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Multiobjective Static Output Feedback Control Design for Vehicle Suspensions*

Haiping DU** and Nong ZHANG**

**Faculty of Engineering, University of Technology, Sydney,
PO Box 123, Broadway, NSW 2007, Australia
E-mail: hdu@eng.uts.edu.au
E-mail: nong.zhang@uts.edu.au

Abstract

This paper presents an approach to design multiobjective static output feedback $H_2/H_\infty/GH_2$ controller for vehicle suspensions by using linear matrix inequalities (LMIs) and genetic algorithms (GAs). A quarter-car model with active suspension system is studied in this paper and three main performance requirements for an advanced vehicle suspension are considered. Among these requirements, the ride comfort performance is optimized by minimizing the H_2 norm from the road disturbance to the sprung mass acceleration, the road holding performance is improved by constraining the H_∞ norm from the road disturbance to the tyre deflection to be less than a given value, and the suspension deflection is guaranteed to be less than its hard limit by constraining the generalized H_2 norm from the road disturbance to the suspension deflection. In addition, the controller gain can be constrained naturally in GAs, which can avoid the actuator saturation problem. A static output feedback controller, which only uses the available sprung velocity and suspension deflection signals as feedback signals, is obtained. This multiobjective controller is realized by using GAs to search for the possible control gain matrix and then to resolve the LMIs together with the minimization optimization problem. The approach is validated by numerical simulation which shows that the designed static output feedback controller can achieve good active suspension performances in spite of its simplicity.

Key words: Multiobjective Control, Active Suspension, Static Output Feedback, Genetic Algorithms, Linear Matrix Inequalities

1. Introduction

Many different performance requirements are often considered by auto makers for an advanced vehicle suspension system. These requirements include ride comfort, handling or road holding capability, and suspension deflection limitation, etc. To meet these conflicting demands, many types of suspension systems, ranging from passive, semi-active to active suspensions, are currently employed and studied. The use of active suspensions has been considered for many years and various approaches have been proposed to improve the performance of active suspensions design [1]. More recently, multiobjective functional (the combination of H_2, H_∞, GL_2, GH_2, etc.) control of vehicle suspensions attracts more attention [2]-[8] because it can reduce the conservatism of the approach that minimizes different performance requirements in a single objective functional (H_2 or H_∞). In particularly, in most multiobjective active suspensions, H_2 or H_∞ norm is often used to specify the ride comfort performance; generalized H_2 (GH_2) norm is used to constrain the suspension deflection and H_∞ norm is used to specify the road holding performance, etc.

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The combination of these performances optimization can emerge as, for example, minimizing $\alpha_1 H_2 + \alpha_2 GH_2$ subject to $H_\infty < \gamma_\infty$, where α_1 and α_2 are positive weighting coefficients, $\gamma_\infty > 0$ is a performance index; minimizing H_∞ (or H_2) subject to hard constrains (e.g., suspension deflection, tyre deflection, actuator saturation, etc.); minimizing H_2 subject to $H_\infty < \gamma_\infty$, etc.

Although the aforementioned multiobjective control strategies can improve the performance and robustness of active suspensions to some extent, there are still some disadvantages. One disadvantage is that the multiobjective control problem is in general a non-convex optimization problem and is very hard to solve. Therefore, mixed control problem, which requires the Lyapunov matrix to be same in all matrix inequalities, is often considered to replace the multiobjective control problem. Therefore, the conservatism is introduced due to the same Lyapunov matrix requirement. The other disadvantage is that only static state feedback control or dynamic output feedback control is studied in the multiobjective control problem by now. The static state feedback control normally requires that all the state variables can be measured. However, for a suspension system, some variables, such as tyre deflection, etc., can not or are not easy to obtain in practice. Hence, observer has to be constructed to obtain the tyre deflection, which often causes difficulties in realistic implementation. On the other hand, the dynamic output feedback control always requires the controller's order to be higher than or as high as the generalized plant, which makes the control system structure complex and increases the expense in hardware realization. Therefore, in order to design active vehicle suspensions for use in realistic situations, the static output feedback controller design approach by only using measurable signals for suspensions, such as suspension deflection, suspension travel velocity, etc., as feedback signals is necessary.

