\eta})[\ddot{\boldsymbol{x}} - \dot{\boldsymbol{J}}(\boldsymbol{\eta})\boldsymbol{J}^{\dagger}(\boldsymbol{\eta})\dot{\boldsymbol{x}}]\\ &+\boldsymbol{C}(\boldsymbol{\nu},\boldsymbol{\eta})\boldsymbol{J}^{\dagger}(\boldsymbol{\eta})\dot{\boldsymbol{x}}+\boldsymbol{C}(\boldsymbol{\nu},\boldsymbol{\eta})\boldsymbol{J}^{\dagger}(\boldsymbol{\eta})\dot{\boldsymbol{x}}+\boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{\tau},\end{aligned}$$

where the superscript † denotes matrix inverse.

Multiplying both sides of the above equation by $\boldsymbol{J}^{\dagger T}(\boldsymbol{\eta})$ leads to the dynamics of the aggregated task,

$$\boldsymbol{M}_t(\boldsymbol{\nu},\boldsymbol{\eta},\boldsymbol{J})\ddot{\boldsymbol{x}}+\boldsymbol{C}_t(\boldsymbol{\nu},\boldsymbol{\eta},\boldsymbol{J})\dot{\boldsymbol{x}}+\boldsymbol{C}_t(\boldsymbol{\nu},\boldsymbol{\eta},\boldsymbol{J})\dot{\boldsymbol{x}}+\boldsymbol{g}_t(\boldsymbol{\eta},\boldsymbol{J})=\boldsymbol{\tau}_t, \tag{5}$$

where

$$\begin{aligned}\boldsymbol{M}_t(\boldsymbol{\nu},\boldsymbol{\eta},\boldsymbol{J}) &= \boldsymbol{J}^{\dagger T}(\boldsymbol{\eta})\boldsymbol{M}(\boldsymbol{\nu},\boldsymbol{\eta})\boldsymbol{J}^{\dagger}(\boldsymbol{\eta}),\\ \boldsymbol{C}_t(\boldsymbol{\nu},\boldsymbol{\eta},\boldsymbol{J}) &= \boldsymbol{J}^{\dagger T}(\boldsymbol{\eta})\boldsymbol{C}(\boldsymbol{\nu},\boldsymbol{\eta})\boldsymbol{J}^{\dagger}(\boldsymbol{\eta})\\ &\quad - \boldsymbol{J}^{\dagger T}(\boldsymbol{\eta})\boldsymbol{M}(\boldsymbol{\nu},\boldsymbol{\eta})\boldsymbol{J}^{\dagger}(\boldsymbol{\eta})\dot{\boldsymbol{J}}(\boldsymbol{\eta})\boldsymbol{J}^{\dagger}(\boldsymbol{\eta}),\\ \boldsymbol{D}_t(\boldsymbol{\nu},\boldsymbol{\eta},\boldsymbol{J}) &= \boldsymbol{J}^{\dagger T}(\boldsymbol{\eta})\boldsymbol{D}(\boldsymbol{\nu},\boldsymbol{\eta})\boldsymbol{J}^{\dagger}(\boldsymbol{\eta}),\\ \boldsymbol{g}_t(\boldsymbol{\eta},\boldsymbol{J}) &= \boldsymbol{J}^{\dagger T}(\boldsymbol{\eta})\boldsymbol{g}(\boldsymbol{\eta}),\\ \boldsymbol{\tau}_t &= \boldsymbol{J}^{\dagger T}(\boldsymbol{\eta})\boldsymbol{\tau},\end{aligned}$$

and $\boldsymbol{M}_t \in \mathbb{R}^{n\times n}$, $\boldsymbol{C}_t \in \mathbb{R}^{n\times n}$, $\boldsymbol{D}_t \in \mathbb{R}^{n\times n}$, $\boldsymbol{g}_t \in \mathbb{R}^n$, and $\boldsymbol{\tau}_t$, denote the inertial matrix, the Coriolis and centripetal matrix, hydrodynamic damping effects, gravitational and buoyant generalized forces, and the generalized driving force, respectively. According to the property of linear-in-parameters [Antonelli *et al.*, 2004], the robot dynamics in task space can be rewritten as

$$\boldsymbol{\tau} = \boldsymbol{\Phi}(\boldsymbol{J},\boldsymbol{a},\boldsymbol{\nu},\boldsymbol{\eta},\dot{\boldsymbol{x}})\boldsymbol{\theta}, \tag{6}$$

where $\boldsymbol{\theta}$ denotes the unknown parameter vector.

