# Comments on "Cross-Tier Cooperation for Optimal Resource Utilization in Ultra-Dense Heterogeneous Networks"

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***Abstract*—Two adaptive dedicated channel allocation algorithms, namely dynamic dedicated channel partitioning ($D^2CP$) and dynamic dedicated channel partitioning with cooperation ($D^2CP{-}C$), were proposed in [1] to improve the system throughput of ultra-dense networks (UDN). However, due to the incorrect use of the Geometric-Arithmetic Mean Inequality theorem, the average system throughput could not be guaranteed to be optimal. In this letter, we study the proposed $D^2CP$ and $D^2CP{-}C$ algorithms in UDN and deduce the average system throughput. Consequently, we prove that the equal resource allocation strategy proposed in [1] is strictly not optimal.**

***Index Terms*—Ultra-dense networks (UDN), cross-tier cooperation, dedicated channel, system throughput**

## I. Introduction

We appreciate the authors of [1] for citing our paper [2]. Actually, we use the work of Prof. Wanjiun Liao's for reference frequently. Moreover, we admire her excellent contributions in the IEEE communication society. However, in [1], the authors misused the Geometric-Arithmetic Mean Inequality theorem unfortunately, which eventually led to misunderstandings in the analysis and simulation. In this letter, we prove that the optimal system throughput could not always be acquired when each user has the equal throughput in all the cases studied in [1]. Hence, the resource allocation strategy proposed in [1] could not be treated as the optimal one. Nevertheless, from the perspective of Max-Min fairness (MMF) [3], which is a widely used criterion for keeping fairness between users, the users are allocated with the same throughput to maximize the minimum user throughput. Generally, MMF is applicable in situations where it is desirable to achieve an equitable distribution of certain resources, shared by competing demands. The name "max-min" comes from the idea of making the rate of the smallest (or minimum) flow as large as possible (maximized). Thus, we give higher relative priority to the smaller flows. Therefore, when adopting the MMF criterion (i.e., resources are allocated in a manner to maximize the minimum throughput of the users), all theorems deduced in [1] will hold.

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TABLE I
Table of Notations in [1]

| Symbol | Description |
|---|---|
| $\tau_i$ | Throughput of user i |
| $\tau_{SLA}$ | Expected throughput of a user |
| $N_M$, $N_P$, $N_S$ | Number of macro-only, potential and small cell users |
| $N_T$ | Number of total users ($N_T{=}N_M{+}N_P{+}N_S$) |
| $S$, $E_T$ | Number of small cells and total resource blocks |
| $\rho_M$, $\rho_S$ | Average spectrum efficiency for a user in the macro cell and in the small cell |
| $\rho_{Coop}$ | Average spectrum efficiency for a user served by cross-tier cooperation |
| $\tau_{D^2CP}$ | Average system throughput of $D^2CP$ in Case 1 |
| $\tau_{D^2CP-C}$ | Average system throughput of $D^2CP{-}C$ in Case 1 |
| $\tau''_{D^2CP}$ | Average system throughput of $D^2CP$ in Case 3 |
| $\tau''_{D^2CP-C}$ | Average system throughput of $D^2CP{-}C$ in Case 3 |
| $E_{Max}^{D^2CP}$ | Number of maximal occupied resource blocks among small cells of $D^2CP$ in Case 3 |
| $E_{Max}^{D^2CP-C}$ | Number of maximal occupied resource blocks among small cells of $D^2CP{-}C$ in Case 3 |
| $\beta$, $1-\beta$ | Portion of resource blocks for macro-only users and potential users |

## II. Problem Description

In the beginning, we give the correct form of the Geometric-Arithmetic Mean Inequality theorem below. Moreover, a comprehensive proof was given in [4].

**Theorem 1** (Geometric-Arithmetic Mean Inequality)**.** *For $n$ arbitrary positive numbers $a_1, a_2, ..., a_n$, we have $A_n = \frac{1}{n}\sum_{i=1}^{n} a_i \geq \sqrt[n]{\prod_{i=1}^{n} a_i} = G_n$, and equality holds if and only if $a_1 = a_2 = ... = a_n$.*

Therefore, Theorem 6 in [1] must be rewritten as

$$\frac{1}{N_T}\sum_{\forall i}\tau_i \geq \sqrt[N_T]{\prod_{\forall i}\tau_i}. \tag{1}$$

With inequality reversed, the Geometric-Arithmetic Mean Inequality theorem was incorrectly used in [1]. As a result, the authors believed that the average system throughput would be maximal when every user throughput equals ($\tau_1 = \tau_2 = ... = \tau_{N_T}$). Unfortunately, this conclusion does not work well in most cases.