Following previous discussion, this paper will mainly concern with the multiobjective static output feedback controller designs for active vehicle suspension systems. Three main performance requirements for advanced vehicle suspensions (ride comfort, road holding capability, and suspension deflection limitation) are considered by constructing an appropriate static output feedback $H_2 / H_\infty / GH_2$ controller to provide a trade-off between these requirements. Among these requirements, the ride comfort performance is optimized by minimizing the H_2 norm from the road disturbance to the sprung mass acceleration, the road holding performance is improved by constraining the H_∞ norm from the road disturbance to the tyre deflection to be less than a given value, and the suspension deflection is guaranteed by constraining the generalized H_2 norm from the road disturbance to the suspension deflection to be less than a given value. Here, H_∞ norm is used to optimize the tyre deflection because it is good measure of the wheel performance but may not be good measure of the body performance [2]. A feasible solution for such a static output feedback controller is obtained by a new procedure based on linear matrix inequalities (LMIs) and genetic algorithms (GAs) and the control gain can be constrained naturally in GAs, which can avoid the actuator saturation problem. A quarter-car model is used in this study. Numerical simulation shows that the designed static output feedback controller can achieve good active suspension performances in spite of its simplicity.

2. Problem Fomulation

A quarter-car model consists of one-fourth of the body mass, suspension components and one wheel as shown in Figure 1.

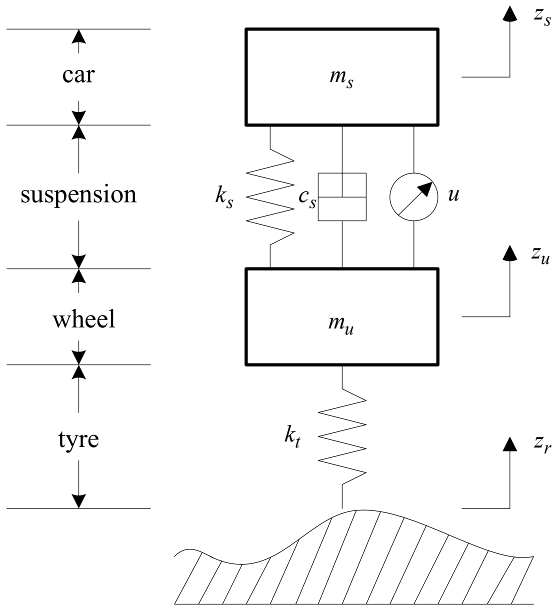

Fig. 1 Quarter-car model with active suspension

The governing equations of motion for the sprung and unsprung masses of the quarter-car model are given by

$$m_s\ddot{z}_s + c_s[\dot{z}_s - \dot{z}_u] + k_s[z_s - z_u] = u, \qquad (1)$$

$$m_u\ddot{z}_u + c_s[\dot{z}_u - \dot{z}_s] + k_s[z_u - z_s] + k_t[z_u - z_r] = -u, \qquad (2)$$

where m_s is the sprung mass, which represents the vehicle chassis; m_u is the unsprung mass, which represents the wheel assembly; c_s and k_s are damping and stiffness of the uncontrolled suspension system, respectively; k_t serves to model the compressibility of the pneumatic tyre; z_s and z_u are the displacements of the sprung and unsprung masses, respectively; z_r is the road displacement input; u represents the active input of the suspension system. Note that z_s, z_u, z_r, u are all time dependent variables, i.e., $z_s(t)$, $z_u(t)$, $z_r(t)$, $u(t)$. For brevity, the time variable t is omitted throughout this paper.

Selecting the state variables as:

$$x_1 = z_s - z_u, \quad x_2 = z_u - z_r, \quad x_3 = \dot{z}_s, \quad x_4 = \dot{z}_u,$$

where x_1 denotes the suspension deflection, x_2 is the tyre deflection, x_3 is the sprung mass speed, x_4 denotes the unsprung mass speed, and defining

$$x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T, \quad w = \dot{z}_r,$$

where w represents the disturbance caused by road roughness, we write (1)-(2) in state-space form as

$$\dot{x} = Ax + B_1 w + B_2 u, \qquad (3)$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -k_s/m_s & 0 & -c_s/m_s & c_s/m_s \\ k_s/m_u & k_t/m_u & c_s/m_u & -c_s/m_u \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 1/m_s \\ -1/m_u \end{bmatrix},$$

are constant matrices.

