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Research Article

Improved Results on Fuzzy $H\infty$ Filter Design for T-S Fuzzy Systems

Jiyao An,[1,2] Guilin Wen,[1] and Wei Xu[3]

[1] *State Key Laboratory of Advanced Design and Manufacture for Vehicle Body, Institute of Space Technology, Hunan University, Changsha 410082, China*
[2] *School of Computer and Communication, Hunan University, Changsha 410082, China*
[3] *Faculty of Engineering and Information Technology, University of Technology Sydney, NSW 2007 Sydney, Australia*

Correspondence should be addressed to Guilin Wen, wenguilin@yahoo.com.cn

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The fuzzy $H\infty$ filter design problem for T-S fuzzy systems with interval time-varying delay is investigated. The delay is considered as the time-varying delay being either differentiable uniformly bounded with delay derivative in bounded interval or fast varying (with no restrictions on the delay derivative). A novel Lyapunov-Krasovskii functional is employed and a tighter upper bound of its derivative is obtained. The resulting criterion thus has advantages over the existing ones since we estimate the upper bound of the derivative of Lyapunov-Krasovskii functional without ignoring some useful terms. A fuzzy $H\infty$ filter is designed to ensure that the filter error system is asymptotically stable and has a prescribed $H\infty$ performance level. An improved delay-derivative-dependent condition for the existence of such a filter is derived in the form of linear matrix inequalities (LMIs). Finally, numerical examples are given to show the effectiveness of the proposed method.

1. Introduction

During the last decades, the filtering problem has attracted many researchers to study through various methodologies, see, for example, [1–20] and the references therein, in which these methods mostly consist of two main approaches, namely, the Kalman filtering approach [1–3] and the $H\infty$ filtering approach [4–17]. In contrast with the Kalman filtering, the $H\infty$ filtering approach does not require the exact knowledge of the statistics of the external noise signals and it is insensitive to the uncertainties both in the exogenous signa statistics and in dynamic models. This advantage renders the $H\infty$ filtering approach very appropriate to some practical applications. Recently, the filter design contains two cases of filtering technique, that is, $L_2 - L_\infty$ filtering technique [18–20] and the $H\infty$ filtering technique [4–17].

On the other hand, Takagi-Sugeno (T-S) fuzzy model can provide an effective way to represent a complex nonlinear system into a weighted sum of some simple linear subsystems [8, 21, 22], which has been an increasing interest in the study of T-S fuzzy systems. In recent years, T-S fuzzy model approach has been extended to $H\infty$ filter or controller design [4–6, 9, 10, 12, 15–21, 23–35]. For instance, the stability analysis and stabilization synthesis problems of T-S fuzzy systems were studied in [21, 29, 30, 33–35], while fuzzy controllers were designed in [23–28]. One set of fuzzy $H\infty$ filters for a class of T-S fuzzy systems was designed in [32]. However, the above-mentioned works use common Lyapunov-Krasovskii functional, and the results under a common Lyapunov method are quite conservative. To reduce the conservatism, a fuzzy weighting-dependent Lyapunov method has been proposed in [6], which is effective in reducing conservatism of previous results on fuzzy systems. More recently, Lin et al. [4] and Su et al. [5] have concerned with $H\infty$ filtering of nonlinear continuous-time state-space models with time-varying delays via T-S fuzzy model approach. However, some negative semidefinite terms are ignored and the lower bound of time delay is restricted to be zero, see, for example, [4–6] and the references therein. Qiu et al. [36] investigated the problem of delay-dependent robust stability and $H\infty$ filtering design for a class of uncertain continuous-time nonlinear systems with time-varying state delay represented by T-S fuzzy models. However, there is room for further investigation to reduce the conservativeness of the filter design. This motivates the current research.

In this paper, we discuss the fuzzy $H\infty$ filter design problem for T-S fuzzy systems with interval time-varying delay. Our aim is to design a suitable fuzzy filter, which ensures both the fuzzy stability and a prescribed performance level of the filter error system. By constructing a Lyapunov-Krasovskii functional, estimating the time derivative of the Lyapunov-Krasovskii functional less conservatively, and adopting convex optimization approach, an improved delay-derivative-dependent condition for the solvability of fuzzy $H\infty$ filter design problem is proposed in terms of linear matrix inequalities (LMIs). Two examples are used to compare with the previous literatures and demonstrate the effectiveness of the proposed method.

The rest of this paper is organized as follows: The fuzzy $H\infty$ filtering problem is formulated in Section 2; the fuzzy $H\infty$ performance analysis is derived in Section 3; and fuzzy $H\infty$ filter design is addressed in Section 4. Numerical examples are provided in Section 5, and Section 6 concludes this paper.