Here, we try to prove that the average system throughput cannot approach the maximum value with the two proposed algorithms, $D^2CP$ and $D^2CP{-}C$. As has been verified in [1], the average system throughput of $D^2CP$ in Case 2 is the same as the one in Case 1. So is the average system throughput of $D^2CP{-}C$. Thus, in the subsequent parts, we just give the detailed deductions of the non-optimality in Case 1 and Case 3.

### A. Case 1: Macro Cell Underutilization

$$(N_M + N_P)\frac{\tau_{SLA}}{\rho_M} < \frac{E_T}{2}$$

#### 1) $D^2CP$:

Assume $\tau_1 = \alpha\overline{\tau}$ $(0 \leq \alpha < 1)$, $\underbrace{\tau_1 = ... = \tau_{N_M+N_P}}_{N_M+N_P} < \overline{\tau} < \underbrace{\tau_{N_M+N_P+1} = ... = \tau_{N_T}}_{N_S}$, where $\overline{\tau} = \frac{1}{N_T}\sum_{i=1}^{N_T}\tau_i$ is the average system throughput and $\alpha$ is the portion factor.

Obviously,

$$\tau_{N_T} = \overline{\tau} + \frac{(\overline{\tau} - \alpha\overline{\tau})\,(N_M + N_P)}{N_S}. \tag{2}$$

When $S\rho_S > \rho_M$, we have

$$E_T = (N_M + N_P)\,\frac{\alpha\overline{\tau}}{\rho_M} + \frac{N_S\tau_{N_T}}{S\rho_S}. \tag{3}$$

Thus,

$$\overline{\tau} = \frac{E_T}{\frac{N_S}{S\rho_S} + (N_M + N_P)\left(\frac{\alpha}{\rho_M} + \frac{1-\alpha}{S\rho_S}\right)} > \tau_{D^2CP}, \tag{4}$$

where $\tau_{D^2CP} = \frac{E_T S\rho_S\rho_M}{N_S\rho_M + N_M S\rho_S + N_P S\rho_S}$ [1]. Given this, $\tau_{D^2CP}$ is strictly not the optimal.

#### 2) $D^2CP{-}C$:

Assume $\tau_1 = \alpha\overline{\tau}$ $(0 \leq \alpha < 1)$, $\underbrace{\tau_1 = ... = \tau_{N_M+N_P}}_{N_M+N_P} < \overline{\tau} < \underbrace{\tau_{N_M+N_P+1} = ... = \tau_{N_T}}_{N_S}$.

Then

$$\tau_{N_T} = \overline{\tau} + \frac{(\overline{\tau} - \alpha\overline{\tau})\,(N_M + N_P)}{N_S}. \tag{5}$$

Consider $\frac{N_M}{\rho_M} + \frac{N_P}{\rho_{Coop}} > \frac{N_M+N_P}{S\rho_S}$, we have

$$E_T = N_M\frac{\tau_1}{\rho_M} + N_P\frac{\tau_{N_M+N_P}}{\rho_{Coop}} + \frac{N_S\tau_{N_T}}{S\rho_S}. \tag{6}$$

Hence,

$$\overline{\tau} = \frac{E_T}{\frac{\alpha N_M}{\rho_M} + \frac{\alpha N_P}{\rho_{Coop}} + \frac{(N_S + (1-\alpha)(N_M+N_P))}{S\rho_S}} > \tau_{D^2CP-C}, \tag{7}$$

where $\tau_{D^2CP-C} = \frac{E_T S\rho_S}{N_S + N_M S\frac{\rho_S}{\rho_M} + N_P S\frac{\rho_S}{\rho_{Coop}}}$ [1]. Therefore, $\tau_{D^2CP-C}$ is strictly not the maximal average system throughput.