Three main performance requirements for a vehicle suspension are ride comfort, road holding capability, and suspension deflection limitation. It is confirmed that ride comfort performance is closely related with the vertical acceleration of the car body. Consequently, to improve ride comfort performance amounts to keep the transfer characteristic from road disturbance to car body (sprung mass) acceleration small over the frequency range of 0-65 rad/s [9]. The vehicle handling or road holding capability is related to the tyre deflection, which needs to be small to keep a firm uninterrupted contact of wheels to road. To ensure good road holding, it is also required that the transfer function from road disturbance to tyre deflection $z_u - z_r$ should be small. The structural feature of the vehicle constrains the amount of suspension deflection $z_s - z_u$ with a hard limit. Hitting the deflection limit not only results in the rapid deterioration on the ride comfort, but at the same time increases the wear of the vehicle. Hence, it is also important to keep the transfer function from road disturbance to suspension deflection $z_s - z_u$ small to prevent excessive suspension bottoming.

In accordance with the aforementioned requirements, we formulate a multiobjective $H_2 / H_\infty / GH_2$ control problem to deal with the three different objectives for vehicle suspensions. In order to satisfy performance requirements, the controlled output is composed of $z_1 = \ddot{z}_s$, $z_2 = z_u - z_r$, and $z_3 = z_s - z_u$, respectively, for the quarter-car model. Hence, the vehicle suspensions control system can be described by equation of the form

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z_1 &= C_1 x + D_{12} u \\ z_2 &= C_2 x \\ z_3 &= C_3 x \\ y &= Cx \end{aligned} \tag{4}$$

where y is the measured output,

$$C_1 = \begin{bmatrix} -k_s / m_s & 0 & -c_s / m_s & c_s / m_s \end{bmatrix},$$

$$D_{12} = 1/m_s \; C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \; C_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

In this paper, the multiobjective static output feedback $H_2 / H_\infty / GH_2$ control problem of vehicle suspension is stated as: find the control gain matrix K such that the closed-loop system with control input $u = Ky$ is stable and the performance $\|T_{z_1 w}\|_{H_2}$ is minimized subject to $\|T_{z_2 w}\|_{H_\infty} < \gamma_\infty$ and $\|T_{z_3 w}\|_{GH_2} < \alpha$, where $T_{z_i w}$ denotes the closed-loop transfer function from w to z_i for $i = 1,2,3$; $\gamma_\infty > 0$ and $\alpha > 0$ are performance indices; and the performances, $\|T_{z_1 w}\|_{H_2}$, $\|T_{z_2 w}\|_{H_\infty}$, $\|T_{z_3 w}\|_{GH_2}$ are defined in the next section.

3. Multiobjective Design

In this section, the different performances are expressed by matrix inequalities, and the control gain matrix K is obtained by combining the solution of LMIs and the randomized search of GAs.

3.1 H_2 Performance

The H_2 norm of $T_{z_1 w}$ is defined by [10]

$$\|T_{z_1 w}\|_{H_2} := tr\sqrt{1/2\pi \int_{-\infty}^{+\infty} T_{z_1 w}(j\omega) T_{z_1 w}(j\omega) d\omega},$$

which corresponds to the asymptotic variance of the output z_1 when the system is driven by white noise w. The static output feedback H_2 problem for system (4) is presented by finding matrices $P_2 > 0$, $R > 0$ and control gain matrix K, while realizing the control objective

$\|T_{z_1w}\|_{H_2} < \gamma_2$ for $\gamma_2 > 0$, such that the following inequalities hold

$$\begin{bmatrix} (A+B_2KC)P_2 + P_2(A+B_2KC)^T & B_1 \\ B_1^T & -I \end{bmatrix} < 0 \tag{5}$$

$$\begin{bmatrix} R & (C_1+D_{12}KC)P_2 \\ P_2(C_1+D_{12}KC)^T & P_1 \end{bmatrix} > 0 \tag{6}$$

$$trace\,(R) < \gamma_2 \tag{7}$$

3.2 H_∞ Performance

The H_∞ norm gives the system input-output gain when both the input and the output are measured in the finite energy. The H_∞ norm of T_{z_2w} can be calculated from

$$\|T_{z_2w}\|_{H_\infty} := \sup_\omega \bar{\sigma}\left[T_{z_2w}(j\omega)\right].$$

The static output feedback H_∞ problem for system (4) is presented as to find matrix $P_\infty > 0$ and control gain matrix K, while realizing the control objective $\|T_{z_2w}\|_\infty < \gamma_\infty$ for $\gamma_\infty > 0$, such that the following inequality holds

$$\begin{bmatrix} (A+B_2KC)P_\infty + P_\infty(A+B_2KC)^T & B_1 & P_2C_2^T \\ B_1^T & -\gamma_\infty I & 0 \\ C_2 & 0 & -\gamma_\infty I \end{bmatrix} < 0 \tag{8}$$

