Less-Expensive Pricing and Hedging of Extreme-Maturity Interest Rate Derivatives and Equity Index Options under the Real-World Measure

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Certificate of Original Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as part of the collaborative doctoral degree and/or fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Notation

The following list provides the meaning of symbols and notation used throughout this thesis.

Symbol	Meaning
$\mathcal{A}_t$	Information available at time t written as a σ-algebra
$\underline{\mathcal{A}}$	Filtration or evolution of the flow of information over time
$A^{+}_{\bar{T},K}(t,U)$	Price of an asset binary call option on an underlying asset having price process U with expiry time $\bar{T}$ and strike price K
$A^{-}_{\bar{T},K}(t,U)$	Price of an asset binary put option on an underlying asset having price process U with expiry time $\bar{T}$ and strike price K
AIC	Akaike Information Criterion
$\bar{\alpha}_0$	Initial drift of the discounted GOP
$\bar{\alpha}_t$	Drift of the discounted GOP
$B^{+}_{\bar{T},K}(t,U)$	Price of a bond binary call option on an underlying asset having price process U with expiry time $\bar{T}$ and strike price K
$B^{-}_{\bar{T},K}(t,U)$	Price of a bond binary put option on an underlying asset having price process U with expiry time $\bar{T}$ and strike price K
B_t	Value of savings account at time t
BA	Benchmark approach
BS	Black-Scholes
$c_{\bar{T},K}(t,U)$	Price of a call option on an underlying asset having price process U with expiry time $\bar{T}$ and strike price K
$\mathbf{cap}_{\mathcal{T},\mathcal{K}}(t)$	Price of a cap in respect of a start date and subsequent payment dates in the set $\mathcal{T} = \{T_0, T_1, \ldots, T_n\}$
$\mathbf{caplet}_{\bar{T},T,K}(t)$	Price of a caplet with start time $\bar{T}$ and end time T

Symbol	Meaning
CEV	Constant elasticity of variance
CIR	Cox-Ingersoll-Ross
δ_*	Strategy associated with the growth optimal portfolio
$\mathrm{E}(U)$	Expectation of a random variable U
$\mathrm{E}(U \mid \mathcal{A}_t)$	Expectation of a random variable U given information available at time t
$f_T(t)$	Instantaneous T-forward rate at time t
$f_\infty(t)$	Asymptotic forward rate as at time t
$F_{\bar{T},T}(t)$	Discrete $[\bar{T}, T]$-forward rate at time t
$\mathbf{floor}_{\mathcal{T},\mathcal{K}}(t)$	Price of a floor in respect of a start date and subsequent payment dates in the set $\mathcal{T} = \{T_0, T_1, \ldots, T_n\}$
$\mathbf{floorlet}_{\bar{T},T,K}(t)$	Price of a floorlet with start time $\bar{T}$ and end time T
$F_T(t, U)$	T-Forward price of an asset having price process U
FRA	Forward rate agreement
GOP	Growth optimal portfolio
$g_T(t)$	Contribution of the short rate to the instantaneous T-forward rate at time t
$g_\infty(t)$	Contribution of the short rate to the asymptotic instantaneous forward rate as at time t
$G_T(t)$	Contribution of the short rate to the T-maturity zero-coupon bond price
$G(\alpha, \gamma)$	Gamma distribution with shape parameter α and scale parameter γ
$G(x; \alpha, \gamma)$	Cumulative distribution function of the gamma distribution
GBM	Geometric Brownian motion
GIG	Generalised inverse Gaussian
GMMM	Generalised minimal market model
$\Gamma(x)$	Gamma function of x given by $\int_0^\infty u^{x-1} \mathrm{e}^{-u}\, du$
$h_T(t)$	Contribution of the short rate to the T-maturity zero-coupon bond yield at time t
$h_\infty(t)$	Contribution of the short rate to the long zero-coupon bond yield as at time t
$\mathbf{1}_X$	Indicator function, equalling 1 if the statement X is true and 0 otherwise
$\mathcal{I}$	Fisher's information matrix
$I_\nu(x)$	Modified Bessel function of the first kind with index ν
η	Net market growth rate of the GOP
κ	Speed of mean reversion associated with a short rate model
K	Strike price of an option
$K_\lambda(\omega)$	Modified Bessel function of the third kind with index λ

Symbol	Meaning
$\ell(\Theta)$	Logarithm of likelihood function of model parameters Θ
$L(\Theta)$	Likelihood function of model parameters Θ
LHS	Left hand side
$LN(\mu, \sigma)$	Lognormal distribution with location parameter μ and scale parameter σ
$LN(x; \mu, \sigma)$	Cumulative distribution function of the lognormal distribution
$m_T(t)$	Contribution of the discounted GOP to the T-forward rate at time t
$M_T(t)$	Contribution of the discounted GOP to the T-maturity zero-coupon bond price
$M(\alpha, \gamma, z)$	Confluent hypergeometric function
MGF	Moment generating function
$MGF_X(t)$	Moment generating function of the random variable X
MLE	Maximum likelihood estimate
MMM	Minimal market model
MSCI	Morgan Stanley Capital International
$n(x)$	Probability density function for the standard normal distribution
$N(x)$	Cumulative distribution function for the standard normal distribution
$N(\mu, \sigma^2)$	Normal distribution having mean μ and variance σ^2
$n_T(t)$	Contribution of the discounted GOP to the T-maturity zero-coupon bond yield at time t
$NCG(\alpha, \gamma, \lambda)$	Non-central gamma distribution having scale parameter γ, shape parameter α and non-centrality parameter λ
$NCG(x; \alpha, \gamma, \lambda)$	Cumulative distribution function of the non-central gamma distribution
OTC	Over the counter
$p_{\bar{T},K}(t, U)$	Price of a put option on an underlying asset having price process U with expiry time $\bar{T}$ and strike price K
$P(t, T)$	Price of T-maturity zero-coupon bond at time t
$P_{\mathcal{T},c}(t)$	Price at time t of a coupon bond having unit notional, coupon rate c and most recent coupon payment date and subsequent coupon payment dates in the set $\mathcal{T} = \{T_0, T_1, \ldots, T_n\}$
$\mathbf{payerswaption}_{\mathcal{T},K,N}(t)$	Price at time t of a payer swaption having strike rate K and underlying swap with notional N and start date (same as expiry date of swaption) and subsequent payment dates in the set $\mathcal{T} = \{T_0, T_1, \ldots, T_n\}$
PDE	Partial differential equation
$Poi(\lambda)$	Poisson distribution having rate parameter λ
QV	Quadratic variation
$[X]_t$	Quadratic variation of a stochastic process X