4.2 Adaptive control under switched tasks

Resolved acceleration control is used instead of approaches through force aggregation in null space, such as the operational space formulation [Khatib, 1987]. It is because that adding the acceleration of secondary tasks in the null space of the primary task relies only on the jacobian matrix and the matrix's time derivative, shown in eq. (4), both of which can be precisely known.

Let the sliding error be defined as

$$\boldsymbol{s} = \dot{\tilde{\boldsymbol{x}}} + \boldsymbol{\Lambda}\tilde{\boldsymbol{x}}, \tag{7}$$

where $\boldsymbol{\Lambda} \in \mathbb{R}^{n\times n}$ is positive definite and

$$\dot{\tilde{\boldsymbol{x}}} = \dot{\boldsymbol{x}}_d - \dot{\boldsymbol{x}},\ \tilde{\boldsymbol{x}} = \boldsymbol{x}_d - \boldsymbol{x},\ \boldsymbol{x}_d = [(\boldsymbol{x}_p^d)^T\ (\boldsymbol{x}_s^d)^T]^T. \tag{8}$$

The sliding error $\boldsymbol{s}$ is used to define the controller and parameter learning rule. Notice that the error and time derivative of error in orientation is different than that in position. Without causing ambiguity, we use " $-$ " to define a generalized operator for calculating difference [Fossen, 1994].

Let the desired acceleration in the aggregated task be designated as

$$\boldsymbol{a}_r = \ddot{\boldsymbol{x}}_d + \boldsymbol{\Lambda}\dot{\tilde{\boldsymbol{x}}}. \tag{9}$$

The control is given as

$$\boldsymbol{\tau} = \boldsymbol{\Phi}(\boldsymbol{J}, \boldsymbol{a}_r, \boldsymbol{\nu}, \boldsymbol{\eta}, \dot{\boldsymbol{x}}_r, \dot{\boldsymbol{x}})\hat{\boldsymbol{\theta}} + \boldsymbol{K}\boldsymbol{s} + \boldsymbol{K_p}\tilde{\boldsymbol{x}}, \tag{10}$$

where $\boldsymbol{K}$ and $\boldsymbol{K}_p \in \mathbb{R}^{n\times n}$ are positive definite. Notice that $\dot{\boldsymbol{x}}_r = \dot{\boldsymbol{x}} - \boldsymbol{s}$ is the value assigned to variable $\dot{\boldsymbol{x}}$ that is multiplied with $\boldsymbol{C}_t(\boldsymbol{\nu},\boldsymbol{\eta},\boldsymbol{J})$ in eq. (5). The other $\dot{\boldsymbol{x}}$ is substituted by its true value. The estimation of $\boldsymbol{\theta}$, denoted as $\hat{\boldsymbol{\theta}}$, is incrementally updated based on the modeling error presented by the mismatch $\boldsymbol{s}$.

Typical implementation of adaptive controllers involves a gradient based learning algorithm for updating estimation of unknown parameters (also known as weights). When the secondary task switches, the projection operator has to be used to bound the parameters, promising ultimately uniform boundedness of the task tracking error and adaptive parameters [Hovakimyan and Cao, 2010]. Similar to most adaptive control approaches, the convergence of $\hat{\boldsymbol{\theta}}$ to its true value can not be guaranteed. However, the stability of the adaptive control of this switched system can be established and is given in Theorem 1.

The update rule of parameters is given as follows,

$$\dot{\hat{\boldsymbol{\theta}}} = \mathbf{Proj}[\boldsymbol{\Gamma}^{-1}\boldsymbol{\Phi}^T(\boldsymbol{J}, \boldsymbol{a}_r, \boldsymbol{\eta}, \boldsymbol{\nu}, \dot{\boldsymbol{x}}_r, \dot{\boldsymbol{x}})\boldsymbol{s}, \boldsymbol{\Theta}], \tag{11}$$

where $\boldsymbol{\Gamma} \in \mathbb{R}^{n\times n}$ is positive definite and $\mathbf{Proj}$ is the operator defined in [Hovakimyan and Cao, 2010]. In addition, $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ and $\boldsymbol{\Theta}$ is assumed known.