2. Problem Formulation

Consider a nonlinear system with interval time-varying delay which could be approximated by a class of T-S fuzzy systems with interval time-varying delays. The T-S fuzzy model with r plant rules can be described by:

Plant rule i: IF $\theta_1(t)$ is N_{i1} and$\cdots$and $\theta_p(t)$ is N_{ip}, **THEN**

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + A_{\tau i} x(t-\tau(t)) + B_i w(t), \\ y(t) &= C_i x(t) + C_{\tau i} x(t-\tau(t)) + D_i w(t), \\ z(t) &= L_i x(t) + L_{\tau i} x(t-\tau(t)) + G_i w(t), \\ x(t) &= \phi(t), \quad \forall t \in [-h_b, 0], \end{aligned} \tag{2.1}$$

where $0 \le h_a \le \tau(t) \le h_b$, and $x(t) \in \mathbb{R}^n$ is the state vector; $y(t) \in \mathbb{R}^m$ is the measurements vector; $w(t) \in \mathbb{R}^q$ is the disturbance signal vector which belongs to $L_2[0,\infty)$; $z(t) \in \mathbb{R}^p$ is the signal vector to be estimated; $\phi(t)$ is the continuous initial vector function defined on $[-h_b, 0]$; The system coefficient matrices are constant real matrices with appropriate dimensions, where $i = 1, 2, \ldots, r$ and r is the number of IF-THEN rules; $\theta_j(t), (j = 1, 2, \ldots, p)$ are the premise variables; $N_{i1}, N_{i2}, \ldots, N_{ip}$ are the fuzzy sets. For the sake of convenience, we denote $\delta_h = h_b - h_a$.

The time-varying delay $\tau(t)$ is assumed to be either differentiable with

$$d_1 \le \dot{\tau}(t) \le d_2, \tag{2.2}$$

where d_1 and d_2 are given bounds, or fast-varying (with no restrictions on the delay derivative).

The fuzzy system (2.1) is supposed to have singleton fuzzifier, product inference and centroid defuzzifier. The final output of the fuzzy system is inferred as follows:

$$\begin{gathered}
\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))[A_i x(t) + A_{\tau i} x(t-\tau(t)) + B_i w(t)], \\
y(t) = \sum_{i=1}^{r} h_i(\theta(t))[C_i x(t) + C_{\tau i} x(t-\tau(t)) + D_i w(t)], \\
z(t) = \sum_{i=1}^{r} h_i(\theta(t))[L_i x(t) + L_{\tau i} x(t-\tau(t)) + G_i w(t)], \\
x(t) = \phi(t), \quad \forall t \in [-h_b, 0],
\end{gathered} \tag{2.3}$$

where for $i = 1, 2, \ldots, r$,

$$h_i(\theta(t)) = \frac{\mu_i(\theta(t))}{\sum_{i=1}^{r} \mu_i(\theta(t))}, \qquad \mu_i(\theta(t)) = \prod_{j=1}^{p} N_{ij}(\theta_j(t)) \tag{2.4}$$

and $N_{ij}(\theta_j(t))$ is the membership function of $\theta_j(t)$ in N_{ij}. Here $\mu_i(\theta(t)) \ge 0$. Here, we assume that $\mu_i(\theta(t)) > 0$, and $\sum_{i=1}^{r} h_i(\theta(t)) = 1$.

Our aim is to design the following fuzzy filter.

Rule i: IF $\theta_1(t)$ is N_{i1} and$\cdots$and $\theta_p(t)$ is N_{ip}, **THEN**

$$\begin{aligned}
\dot{\hat{x}}(t) &= A_{fi}\hat{x}(t) + B_{fi}y(t), \qquad \hat{x}(0) = 0, \\
\hat{z}(t) &= C_{fi}\hat{x}(t) + D_{fi}y(t),
\end{aligned} \qquad (i = 1, 2, \ldots, r), \tag{2.5}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the filter state, $\hat{z}(t) \in \mathbb{R}^p$ is the estimation of $z(t)$ in fuzzy system (2.1), the constant matrices $A_{fi} \in \mathbb{R}^{n\times n}$, $B_{fi} \in \mathbb{R}^{n\times m}$, $C_{fi} \in \mathbb{R}^{p\times n}$, $D_{fi} \in \mathbb{R}^{p\times m}$ are the filter matrices to be determined. The final fuzzy filter of fuzzy system (2.1) is thus inferred as follows

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i(\theta(t))\left[A_{fi}\hat{x}(t) + B_{fi}y(t)\right], \qquad \hat{x}(0) = 0,\\ \hat{z}(t) &= \sum_{i=1}^{r} h_i(\theta(t))\left[C_{fi}\hat{x}(t) + D_{fi}y(t)\right].\end{aligned} \tag{2.6}$$