3.3 GH_2 Performance

If the input is quantified by its energy and the peak amplitude of the output is kept to a certain level, this leads to the so-called generalized H_2 (GH_2) control problem or energy-to-peak control problem. The GH_2 norm of system T_{z_3w} is defined by [6]

$$\|T_{z_3w}\|_{GH_2} := \lambda_{\max}\sqrt{1/2\pi\int_{-\infty}^{+\infty} T_{z_3w}(j\omega)T_{z_3w}(j\omega)d\omega}\,.$$

The static output feedback GH_2 problem for system (4) is commonly presented as finding matrix $P_{G2} > 0$ and control gain matrix K, while realizing the control objective $\|T_{z_3w}\|_{GH_2} < \alpha$ for $\alpha > 0$, such that the following inequalities hold

$$\begin{bmatrix} P_{G2}(A+B_2KC)^T + (A+B_2KC)P_{G2} & B_1 \\ B_1^T & -I \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} P_{G2} & P_{G2}C_3^T \\ C_3P_{G2} & -\alpha I \end{bmatrix} > 0. \tag{10}$$

3.4 Multiobjective Controller Design via GA

The multiobjective static output feedback $H_2/H_\infty/GH_2$ problem presented in this paper is to find matrices $P_2 > 0$, $R > 0$, $P_\infty > 0$, $P_{G2} > 0$ and control gain matrix K such that $\|T_{z_1w}\|_{H_2}$ is minimized subject to $\|T_{z_2w}\|_{H_\infty} < \gamma_\infty$ and $\|T_{z_3w}\|_{GH_2} < \alpha$. This requires that the inequalities (5)-(10) are satisfied simultaneously. Normally, for inequalities (5)-(10), mixed state feedback control problem (where C should be identity matrix) is to set $P_2 = P_\infty = P_{G2} = P > 0$ and define $Q = KP$, and to find P and Q to satisfy (5)-(10). It is convex optimization problem and can be solved by Matlab LMI toolbox in spite of its conservatism. However, when considering the static output feedback problem, (5)-(10) are bilinear matrix inequalities (BMIs) and cannot be solved by numerically tractable methods. Therefore, genetic algorithm is presented in this paper to find the solutions based on its stochastic search capability. Similar to the approach presented in [11] to design a static output feedback controller based on GA, the multiobjective $H_2/H_\infty/GH_2$ static output feedback controller design problem is resolved by a binary-coded GA approach via the following minimization problem:

$$\min_{K\in\mathbf{K}} \left\|T_{z_1w}\right\|_{H_2} \; s.t. \; \left\|T_{z_2w}\right\|_{H_\infty} < \gamma_\infty \quad and \quad \left\|T_{z_3w}\right\|_{GH_2} < \alpha \tag{11}$$

where $\mathbf{K} := \{K \mid T_{z_iw}(s;K) \;\; is \;\; stable, i = 1,2,3\}$.

The GA based scheme is outlined as below (more detailed explanation about the genetic algorithms can be found in [12] and references therein, and they are omitted here for brevity):

Step 1: Parameter Encoding. The feedback gain matrix K in the search range (space) is converted into a row vector. Each element is then coded as a binary string. Here, we can define the search range (space) in a small range to constrain the control gain, which can be used to avoid the actuator saturation problem naturally.

Step 2: Population Initialization. Randomly generate an initial population of N_p chromosomes.

Step 3: Objective Function Evaluation and Fitness Assignment. Decode the initial population produced in **Step 2** into real values for every controller gain matrix $K_j, j = 1,2,\dots,N_p$. If the closed-loop system with K_j is stable, then determine $\gamma_{2_j} = \min\left\|T_{z_1w}\right\|_{H_2}, j = 1,2,\dots,N_p$ by solving LMIs (5)-(10) (note that inequalities (5)-(10) are LMIs for $R > 0$, $P_2 > 0$, $P_\infty > 0$, $P_{G2} > 0$ once the control gain matrix K_j is known, and these LMIs can be solved by using Matlab LMI toolbox), and take every γ_{2_j} as the objective value corresponding to K_j and associate every K_j with a suitable fitness value according to rank-based fitness assignment approach, and then go to Step 4.

Step 4: Tournament Selection. According to the assigned fitness in **Step 3**, the offspring will be chosen for next crossover and mutation steps by using tournament selection approach.