Consider the following Lyapunov function candidate

$$\mathcal{V} = \frac{1}{2}\boldsymbol{s}^T\boldsymbol{M}_t\boldsymbol{s} + \frac{1}{2}\tilde{\boldsymbol{x}}^T\boldsymbol{K}\tilde{\boldsymbol{x}} + \frac{1}{2}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^T\boldsymbol{\Gamma}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}).$$

Let

$$V_\theta = \frac{1}{2}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^T\boldsymbol{\Gamma}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}) \tag{12}$$

and $V_\theta \leq V_\theta^U$, where V_θ^U is a constant upper bound of V_θ. Let i denote the reference input index of the secondary task,

$$V_i = \frac{1}{2}\boldsymbol{s}^T\boldsymbol{M}_t\boldsymbol{s} + \frac{1}{2}\tilde{\boldsymbol{x}}^T\boldsymbol{K}\tilde{\boldsymbol{x}}, \tag{13}$$

and $V_i \leq V_i^U$, where V_i^U is a constant upper bound of $\boldsymbol{s}$ given the ith reference input. We have the following stability theorem, the proof of which is given in Appendix.

Theorem 1. *For any $\delta > 0$, after time $(\delta + 1)\ln\delta$ following each switch of the task, the robot system, defined over eq. (6), eq. (10), and eq. (11), can reach position error $|\tilde{\boldsymbol{x}}| \leq \sqrt{\frac{V_\theta^U + V_i^U}{\delta|\boldsymbol{K}|}}$.*

5 Decision by Subspace Discrepancy Measure

For linear time-invariant systems, when the reference signal has $k < N$ spectral lines in its spectrum, N being the number of the unknown parameters, the parameter vector can be shown to converge to a subspace of dimension $N - k$ [Boyd and Sastry, 1986]. When the primary and secondary tasks converge, i.e., the tracking errors from both tasks converge to zero, the parameter vector converges to a subspace of unknown dimensions. Therefore, parameters' partial convergence can be detected by the tracking error of tasks, as shown in eq. (5.7b) in [Boyd and Sastry, 1986]. Numerical simulations in Section 6 validates this observation.

Once the parameters partially converge, the parameter estimations are collected for subspace detection. Let $\mathcal{M}_i$ denote the data on these parameter estimations given the ith reference input to the secondary task. The detection of subspaces are done through PCA. Let $\mathcal{P}_i \in \mathbb{R}^{D\times d}$ denote the subspaces based on $\mathcal{M}_i$, where D is the number of parameters and $d \leq D$ is the dimensions of the subspace $\mathcal{P}_i$. The number of subspace dimension d is determined by a threshold on its principal values.

The SDM between two subspaces of same dimensions is introduced first and then this concept is extended to subspaces of different dimensions. Assume we have detected subspaces $\mathcal{P}_i$ and $\mathcal{P}_j$ by using the ith and jth reference inputs, respectively. Both of them are in the space $\mathbb{R}^{D\times d}$. Intuitively, if the two subspaces are similar or close, then they should not be too far away between each other in the Grassmannian. Then, the SDM is defined in the terms of the principal angles

$$D(\mathcal{P}_i, \mathcal{P}_j) = \sum_{k=1}^{d} \alpha_k, \tag{14}$$

where $\alpha_k \geq 0$ denotes the kth principal angle between the $\mathcal{P}_i$ and $\mathcal{P}_j$, given by $\mathcal{P}_i^T\mathcal{P}_j = \mathcal{U}\cos(\boldsymbol{\Sigma})\mathcal{V}^T$, and $\boldsymbol{\Sigma} = \mathrm{diag}[\alpha_1, \ldots, \alpha_d]$. They measure the discrepancy that subspaces have. Moreover, note that $D(d)$ is at most $d\pi/2$. A small value of $D(\mathcal{P}_i, \mathcal{P}_j)$ indicates $\mathcal{P}_i$ and $\mathcal{P}_j$ are aligned.