Defining the augmented state vector $\tilde{x}(t) := \text{col}\{x(t)\ \ \hat{x}(t)\}$, $e(t) := z(t) - \hat{z}(t)$, from (2.3) and (2.6), we can then obtain the following filtering error system:

$$\begin{aligned}\dot{\tilde{x}}(t) &= \tilde{A}(t)\tilde{x}(t) + \tilde{A}_\tau(t)E\tilde{x}(t-\tau(t)) + \tilde{B}(t)w(t),\\ e(t) &= \tilde{C}(t)\tilde{x}(t) + \tilde{C}_\tau(t)E\tilde{x}(t-\tau(t)) + \tilde{D}(t)w(t),\\ \tilde{x}(t) &= \left[\phi^T(t)\ \ 0\right]^T, \quad \forall t \in [-h_b, 0],\end{aligned} \tag{2.7}$$

where

$$\begin{aligned}\tilde{A}(t) &= \sum_{i=1}^{r} h_i(\theta(t))\sum_{j=1}^{r} h_j(\theta(t))\begin{bmatrix} A_j & 0\\ B_{fi}C_j & A_{fi}\end{bmatrix} := \begin{bmatrix} A(t) & 0\\ B_f(t)C(t) & A_f(t)\end{bmatrix},\\ \tilde{A}_\tau(t) &= \sum_{i=1}^{r} h_i(\theta(t))\sum_{j=1}^{r} h_j(\theta(t))\begin{bmatrix} A_{\tau j}\\ B_{fi}C_{\tau j}\end{bmatrix} := \begin{bmatrix} A_\tau(t)\\ B_f(t)C_\tau(t)\end{bmatrix},\\ \tilde{B}(t) &= \sum_{i=1}^{r} h_i(\theta(t))\sum_{j=1}^{r} h_j(\theta(t))\begin{bmatrix} B_j\\ B_{fi}D_j\end{bmatrix} := \begin{bmatrix} B(t)\\ B_f(t)D(t)\end{bmatrix}, \quad E = \begin{bmatrix} I & 0\end{bmatrix},\\ \tilde{C}(t) &= \sum_{i=1}^{r} h_i(\theta(t))\sum_{j=1}^{r} h_j(\theta(t))\left[L_j - D_{fi}C_j\ \ -C_{fi}\right] := \left[L(t) - D_f(t)C(t)\ \ -C_f(t)\right],\\ \tilde{C}_\tau(t) &= \sum_{i=1}^{r} h_i(\theta(t))\sum_{j=1}^{r} h_j(\theta(t))\left[L_{\tau j} - D_{fi}C_{\tau j}\right] := L_\tau(t) - D_f(t)C_\tau(t),\\ \tilde{D}(t) &= \sum_{i=1}^{r} h_i(\theta(t))\sum_{j=1}^{r} h_j(\theta(t))\left[G_j - D_{fi}D_j\right] := G(t) - D_f(t)D(t).\end{aligned} \tag{2.8}$$

So far, *the fuzzy $H\infty$ filter design problem* for fuzzy system (2.3) can be stated as follows. Given a scalar $\gamma > 0$, design a suitable fuzzy filter in the form of (2.5) such that the filtering error system (2.7) has a prescribed $H\infty$ performance γ, and the following two purposes are satisfied:

(i) the system (2.7) with $w(t) = 0$ is asymptotically stable;

(ii) the $H\infty$ performance $\|e\|_2 < \gamma\|w\|_2$ is guaranteed for all nonzero $w(t) \in L_2[0,\infty)$ and a prescribed $\gamma > 0$ under the condition $\tilde{x}(t) = 0$, for all $t \in [-h_b, 0]$. If this is the case, we say that the fuzzy $H\infty$ filter design problem is solved.

3. Fuzzy $H\infty$ Performance Analysis

In this section, we propose the sufficient criterion for the filter error system (2.7) satisfying a prescribed $H\infty$ performance level for fuzzy system (2.1) or (2.3).

Theorem 3.1. *Given scalars $0 \le h_a \le h_b$, $d_1 \le d_2$ and $\gamma > 0$, the $H\infty$ filter error system* (2.7)*, for all differentiable delay $\tau(t) \in [h_a, h_b]$ with $d_1 \le \dot{\tau}(t) \le d_2$, is asymptotically stable and has a prescribed $H\infty$ performance level γ if there exist real symmetry matrices $R_0 > 0$, $R_\delta > 0$, $Q_0 > 0$, $Q_\delta > 0$, $P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0$, $R_\tau \ge 0$, $P_\tau \ge 0$, and real matrices $X_{ij}(t)$, $(i = 1, 2; j = 1, 2, \ldots, 6)$ with appropriate dimensions such that the two LMIs* (3.1) *where $\dot{\tau}(t) = d_1, d_2$, are feasible.*

$$\Xi_i(t) := \begin{bmatrix} \Omega(t) + \left[-I_i^T Q_\delta I_i + \delta_h X_i(t) I_i + \delta_h I_i^T X_i^T(t)\right] & h_a \Gamma_1^T(t) Q_0 & \delta_h \Gamma_1^T(t) Q_\delta & \delta_h X_i(t) & \Gamma_3^T(t) \\ * & -Q_0 & 0 & 0 & 0 \\ * & * & -Q_\delta & 0 & 0 \\ * & * & * & -Q_\delta & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (i = 1, 2), \tag{3.1}$$