Step 5: Uniform Crossover. The newly selected chromosomes in the new population are randomly paired together. In each pair of chromosomes, the bits are probabilistically and independently swapped at each bit position with crossover probability p_c to produce new pair of chromosomes (offspring).

Step 6: Bit Mutation. The mutation operation simply flips each bit (changing a 1 to a 0 and vice versa) in the population of chromosomes with a small mutation probability p_m.

Step 7: Elitist Reinsertion. Elitist reinsertion guarantees that the best chromosomes in the population always survives and is retained in the next generation.

Steps 3 to **7** correspond to one generation. The evolution process will repeat for N_g generations or being ended when the search process converges with a given accuracy. The best chromosome is decoded into real values to produce again the control gain matrix K. In this approach, we do not require that $P_2 = P_\infty = P_{G2} = P > 0$, and hence, conservatism of the mixed control problem is reduced.

Remark 1: When the disturbance is zero, i.e., $w = 0$, the closed-loop system is expressed as $\dot{x} = (A + B_2KC)x$. From Lyapunov stability theory we know that the closed-loop system matrix $A + B_2KC$ is stable if and only if K satisfies the matrix inequality $(A + B_2KC)^T P + P(A + B_2KC) < 0$ for some $P > 0$ [13]. Now, P can be P_2, P_∞, or P_{G2}. So we can use the proposed design procedure to find K, and then to guarantee the closed-loop system stability no matter what the initial conditions of the systems are.

Remark 2: The efficiency of the proposed approach will be evaluated by simulations in the next section. Although it is only applied to a quarter-car model in this paper, the approach can be applied to more complicated suspension models [14]. Certainly, the computational complexity and time will be different.

Remark 3: The existence of the static output feedback control gain K for a given system can be checked by the theorem presented in [13]. The proposed approach only tries to find the possibly existing controller gain. However, it does not guarantee to find the solutions all the time without any constraint conditions. To increase the opportunity to find the feasible solutions, two methods can be used. If the approach does not find a candidate solution which can stabilize the closed-loop system at Step 3, a value which is related to the feasibility solution of LMIs can be used as fitness value to this candidate so that it can be evolved to find the feasible solution at last. The second method is to try different parameters setting for GA especially the search range of the controller gain to find a possible solution. Nevertheless, as will be shown in the next section, the feasible solution is easily found for the given example without resorting to these two methods

Remark 4: In practice, there are always parameters uncertainties in the system due to the modeling problem and components aging etc. When these uncertainties exist, the system equation can be expressed as $\dot{x} = (A + \Delta A)x + (B_1 + \Delta B_1)w + (B_2 + \Delta B_2)u$, where $\Delta A, \Delta B_1, \Delta B_2$ are real-valued unknown matrices representing parameter uncertainties and are assumed to be the form of $\Delta A = H_A F E_A, \Delta B_1 = H_1 F E_1, \Delta B_2 = H_2 F E_2$, where $H_A, H_1, H_2, E_A, E_1, E_2$ are known real constant matrices. F is unknown matrix satisfying $F^T F \le I$. Then, the above mentioned three performances can be expressed by LMIs with some derivations and the presented controller design approach can still be applied to find the appropriate static output feedback controller to stabilize the system with required performances. For brevity, these contents will not be studied in this paper.

4. Design Results

In this section, we will apply the proposed approach to design the static output feedback controllers based on the quarter-car model described in Section 2. The quarter-car model parameters have the following values

m_s=504.5 kg, m_u=62 kg, k_s=13100 N/m, c_s=400 Ns/m, k_t=252000 N/m.

For comparison purpose, we first design a full state feedback controller by setting $P_2 = P_\infty = P_{G2} = P > 0$, defining $Q = KP$, and finding P and Q to satisfy LMIs (5)-(10). And for brevity, we denote the active suspension realized by this state feedback controller as Active Suspension I. Then, to show the effectiveness of the presented approach, we take the case that assumes only the suspension deflection $z_s - z_u$ and the velocity of sprung mass $\dot{z}_s$ are measurements available as an example, and we use the approach presented in Section 3 to design the static output feedback controller via LMIs and GA. For brevity, we denote the active suspension realized by this static output feedback controller as Active Suspension II.