### B. Case 3: Macro Cell Serious Overutilization

$$(N_M + N_P)\frac{\tau_{SLA}}{\rho_M} > E_T$$

In order to prevent the number of resource blocks used in the macro cell from exceeding the maximum, we use $E^{D^2CP}$ and $E^{D^2CP-C}$ to denote the numbers of maximal resource blocks used in small cells for $D^2CP$ and $D^2CP{-}C$, respectively.

#### 1) $D^2CP$:

Assume $\underbrace{\tau_1 = ... = \tau_{N_S}}_{N_S} < \overline{\tau} < \underbrace{\tau_{N_S+1} = ... = \tau_{N_T}}_{N_M+N_P}$.

Thus,

$$E^{D^2CP} < E^{D^2CP}_{Max}. \tag{8}$$

When $S\rho_S < \rho_M$, the average system throughput can be obtained by

$$\overline{\tau} = \frac{1}{N_T}\left[E^{D^2CP}S\rho_S + \left(E_T - E^{D^2CP}\right)\rho_M\right] > \tau''_{D^2CP}, \tag{9}$$

where $\tau''_{D^2CP} = \frac{1}{N_T}[(E_T - E^{D^2CP}_{Max})\rho_M + E^{D^2CP}_{Max}S\rho_S]$ [1]. Therefore, we can deduce that $\tau''_{D^2CP}$ is strictly not the maximal average system throughput. Particularly, as shown in Fig.10 of [1], macro-only users and potential users occupied almost all resource blocks in Case 3. Thus, the resource blocks could hardly be allocated to small cell users. For this reason, the "equality" could not hold between the macro and small cell users.

#### 2) $D^2CP{-}C$:

Assume $\underbrace{\tau_1 = ... = \tau_{N_S}}_{N_S} < \overline{\tau} < \underbrace{\tau_{N_S+1} = ... = \tau_{N_T}}_{N_M+N_P}$.

Then

$$E^{D^2CP-C} < E^{D^2CP-C}_{Max}. \tag{10}$$

Consider $\frac{N_P}{\rho_{Coop}} + \frac{N_M}{\rho_M} < \frac{N_M+N_P}{S\rho_S}$, the system throughput can be obtained by

$$\begin{aligned}\overline{\tau} = &\tfrac{1}{N_T}[E^{D^2CP-C}S\rho_S + \left(E_T - E^{D^2CP-C}\right)\beta\rho_M \\ &+ \left(E_T - E^{D^2CP-C}\right)(1-\beta)\,\rho_{Coop}] > \tau''_{D^2CP-C},\end{aligned} \tag{11}$$

where $\tau''_{D^2CP-C} = \frac{1}{N_T}[(E_T - E^{D^2CP-C}_{Max})\beta\rho_M + (E_T - E^{D^2CP-C}_{Max})(1-\beta)\rho_{Coop} + E^{D^2CP-C}_{Max}S\rho_S]$ [1]. In this way, we prove that $\tau''_{D^2CP-C}$ is strictly not the optimal.

## III. Conclusion

Overall, after the above discussions, we have come to the conclusion that the average system throughput is strictly not the maximum one when the throughput of each user equals. Thus, the resource allocation strategy proposed in [1] has been proved not optimal. Moreover, the resource block allocation for achieving the optimal average system throughput in the cross-tier cooperation system could be viewed as an equivalent NP-hard problem, and no straightforward method could be utilized to solve it.

In addition, we also pay some efforts on fixing the flaws to improve the system throughput under the system model in [1]. Considering users suffer from the variant interference from randomly deployed small cells, we can model this problem as a nonconvex resource allocation optimization [5]. Further, we could decompose it into separate subcarrier allocation and power allocation steps to reduce complexity. In the first step, the dynamic subcarrier allocation scheme proposed in [6], which takes both maximum signal to interference plus noise ratio and fairness into consideration, could be extended to realize sub-optimal subcarrier allocation. In the second step, two problems in the power allocation should be settled. 1) Since the number of users in each small cell is random, we need to estimate the fluctuating co-tier interference. 2) Considering the cross-tier cooperation involved in the system model, we need to develop an appropriate power allocation scheme to balance between the improved spectrum utilization and the correlated severe interference.

However, we should emphasize that all theorems acquired in [1] hold when the MMF criterion is assumed.

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