where

$$\Omega(t) = \begin{bmatrix} \varphi_{11} & \varphi_{12} & Q_0 & 0 & \varphi_{15} & \varphi_{16} \\ * & \varphi_{22} & 0 & 0 & \varphi_{25} & \varphi_{26} \\ * & * & R_\tau - R_0 + R_\delta - Q_0 - Q_\delta & 0 & Q_\delta & 0 \\ * & * & * & -R_\delta - Q_\delta - P_\tau & Q_\delta & 0 \\ * & * & * & * & -(1-\dot{\tau}(t))(R_\tau - P_\tau) - 2Q_\delta & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\begin{gathered} \varphi_{11} = P_1 A(t) + A^T(t) P_1 + P_2 B_f(t) C(t) + C^T(t) B_f^T(t) P_2^T + R_0 - Q_0, \\ \varphi_{12} = P_2 A_f(t) + A^T(t) P_2^T + C^T(t) B_f^T(t) P_3, \qquad \varphi_{22} = P_3 A_f(t) + A_f^T(t) P_3, \\ \varphi_{15} = P_1 A_\tau(t) + P_2 B_f(t) C_\tau(t), \qquad \varphi_{25} = P_2^T A_\tau(t) + P_3 B_f(t) C_\tau(t), \\ \varphi_{16} = P_1 B(t) + P_2 B_f(t) D(t), \qquad \varphi_{26} = P_2^T B(t) + P_3 B_f(t) D(t), \\ I_1 = \begin{bmatrix} 0 & 0 & 0 & -I & I & 0 \end{bmatrix}, \qquad I_2 = \begin{bmatrix} 0 & 0 & I & 0 & -I & 0 \end{bmatrix}, \\ X_i(t) := \operatorname{col}\left\{X_{i1}(t) \ \ X_{i2}(t) \ \ X_{i3}(t) \ \ X_{i4}(t) \ \ X_{i5}(t) \ \ X_{i6}(t)\right\}, \quad (i = 1, 2), \end{gathered} \tag{3.2}$$

$$\begin{gathered} \Gamma_1(t) := \begin{bmatrix} A(t) & 0 & 0 & 0 & A_\tau(t) & B(t) \end{bmatrix}, \\ \Gamma_2(t) := \begin{bmatrix} B_f(t) C(t) & A_f(t) & 0 & 0 & B_f(t) C_\tau(t) & B_f(t) D(t) \end{bmatrix}, \\ \Gamma_3(t) := \begin{bmatrix} L(t) - D_f(t) C(t) & -C_f(t) & 0 & 0 & L_\tau(t) - D_f(t) C_\tau(t) & G(t) - D_f(t) D(t) \end{bmatrix}. \end{gathered} \tag{3.3}$$

Proof. First, we show that the error system (2.7) with $w(t) \equiv 0$ is asymptotically stable, and then prove that the second condition of the fuzzy $H\infty$ filter design problem in the previous section can be achieved.

We introduce the following Lyapunov-Krasovskii Functional:

$$V(t, \tilde{x}_t) = V_p(t, \tilde{x}_t) + V_h(t, \tilde{x}_t), \tag{3.4}$$

where $\tilde{x}_t$ denotes the function $\tilde{x}(s)$ defined on $[t - h_b, t]$, $V_p(t, \tilde{x}_t) = \tilde{x}^T(t) P \tilde{x}(t)$ and

$$\begin{aligned} V_h(t, \tilde{x}_t) = &\int_{t-h_a}^{t} \tilde{x}^T(s) E^T R_0 E \tilde{x}(s) ds + \int_{t-h_b}^{t-h_a} \tilde{x}^T(s) E^T R_\delta E \tilde{x}(s) ds \\ &+ \int_{t-\tau(t)}^{t-h_a} \tilde{x}^T(s) E^T R_\tau E \tilde{x}(s) ds + \int_{t-h_b}^{t-\tau(t)} \tilde{x}^T(s) E^T P_\tau E \tilde{x}(s) ds \\ &+ h_a \int_{-h_a}^{0} \int_{t+\theta}^{t} \dot{\tilde{x}}^T(s) E^T Q_0 E \dot{\tilde{x}}(s) ds\, d\theta + \delta_h \int_{-h_b}^{-h_a} \int_{t+\theta}^{t} \dot{\tilde{x}}^T(s) E^T Q_\delta E \dot{\tilde{x}}(s) ds\, d\theta \end{aligned} \tag{3.5}$$

with $R_\delta > 0, Q_\delta > 0$, $R_0 > 0, R_\tau \geq 0$, $P_\tau \geq 0$, $Q_0 > 0$, $P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0$ being real symmetry matrices with appropriate dimensions.