The parameters used in the genetic algorithm are selected as: N_p=80, N_g=100, p_c=0.7, p_m=0.02, and the performance indices are set as γ_∞=1, α=0.3. Note that the parameter selections for γ_∞ and α are made by trial and error in terms of the three performances realized by the state feedback control. Hence, the static output feedback control will also use these parameters to make fair comparison. The parameters for GA are selected according to previous experiences and some simulation results. To show the effects of GA parameters on the design results, several examples are given in Figures 2 and 3, which show, for example, the different settings on population size and search range make different effects on the evolution results. It can be seen that GA is really a random algorithm. Generally, it needs to run many times to get a fair result (20 times in this study). Parameter setting on population size does not affect the results too much. Similar observations are found to the parameter settings on crossover probability and mutation probability. For brevity, these results are not shown. The search range will affect the result largely. However, from the practical point of view, the controller gain cannot be given arbitrarily. Also for fair comparison, the search range, which is given as $[-10^4,\ 10^4]$, is set referring to the state feedback control case. The generation size can be selected as a large number, but it will spend more time for the evolution process. Observing the simulation experiments, the evolution results will converge to small values after 50 generations for most cases, so the generation size is selected as 100 which can satisfy the requirement. Even though the different parameter settings may affect the evolution results, it can be seen from Figures 2-3 that the proposed approach is able to find the desired result very efficiently with the given parameter setting. This validates the efficiency of the proposed approach.

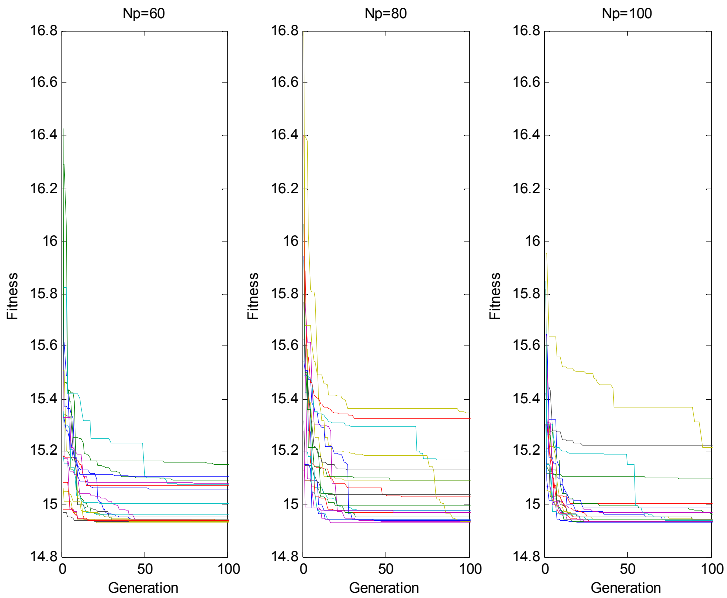

Fig. 2 Evolution process with different population size settings.

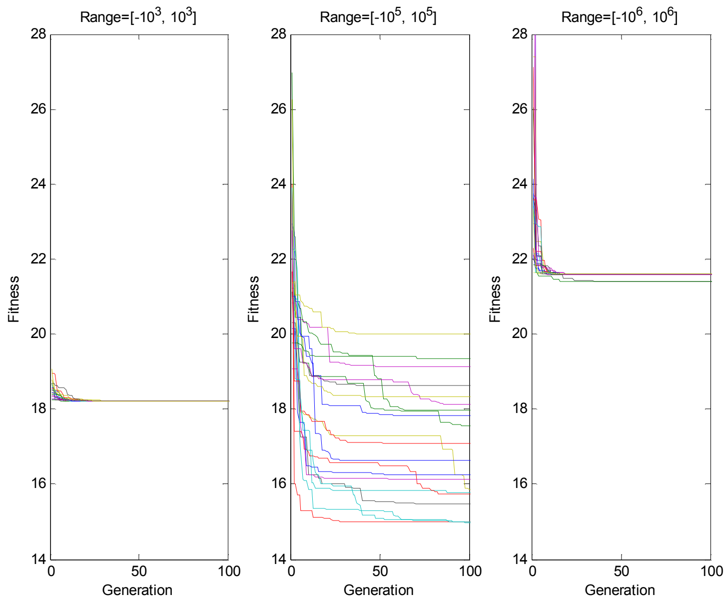

Fig. 3 Evolution process with different parameter search range settings.