As pointed out in [Ye and Lim, 2016], the distance in the case of subspaces with different dimensions can be considered as that of a point $\mathcal{P}_i \in \mathcal{G}$ to a closed set $\mathcal{P}_j \in \mathcal{G}$, where $\mathcal{G}$ is a compact Grassmannian. The distance between subspace $\mathcal{P}_i$ of dimension d_i and $\mathcal{P}_j$ of dimension d_j is given as

$$D(\mathcal{P}_i, \mathcal{P}_j) = \sum_{k=1}^{\min(d_i,d_j)} \alpha_k. \tag{15}$$

given by $\mathcal{P}_i^T\mathcal{P}_j = \mathcal{U}\cos(\boldsymbol{\Sigma})\mathcal{V}^T$, and $\boldsymbol{\Sigma} = \mathrm{diag}[\alpha_1, \ldots, \alpha_{\min(d_i,d_j)}, 0, \cdots, 0]$.

In general, a reference input to secondary task with more signals in spectrum will have a higher probability of providing a PE condition after nonlinear transform from robot state space to task space. However, complex signals may lead to significant control problems and require more power. The proposed algorithm uses SDM to determine if a new reference input should be added to the existing reference inputs through trial-and-evaluation procedures. When the SDM between the new reference input and the existing reference inputs is over a threshold, this new reference input is added to the existing ones.

6 Numerical Simulations and Results

6.1 Partial parameter convergence

In order to visualize the history of parameter estimations and their behavior of converging to a subspace and to show the relationship between such convergence with robotic system tracking errors, a single-input system is used, which is given as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}[u + \boldsymbol{\phi}(\boldsymbol{x})\boldsymbol{\theta}], \tag{16}$$

where the system matrix and input matrix are given as

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{17}$$

The correct values of unknown parameter vector $\boldsymbol{\theta}$ is $[-0.2 \ 0.25 \ 0.1]^T$ and the regressor is given as $\boldsymbol{\phi}(\boldsymbol{x}) = [x(1) \ x(2) \ x(1)x(2)]^T$. The reference trajectory is given as $\begin{bmatrix} sin(0.1 \ t) \\ 0.1cos(0.1 \ t) \end{bmatrix}$. Figures 1 and 2 show that when the tracking errors converge to zeros, the parameters also converge to a subspace, which is a straight line.

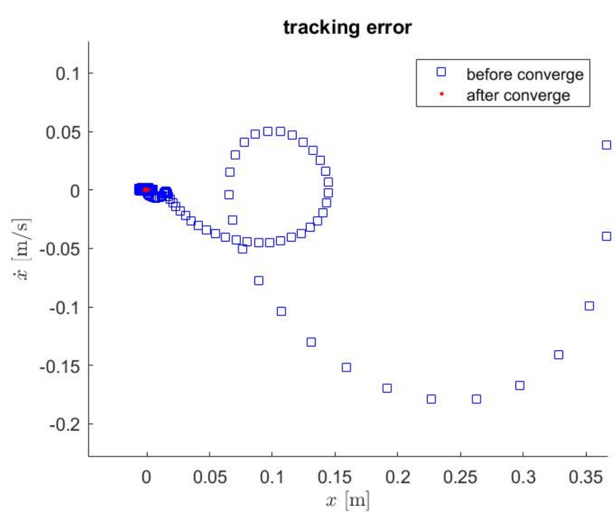

Figure 1: Tracking error.