We employ (3.4) and Jensen's inequality [40] to study the performance analysis for the filter error system (2.7). In doing so, for simplicity, we introduce the following vector:

$$\Upsilon := \text{col}\{ x(t) \;\; \hat{x}(t) \;\; x(t-h_a) \;\; x(t-h_b) \;\; x(t-\tau(t)) \;\; w(t) \} \tag{3.6}$$

Then, rewrite error system (2.7) as

$$\begin{gathered} \dot{\tilde{x}}(t) = \begin{bmatrix} \Gamma_1(t) \\ \Gamma_2(t) \end{bmatrix} \Upsilon, \\ e(t) = \Gamma_3(t) \Upsilon, \\ \tilde{x}(t) = \left[\phi^T(t) \;\; 0 \right]^T, \quad \forall t \in [-h_b, 0], \end{gathered} \tag{3.7}$$

where $\Gamma_i(t)$, $(i = 1, 2, 3)$ are defined in (3.3).

Now, taking the derivative of (3.4) with respect to t along the trajectory of (2.7) yields

$$\dot{V}_p(t,\tilde{x}_t) = 2\tilde{x}^T(t)P\begin{bmatrix}\Gamma_1(t)\\ \Gamma_2(t)\end{bmatrix}\Upsilon, \tag{3.8}$$

$$\begin{aligned}\dot{V}_h(t,\tilde{x}_t) = {}& x^T(t)R_0x(t) - x^T(t-h_a)R_0x(t-h_a) + x^T(t-h_a)R_\delta x(t-h_a) - x^T(t-h_b)R_\delta x(t-h_b)\\ &+ x^T(t-h_a)R_\tau x(t-h_a) - (1-\dot{\tau}(t))x^T(t-\tau(t))R_\tau x(t-\tau(t))\\ &+ (1-\dot{\tau}(t))x^T(t-\tau(t))P_\tau x(t-\tau(t)) - x^T(t-h_b)P_\tau x(t-h_b)\\ &+ h_a^2\dot{x}^T(t)Q_0\dot{x}(t) - h_a\int_{t-h_a}^{t}\dot{x}^T(s)Q_0\dot{x}(s)ds + \delta_h^2\dot{x}^T(t)Q_\delta\dot{x}(t)\\ &- \delta_h\int_{t-h_b}^{t-h_a}\dot{x}^T(s)Q_\delta\dot{x}(s)ds\end{aligned} \tag{3.9}$$

Since $\tau(t)\in[h_a,h_b]$, and defining $\rho(t)=(h_b-\tau(t))/\delta_h$, we apply Jensen's inequality to yield the following inequalities:

$$-h_a\int_{t-h_a}^{t}\dot{x}^T(s)Q_0\dot{x}(s)ds \le \begin{bmatrix}x(t)\\ x(t-h_a)\end{bmatrix}^T\begin{bmatrix}-Q_0 & Q_0\\ * & -Q_0\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-h_a)\end{bmatrix} \tag{3.10}$$

$$\begin{aligned}&-\delta_h\int_{t-h_b}^{t-h_a}\dot{x}^T(s)Q_\delta\dot{x}(s)ds\\ &\quad= -\delta_h\int_{t-h_b}^{t-\tau(t)}\dot{x}^T(s)Q_\delta\dot{x}(s)ds - \delta_h\int_{t-\tau(t)}^{t-h_a}\dot{x}^T(s)Q_\delta\dot{x}(s)ds\\ &\quad= -(h_b-\tau(t))\int_{t-h_b}^{t-\tau(t)}\dot{x}^T(s)Q_\delta\dot{x}(s)ds - (\tau(t)-h_a)\int_{t-\tau(t)}^{t-h_a}\dot{x}^T(s)Q_\delta\dot{x}(s)ds\\ &\qquad- (1-\rho(t))\delta_h\int_{t-h_b}^{t-\tau(t)}\dot{x}^T(s)Q_\delta\dot{x}(s)ds - \rho(t)\delta_h\int_{t-\tau(t)}^{t-h_a}\dot{x}^T(s)Q_\delta\dot{x}(s)ds\\ &\quad\le \begin{bmatrix}x(t-h_a)\\ x(t-h_b)\\ x(t-\tau(t))\end{bmatrix}^T\left(\begin{bmatrix}-Q_\delta & 0 & Q_\delta\\ * & -Q_\delta & Q_\delta\\ * & * & -2Q_\delta\end{bmatrix}\right)\begin{bmatrix}x(t-h_a)\\ x(t-h_b)\\ x(t-\tau(t))\end{bmatrix}\\ &\qquad- (1-\rho(t))\delta_h\int_{t-h_b}^{t-\tau(t)}\dot{x}^T(s)Q_\delta\dot{x}(s)ds - \rho(t)\delta_h\int_{t-\tau(t)}^{t-h_a}\dot{x}^T(s)Q_\delta\dot{x}(s)ds.\end{aligned} \tag{3.11}$$

