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34980 PhD Thesis: Mathematics

Subgradient and duality methods for optimal stochastic control

With applications to real options valuation

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Certificate of original authorship

I, Jeremy Yee, declare that this thesis, is submitted in fulfilment of the requirements for the award of the Doctor of Philosophy, in the School of Mathematical and Physical Sciences at the University of Technology Sydney. This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis. This thesis is the result of a research candidature conducted with CSIRO as part of a collaborative Doctoral degree. This document has not been submitted for qualifications at any other academic institution. This research is supported by the Australian Government Research Training Program.

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Abstract

This thesis presents a subgradient and duality approach towards solving an important class of Markov decision processes. The key assumptions lie in the linear state dynamics and in the convexity of the functions in the Bellman recursion. Approximations of the value functions are then constructed using convex piecewise linear functions formed using operations on tangents. This approach can be efficiently implemented with most of the computational effort reduced to simple matrix additions and multiplications. The quality of the approximations can then be gauged using pathwise duality methods which return confidence intervals for the true unknown value. This thesis will then explore the use of nearest neighbour algorithms in reducing the computational effort. Numerical experiments demonstrate that the subgradient and duality approach returns tight confidence intervals indicating good quality results. These methods are tested on a wide range of applications such as financial option pricing, optimal liquidation problems, battery control, and natural resource extraction. Extension to uncontrolled hidden Markov models is also considered. These methods have all been implemented in a *R* package with scripts posted online to reproduce most of the numerical results within this thesis.

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