Because the ride comfort performance is frequency sensitive and the human body is much sensitive to vertical vibrations in the frequency range 4-8 Hz according to ISO2361, we mainly evaluate the ride comfort performance in frequency domain. The frequency responses (magnitude) for the above designed active suspensions from disturbance to sprung mass acceleration are depicted in Figure 4. For comparison, the frequency response for the passive suspension is plotted in the same figure as well. It can be clearly seen from Figure 4 that the designed active suspensions achieve significant improvement on ride comfort performance for the active suspension systems. The sprung mass accelerations of active suspensions are smaller than the uncontrolled suspension especially in the range of sprung mass resonance. Note that the frequency response of body acceleration is invariant at the second resonance frequency (10.15 Hz) due to no tyre damping being considered in the present model [15]. In spite of the simplicity of active suspension II, active suspension II even realizes a better ride comfort performance than active suspension I. The frequency responses from disturbance to suspension deflection, tyre deflection for both active suspensions and passive suspension are plotted in Figure 5 and Figure 6, respectively. It can be seen that active suspensions improve the suspension deflection and the tyre deflection performances as well compared with passive suspension.

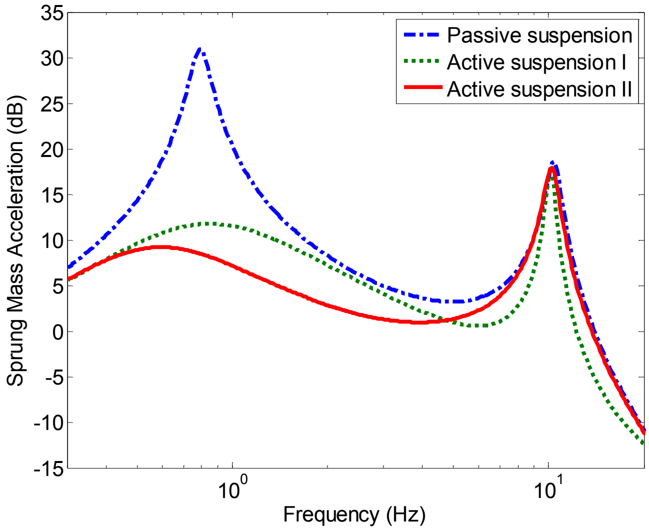

Fig. 4 Frequency response of the sprung mass acceleration.

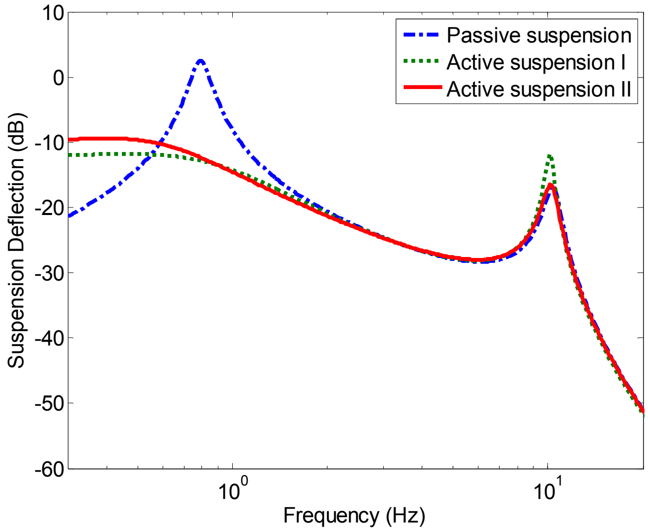

Fig. 5 Frequency response of the suspension deflection.

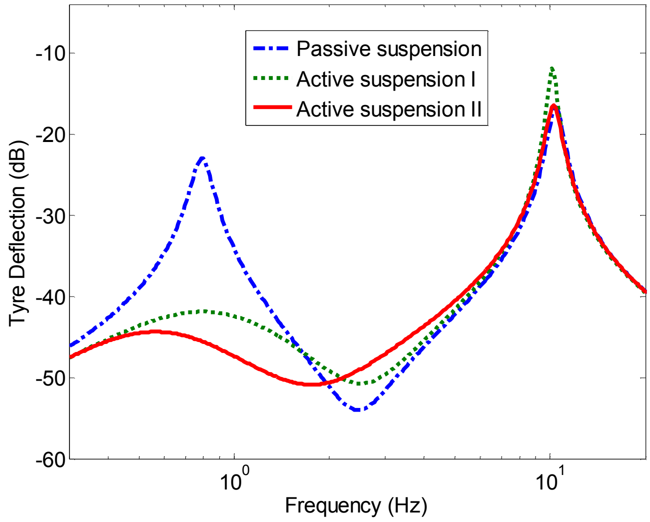

Fig. 6 Frequency response of the tyre deflection.