In addition, by the Leibniz-Newton formula, we obtain the following equation for any real matrices $X_{ij}(t)$, $i = 1,2$; $j = 1,2,\dots,6$ with appropriate dimensions:

$$\begin{aligned}
0 &= 2\delta_h(1-\rho(t))\Upsilon^T X_1(t)\left[x(t-\tau(t)) - x(t-h_b) - \int_{t-h_b}^{t-\tau(t)} \dot{x}(s)ds\right],\\
0 &= 2\delta_h\rho(t)\Upsilon^T X_2(t)\left[x(t-h_a) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-h_a} \dot{x}(s)ds\right],\\
X_i(t) &:= \operatorname{col}\{X_{i1}(t)\ \ X_{i2}(t)\ \ X_{i3}(t)\ \ X_{i4}(t)\ \ X_{i5}(t)\ \ X_{i6}(t)\},\quad i = 1,2.
\end{aligned} \tag{3.12}$$

By adding the right-hand side of (3.12) to (3.11), and combining with (3.8)–(3.11) We yield the following inequality:

$$\begin{aligned}
&\dot{V}(t,\tilde{x}_t) - \gamma^2 w^T(t)w(t)\\
&\quad \le \Upsilon^T\Omega_{\dot{\tau}(t)}\Upsilon - \delta_h(1-\rho(t))\int_{t-h_b}^{t-\tau(t)}\left(\Upsilon^T X_1 + \dot{x}^T(s)Q_\delta\right)Q_\delta^{-1}\left(X_1^T\Upsilon + Q_\delta\dot{x}(s)\right)ds\\
&\qquad - \delta_h\rho(t)\int_{t-\tau(t)}^{t-h_a}\left(\Upsilon^T X_2 + \dot{x}^T(s)Q_\delta\right)Q_\delta^{-1}\left(X_2^T\Upsilon + Q_\delta\dot{x}(s)\right)ds,
\end{aligned} \tag{3.13}$$

where

$$\begin{aligned}
\Omega_{\dot{\tau}(t)} &= (1-\rho(t))\Omega_1(t) + \rho(t)\Omega_2(t),\\
\Omega_i(t) &:= \Omega(t) + \left[-I_i^T Q_\delta I_i + \delta_h X_i(t) I_i + \delta_h I_i^T X_i^T(t)\right]\\
&\quad + h_a^2\Gamma_1^T(t)Q_0\Gamma_1(t) + \delta_h^2\Gamma_1^T(t)Q_\delta\Gamma_1(t) + \delta_h^2 X_i^T(t)Q_\delta^{-1}X_i(t),\quad i = 1,2.
\end{aligned} \tag{3.14}$$

with $X_i(t), (i = 1,2)$ and $\Omega(t)$, I_1, I_2 are defined in (3.12) and (3.2), respectively.

Notice that, since $Q_\delta > 0, \rho(t) \in [0,1]$, (3.13) implies the following:

$$\dot{V}(t,\tilde{x}_t) - \gamma^2 w^T(t)w(t) \le \Upsilon^T\Omega_{\dot{\tau}(t)}\Upsilon. \tag{3.15}$$

Due to $\rho(t) \in [0,1]$, $\Omega_{\dot{\tau}(t)}$ is negative definite only if $\Omega_i(t) < 0, i = 1,2$. According to Schur's complement, $\Omega_i(t) < 0, i = 1,2$ is equivalent to the following LMIs:

$$\widehat{\Xi}_i(t) := \begin{bmatrix} \Omega(t) + \left[-I_i^T Q_\delta I_i + \delta_h X_i(t) I_i + \delta_h I_i^T X_i^T(t)\right] & h_a\Gamma_1^T(t)Q_0 & \delta_h\Gamma_1^T(t)Q_\delta & \delta_h X_i(t)\\ * & -Q_0 & 0 & 0\\ * & * & -Q_\delta & 0\\ * & * & * & -Q_\delta \end{bmatrix} < 0,\quad i = 1,2. \tag{3.16}$$

And $\widehat{\Xi}_1(t) < 0$ leads for $\dot{\tau}(t) = d_i$, $i = 1,2$ to the following:

$$\widehat{\Xi}_{1i}(t) = \widehat{\Xi}_1(t)\Big|_{\dot{\tau}(t)=d_i} < 0, \quad i = 1,2. \tag{3.17}$$

Notice that

$$\widehat{\Xi}_1(t) = \frac{d_2 - \dot{\tau}(t)}{d_2 - d_1}\widehat{\Xi}_{11}(t) + \frac{\dot{\tau}(t) - d_1}{d_2 - d_1}\widehat{\Xi}_{12}(t). \tag{3.18}$$

Therefore, the two LMIs (3.17) imply (3.16), and $\widehat{\Xi}_1(t)$ is thus convex in $\dot{\tau}(t) \in [d_1, d_2]$. Similarly, $\widehat{\Xi}_2(t)$ is also convex in $\dot{\tau}(t) \in [d_1, d_2]$. Then if the two LMIs in (3.16) are feasible, then $\Omega_{\dot{\tau}(t)} < 0$. It follows from (3.15) that

$$\dot{V}(t, \tilde{x}_t) - \gamma^2 w^T(t)w(t) < -\lambda\|\tilde{x}(t)\|^2, \quad \forall\tilde{x}(t) \neq 0, \tag{3.19}$$

where $\lambda = \lambda_{\min}(-\Omega_{\dot{\tau}(t)})$.