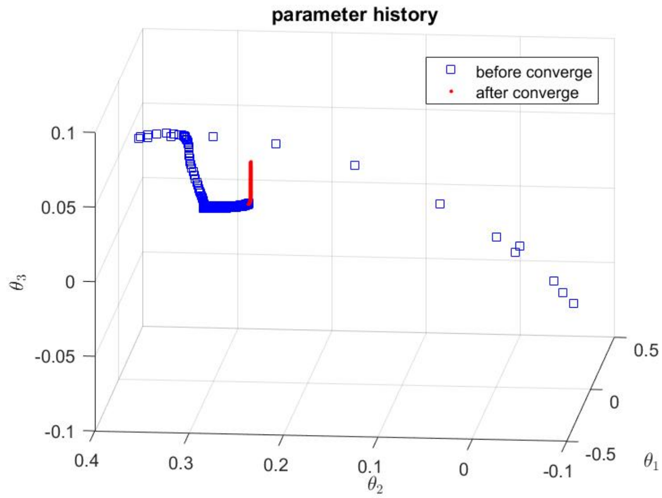

Figure 2: Parameter estimations

6.2 Choosing tasks via SDM

The comparison of the performances on parameter convergence of different references to the secondary task is shown by the following simulations. The simulated robot has a three-DoF manipulator [Antonelli, 2010], and the goal of the end-effector on this manipulator is to be regulated at a fixed pose in inertial frame, $\boldsymbol{x}_p = [0, \ 0, \ 0, \ 0, \ 0, \ 0]^T$. The candidates for the secondary task is given as $\boldsymbol{x}_s^d = [\sin(\omega t{+}0.1) \ \sin(\omega t{+}0.2) \ \sin(\omega t{+}0.3)]^T$ for three joints. The candidate set χ is constructed by choosing $\omega \in \{0.02, \ 0.04, \ 0.08, \ 0.1, \ 0.2, \ 0.5\}$. Each candidate for the secondary task is indexed by 1 to 6, respectively. Six simulations are conducted with the following setting. In each simulation, during the first $0 - 40$ seconds, the secondary tasks are chosen as $[0, 0, 0]^T$, then the secondary task is switched to one of the candidates from χ. The true parameter values are $[50, 50, 50, 5, 5, 5, 0.2, 0.2, 0.2, 0.2, 0.1, 0.2]^T$, while the initial parameter estimations are all zeros.

The relative parameter errors (percentage) and the $L2$ norm of parameter errors are shown in Fig. 3 and Fig. 4, respectively. As shown in Fig. 3, when the secondary task does not introduce any excitation in the first 40 seconds, the parameters do not converge to their true values. In addition, the secondary tasks of higher frequency may be better than the ones of lower frequencies, but not always. The 2nd candidate with $\omega = 0.04$ performs better than 4th candidate with $\omega = 0.1$. The SDM value of each secondary task against this primary task is summarized in Fig. 5, which shows that a secondary task with the higher SDM is better in compensating the primary task for learning unknown parameters. In all cases, the task tracking error converges to zeros as expected from adaptive control approaches, shown in Fig. 6.

Another simulation scenario is considered to test the

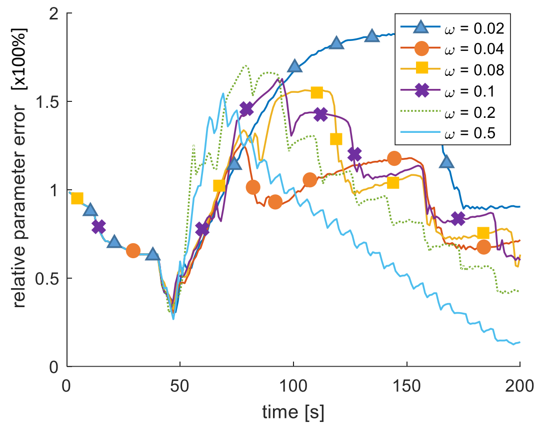

Figure 3: Tracking error: relative (percentage)

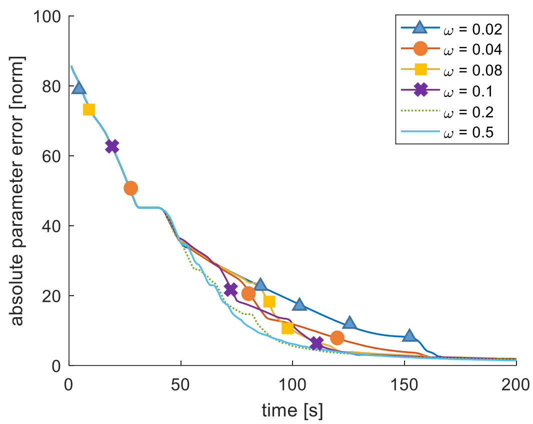

Figure 4: Tracking error: absolute (norm)

proposed algorithm with making decisions on the secondary task via SDM, where a choice of the secondary task has to be made at the 40th, 80th, 120th, 160th seconds, respectively. In addition, the choice at 0th second is set as 1 while the last four choices are determined by two strategies. The first strategy, referred to as "SDM", makes choices on the secondary tasks based on their SDM values. Its performance is compared with the second strategy, referred to as "random", which chooses secondary tasks randomly from χ. Again, a robot with a three-DoF manipulator is simulated, whose primary task is to regulate the end-effector at a desired pose $[0, 0, 0, 0, 0, 0]^T$. If the additional reference input has an SDM value over 0.5 (a threshold) against an existing one, it is added to the secondary task. Otherwise the additional reference input is abandoned. As shown in Fig. 7, the simulations from this scenario demonstrate that the use of SDM can enhance the convergence of parameter estimations by choosing reference inputs with larger SDM online. Note that in all cases the absolute parameter errors converge to zeros and the tracking error converges to zeros as expected from adaptive control approaches (not shown here).