Symbol	Meaning
r	Short rate
$\bar{r}$	Level of mean reversion associated with a short rate model
$\mathbf{receiverswaption}_{\mathcal{T},K,N}(t)$	Price at time t of a receiver swaption having strike rate K and underlying swap with notional N and start date (same as expiry date of swaption) and subsequent payment dates in the set $\mathcal{T} = \{T_0, T_1, \ldots, T_n\}$
$\mathbb{R}$	Real numbers
RHS	Right hand side
$\mathbf{swaprate}_{\mathcal{T}}(t)$	Swap rate at time t in respect of a start date and subsequent payment dates in the set $\mathcal{T} = \{T_0, T_1, \ldots, T_n\}$
s, t	Time
σ	Diffusion parameter of a short rate process
$S_t^{\delta_*}$	Value of the growth optimal portfolio at time t
$\bar{S}_t^{\delta_*}$	Discounted value of the growth optimal portfolio at time t
S&P 500	Standard and Poor's 500 equity index
SDE	Stochastic differential equation
SGH	Symmetric generalised hyperbolic
Skew(U)	Skew of a random variable U
SE($\hat{p}$)	Standard error of an estimate of the parameter p
T, $\bar{T}$	Time of option expiry or bond maturity
θ	Volatility of the discounted GOP
USD	United States Dollar
Var(U)	Variance of a random variable U
Var($U\|\mathcal{A}_t$)	Variance of a random variable U given information available at time t
VaR	Value at risk
W	Wiener process driving the discounted GOP
WSI	Diversified world stock index
χ^2_ν	Chi-squared distribution with ν degrees of freedom
$\chi^2_\nu(x)$	Cumulative distribution function for the chi-squared distribution with ν degrees of freedom
$\chi^2_{\nu,\lambda}$	Non-central chi-squared distribution with ν degrees of freedom and non-centrality parameter λ
$\chi^2_{\nu,\lambda}(x)$	Cumulative distribution function for the non-central chi-squared distribution with ν degrees of freedom and non-centrality parameter λ
$y_T(t)$	Continuously compounded T-maturity zero-coupon bond yield at time t
$y_\infty(t)$	Continuously compounded long zero-coupon bond yield as at time t

Symbol	Meaning
Z	Wiener process driving the short rate
ZCB	Zero-coupon bond
$\mathbf{zcbcall}_{\bar{T},T,K}$	Price of $\bar{T}$-expiry call option on a T-maturity zero-coupon bond with strike price K
$\mathbf{zcbput}_{\bar{T},T,K}$	Price of $\bar{T}$-expiry put option on a T-maturity zero-coupon bond with strike price K
$(x)_n$	Pockhammer function which is shorthand for the product $x\,(x+1)\,(x+2)\,\ldots\,(x+n-1)$, where n is a non-negative integer and we use the convention that the empty product is one

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Abstract

This thesis is practically oriented towards the pricing and hedging of long-dated interest rate derivatives and equity index options under Platen's benchmark approach. It aims to be self-contained for convenience of the reader, including all proofs. Among leading banks and insurance companies there does not appear to exist a generally accepted methodology of accurately pricing and hedging such over-the-counter derivatives. This remains the case, despite significant efforts by academics and market practitioners since the early 1990s. This thesis revisits this problem in the light of empirical evidence in a much wider modelling framework than that provided by the classical risk neutral approach.

The models considered in this thesis are specified by stochastic differential equations that describe the real-world dynamics of two market variables, namely the short rate and the volatility of the growth optimal portfolio (GOP). The latter is essentially a diversified equity index.

This thesis assesses for these models their ability to generate reasonably accurate prices and hedges of typical interest rate term structure derivatives and equity index options. When the discounted GOP is modelled as a time-transformed squared Bessel process, fair prices differ from classical risk neutral prices, resulting in lower prices and lower values-at-risk of long-dated derivatives. Also, such models reflect well empirical market features, such as leptokurtic returns, the leverage effect and a stochastic, yet stationary, volatility structure of the equity index.

The results of this analysis, which are contained in this thesis, have been supplemented by the publications of Fergusson and Platen [2006], Fergusson and Platen [2014a], Fergusson and Platen [2015b], Fergusson [2017a] and Fergusson [2017b] and the research reports of Fergusson and Platen [2013], Fergusson and Platen [2014b] and Fergusson and Platen [2015a]. In addition, as by-products of the work done in this thesis, the following papers have been published: Thompson et al. [2017], Calderin et al. [2017] and the following have been submitted to journals for publication: Fergusson and Platen [2014c], Fergusson and Platen [2017]. Finally, the following working papers are to be submitted to journals shortly: Fergusson [2017c], Fergusson [2017d], Fergusson [2017e].