From the above process, we can obtain the asymptotic stability of error system (2.7) with $w(t) = 0$.

Next, assuming that $\tilde{x}(t) = 0$, for all $t \in [-h_b, 0]$, we prove that the $H\infty$ performance $\|e\|_2 < \gamma\|w\|_2$ is also guaranteed for all nonzero $w(t) \in L_2[0,\infty)$ and a prescribed performance level $\gamma > 0$.

Notice that $e^T(t)e(t) = \Upsilon^T\Gamma_3^T(t)\Gamma_3(t)\Upsilon$, one rewrites (3.15) to the following:

$$\dot{V}(t, \tilde{x}_t) \leq \Upsilon^T\widehat{\Omega}_{\dot{\tau}(t)}\Upsilon - e^T(t)e(t) + \gamma^2 w^T(t)w(t), \tag{3.20}$$

where

$$\begin{aligned}
\widehat{\Omega}_{\dot{\tau}(t)} &= (1 - \rho(t))\widehat{\Omega}_1(t) + \rho(t)\widehat{\Omega}_2(t),\\
\widehat{\Omega}_i(t) &:= \Omega(t) + \left[-\mathrm{I}_i^T Q_\delta \mathrm{I}_i + \delta_h X_i(t)\mathrm{I}_i + \delta_h \mathrm{I}_i^T X_i^T(t)\right]\\
&\quad + h_a^2\Gamma_1^T(t)Q_0\Gamma_1(t) + \delta_h^2\Gamma_1^T(t)Q_\delta\Gamma_1(t) + \delta_h^2 X_i^T(t)Q_\delta^{-1}X_i(t) + \Gamma_3^T(t)\Gamma_3(t), \quad (i = 1,2).
\end{aligned} \tag{3.21}$$

If the LMIs (3.1) are feasible, applying Schur's complement yields $\widehat{\Omega}_{\dot{\tau}(t)} < 0$. Otherwise, similar to (3.16) and (3.17), then $\widehat{\Xi}_i(t)$, $(i = 1,2)$ are also convex in $\dot{\tau}(t) \in [d_1, d_2]$. So far, one has the following:

$$\dot{V}(t, \tilde{x}_t) \leq -e^T(t)e(t) + \gamma^2 w^T(t)w(t). \tag{3.22}$$

Integrating both sides of (3.22) from 0 to ∞ on t, and considering the zero initial condition, one obtains

$$\int_0^{\infty} e^T(t)e(t)dt < \gamma^2 \int_0^{\infty} w^T(t)w(t)dt, \tag{3.23}$$

that is, $\|e\|_2 < \gamma\|w\|_2$. This completes the proof. □

For unknown d_1, only by substituting $\dot{\tau}(t) = d_2$ into (3.1)-(3.2), we can obtain the following Corollary.

Corollary 3.2. *Given scalars $0 \leq h_a \leq h_b, d_2$ and $\gamma > 0$, the $H\infty$ filter error system* (2.7)*, for all differentiable delay $\tau(t) \in [h_a, h_b]$ with $\dot{\tau}(t) \leq d_2$, is asymptotically stable and has a prescribed $H\infty$ performance level γ if there exist matrices $R_0 > 0, R_\delta > 0, Q_0 > 0, Q_\delta > 0, P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0$, $R_\tau \geq 0, P_\tau \geq 0$, and real matrices $X_{ij}(t)$, $(i = 1,2; j = 1,2,\ldots,6)$ with appropriate dimensions such that two LMIs* (3.1) *where $\dot{\tau}(t) = d_2$, with notations in* (3.2) *and* (3.3)*, are feasible.*

Moreover, if the above LMIs are feasible with $R_\tau = 0$, $P_\tau = 0$, then the $H\infty$ filter error system (2.7)*, for all fast-varying delay $\tau(t) \in [h_a, h_b]$, is also asymptotically stable and has a prescribed $H\infty$ performance level γ.*

In addition, when the number of IF-THEN rules is one, and the system is reduced to a simple time delay systems, that is, the system can be described as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t - \tau(t)), \quad t > 0, \\ x(t) &= \phi(t), \quad t \in [-h_b, 0], \end{aligned} \tag{3.24}$$

where

$$\tau(t) \in [h_a, h_b], \qquad d_1 \leq \dot{\tau}(t) \leq d_2. \tag{3.25}$$

According to the similar line of Theorem 3.1, without using the free-weighting matrices technique, one derives the following Corollary.