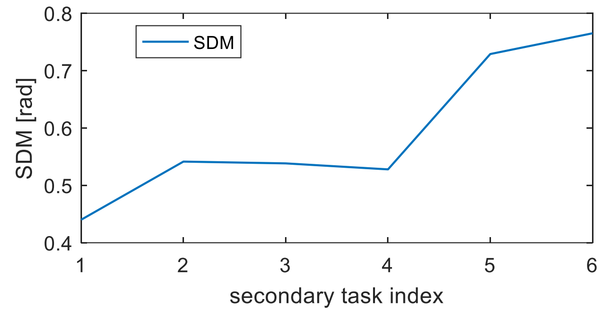

Figure 5: SDM

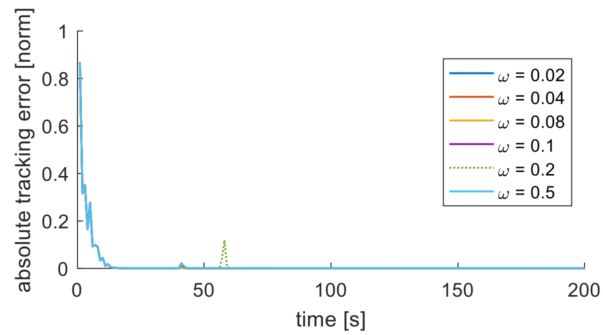

Figure 6: End-effector tracking error

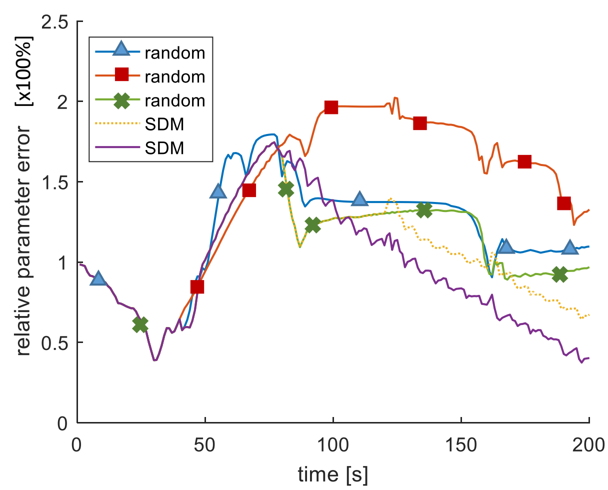

Figure 7: Relative parameter error

7 Conclusion and Future Work

Subspace discrepancy measure in Grassmannian is effective in evaluating the benefit from secondary task against existing tasks, regarding unknown parameter convergence in adaptive control. Numerical simulations have demonstrated that the decision on reference inputs based on SDM outperforms the decision made from a random algorithm. The SDM approach will be further explored in the fields of non-parametric adaptive control and transfer learning, which will potentially enhance data efficiency in learning robot models.

Appendix

The proof of Theorem 1 is given as follows.

Proof. Reconsider the following Lyapunov function candidate

$$\mathcal{V} = \frac{1}{2}\boldsymbol{s}^T\boldsymbol{M}_t\boldsymbol{s} + \frac{1}{2}\tilde{\boldsymbol{x}}^T\boldsymbol{K}\tilde{\boldsymbol{x}} + \frac{1}{2}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})^T\boldsymbol{\Gamma}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}). \quad (18)$$

It is obvious that $\mathcal{V}$ is semi-positive definite. Differentiating $\mathcal{V}$ with respect to time yields

$$\begin{aligned}\dot{\mathcal{V}} =& \frac{1}{2}\boldsymbol{s}^T\dot{\boldsymbol{M}}_t\boldsymbol{s} + \boldsymbol{s}^T\boldsymbol{M}_t\dot{\boldsymbol{s}} + \tilde{\boldsymbol{x}}^T\boldsymbol{K}(\boldsymbol{s} - \boldsymbol{\Lambda}\tilde{\boldsymbol{x}}) \\ &+ (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^T\boldsymbol{\Gamma}(\dot{\boldsymbol{\theta}} - \dot{\hat{\boldsymbol{\theta}}}). \end{aligned} \quad (19)$$