In order to show the time domain performance of the new designed active suspensions (tyre deflection and suspension deflection are normally regarded as time domain performances), the time responses of sprung mass acceleration, tyre deflection, and suspension deflection for a bump road input are plotted in Figure 7, which shows the sprung mass acceleration, tyre deflection and suspension deflection, respectively, for the active suspensions and the passive suspension; and the active force for the active suspensions. The corresponding displacement for the bump road input is given by

$$z_r(t)=\begin{cases}\frac{A}{2}(1-\cos(\frac{2\pi V}{L}t)), & 0\le t\le \frac{L}{V}\\ 0, & t>\frac{L}{V}\end{cases} \tag{12}$$

where A and L are the height and the length of the bump, and A =0.1m, L =5m and the vehicle forward velocity is V =45 km/h.

It can be seen from Figure 7 that the active suspensions experience smaller body acceleration, tyre deflection and suspension deflection, respectively, than those of passive suspension for the same bump disturbance input. It proves that, in spite of its simplicity, the static output feedback controller realizes the active suspension performances very well.

5. Conclusion

This paper presents a multiobjective static output feedback controller design approach with application to vehicle suspensions. This multiobjective control problem is expressed as minimizing the ride comfort performance (H_2 norm) subjected to the tyre deflection (H_∞ norm) and suspension deflection (GH_2 norm) being constrained to given limitations. Due to the difficulties in resolving such a multiobjective control problem, genetic algorithm is used to search for the final result together with the feasible solution of LMIs. The designed static output feedback controller is more applicable in engineering because it just uses the measurable variables for the suspension system. Numerical simulation validates that the vehicle suspension performances are improved with such a controller in spite of its simple structure.

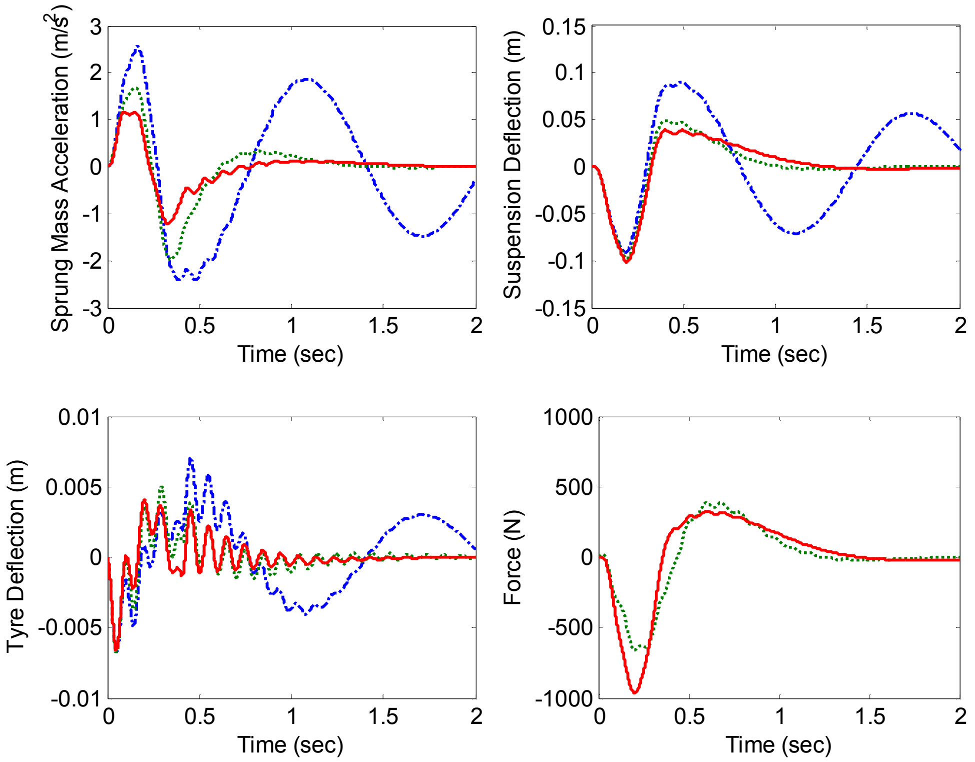

Fig. 7 Bump response: Passive suspension (dot-dash line), Active suspension I (dot line), Active suspension II (solid line).

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