In paper [Li and Ge, 2013], it has been shown that in task space (i) the inertia matrix of the vehicle body system is positive definite and symmetric; (ii) the damping matrix is positive definite; and (iii) the matrix $\dot{\boldsymbol{M}}_t - 2\boldsymbol{C}_t(\boldsymbol{\nu})$ is skew symmetric, i.e., $\boldsymbol{\nu}^T\left[\dot{\boldsymbol{M}}_t - 2\boldsymbol{C}_t(\boldsymbol{\eta},\boldsymbol{\nu})\right]\boldsymbol{\nu} = \boldsymbol{O}$, $\forall\ \boldsymbol{\nu} \in \mathbb{R}^n$. We now have

$$\begin{aligned}\dot{\mathcal{V}} =& \boldsymbol{s}^T[\boldsymbol{M}_t\boldsymbol{a}_r + \boldsymbol{C}_t(\boldsymbol{\nu},\boldsymbol{\eta})\dot{\boldsymbol{x}}_r + \boldsymbol{D}_t(\boldsymbol{\nu},\boldsymbol{\eta})\dot{\boldsymbol{x}} + \boldsymbol{g}_t(\boldsymbol{\eta}) - \boldsymbol{\tau}_t] \\ &+ \tilde{\boldsymbol{x}}^T\boldsymbol{K}(\boldsymbol{s} - \boldsymbol{\Lambda}\tilde{\boldsymbol{x}}) \\ &- (\tilde{\boldsymbol{\theta}})^T\boldsymbol{\Gamma}\boldsymbol{\Gamma}^{-1}\boldsymbol{\Phi}^T(\boldsymbol{J},\boldsymbol{a}_r,\boldsymbol{\nu},\boldsymbol{\eta},\dot{\boldsymbol{x}}_r,\dot{\boldsymbol{x}})\boldsymbol{s} \\ =& -\boldsymbol{s}^T\boldsymbol{K}\boldsymbol{s} - \tilde{\boldsymbol{x}}^T\boldsymbol{K}\boldsymbol{\Lambda}\tilde{\boldsymbol{x}}, \end{aligned} \quad (20)$$

where the actual parameter vector $\boldsymbol{\theta}$ is assumed constant or slowly changing, i.e, $\dot{\boldsymbol{\theta}} = \boldsymbol{0}$.

Thus $\dot{\mathcal{V}} \leq 0$ and the system can then be proved stable using the Barbalat′s Lemma. Since $\mathcal{V}$ is semi-positive definite, $\dot{\mathcal{V}} \leq 0$, and $\dot{\mathcal{V}}$ is uniformly continuous, then $\tilde{\boldsymbol{x}} \to 0, \boldsymbol{s} \to 0$, and $\boldsymbol{x} \to \boldsymbol{x}^d$ as $t \to \infty$.

According to eq.(13) and eq.(12), the Lyapunov candidate under ith reference input is given as

$$V(i) = V_i + V_\theta. \quad (21)$$

Then, by choosing $\boldsymbol{\Lambda}$ large enough, we have

$$\dot{V}(i) \leq -V_i, \quad (22)$$

for ith referent input to the secondary task. Here two complementary cases are considered:

(i) $V_\theta \geq \delta V_i$, then $V_i \leq (V_\theta^U + V_i^U)/\delta$.

(ii) $V_\theta \leq \delta V_i$, then $\dot{V}(i) = \dot{V}_\theta + \dot{V}_i \leq -V_i \leq -\frac{1}{\delta+1}(V_\theta + V_i)$.

Therefore, after $t = (\delta+1)\ln\delta$, we have $V_\theta + V_i \leq \frac{1}{\delta}(V_\theta^U + V_i^U)$, and thus $\tilde{\boldsymbol{x}}_i^T\boldsymbol{K}_i\tilde{\boldsymbol{x}}_i \leq \frac{1}{\delta}(V_\theta^U + V_i^U)$. □

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