Corollary 3.3. *Given scalars $0 \leq h_a \leq h_b$, $d_1 \leq d_2$, the system* (3.24)*, for all differentiable delay $\tau(t) \in [h_a, h_b]$ with $d_1 \leq \dot{\tau}(t) \leq d_2$, is asymptotically stable if there exist real symmetry matrices $R_0 > 0$, $R_\delta > 0$, $Q_0 > 0$, $Q_\delta > 0$, $P > 0$, $R_\tau \geq 0$, $P_\tau \geq 0$ such that the following LMIs, where*

$\dot{\tau}(t) = d_i, (i = 1, 2)$, *are feasible:*

$$\begin{bmatrix} \widehat{\Omega} + \left[\widehat{\mathrm{I}}_i^t Q_\delta \widehat{\mathrm{I}}_i\right] & h_a \widehat{\Gamma}_1^T Q_0 & \delta_h \widehat{\Gamma}_1^T Q_\delta \\ * & -Q_0 & 0 \\ * & * & -Q_\delta \end{bmatrix} < 0, \quad (i = 1, 2), \tag{3.26}$$

where $\widehat{\mathrm{I}}_1 := [0 \;\; 0 \;\; -I \;\; I]$, $\widehat{\mathrm{I}}_2 := [0 \;\; I \;\; 0 \;\; -I]$, $\widehat{\Gamma}_1 := [A \;\; 0 \;\; 0 \;\; A_d]$ *and*

$$\widehat{\Omega} := \begin{bmatrix} PA + A^T P + R_0 - Q_0 & Q_0 & 0 & PA_d \\ * & -R_0 + R_\delta + R_\tau - Q_0 - Q_\delta & 0 & Q_\delta \\ * & * & -R_\delta - P_\tau - Q_\delta & Q_\delta \\ * & * & * & -(1 - \dot{\tau}(t))(R_\tau - P_\tau) - 2Q_\delta \end{bmatrix} \tag{3.27}$$

Remark 3.4. It is worth mentioning that in the previous studies (see [37–39, 41]), some negative terms are ignored when estimating the time derivative of the Lyapunov-Krasovskii functional, which may lose a great amount of useful information and lead to conservative results. Instead, in this paper, those negative terms are effectively used in (3.11). In addition, when constructing the Lyapunov-Krasovskii functional candidate, the information on the lower bound of the delay is taken full advantage of by introducing the terms $\int_{t-h_b}^{t-h_a} \widetilde{x}^T(s) E^T R_\delta E \widetilde{x}(s) ds$ and $\int_{t-\tau(t)}^{t-h_a} \widetilde{x}^T(s) E^T R_\tau E \widetilde{x}(s) ds$ in the Lyapunov-Krasovskii functional. From Example 5.3 below, it is clear to see that our approach is less conservative than the existing ones.

4. Fuzzy $H\infty$ Filter Design

It is worth mentioning that the problem in this paper essentially aims at designing a filter to estimated $z(t)$ based on $H\infty$ norm constraint. The following theorem provides sufficient condition for the existence of fuzzy $H\infty$ filter for fuzzy system (2.3) with interval time-varying delay. And a suitable filter design is obtained from the parameter matrices A_{fi}, B_{fi}, C_{fi}, and D_{fi}, $(i = 1, 2, \ldots, r)$.

Theorem 4.1. *Given scalars* $0 \le h_a \le h_b$, $d_1 \le d_2$ *and* $\gamma > 0$, *the fuzzy* $H\infty$ *filter design problem, for all differentiable delay* $\tau(t) \in [h_a, h_b]$ *with* $d_1 \le \dot{\tau}(t) \le d_2$, *is solvable if there exist matrices* $P_1 > 0$, $U > 0$, $R_\tau \ge 0$, $P_\tau \ge 0$, $R_0 > 0$, $R_\delta > 0$, $Q_0 > 0$, *and* $Q_\delta > 0$, *and real matrices* N_{1i}, N_{2i}, N_{3i}, N_{4i}, $(i = 1, 2, \ldots, r)$, $\widehat{X}_i^k := \mathrm{col}\{X_{i1}^k \;\; \widehat{X}_{i2}^k \;\; X_{i3}^k \;\; X_{i4}^k \;\; X_{i5}^k \;\; X_{i6}^k\}$, *and* $i = 1, 2$; $k = 1, 2, \ldots, r$ *with appropriate dimensions such that the following LMIs: where* $\dot{\tau}(t) = d_1, d_2$, *are feasible:*

$$U - P_1 < 0, \tag{4.1}$$

$$\Pi_i(m, n) + \Pi_i(n, m) < 0, \quad m \le n, \;\; (m, n = 1, 2, \ldots, r), \;\; (i = 1, 2), \tag{4.2}$$

where $\mathrm{I}_1, \mathrm{I}_2$ *is defined in* (3.2), *and*

$$\Pi_i(m,n) := \begin{bmatrix} \widehat{\Omega}_{mn} + \left[-\mathrm{I}_i^T Q_\delta \mathrm{I}_i + \delta_h \widehat{X}_i^m \mathrm{I}_i + \delta_h \mathrm{I}_i^T \left(\widehat{X}_i^m\right)^T\right] & h_a (\Gamma_1^m)^T Q_0 & \delta_h (\Gamma_1^m)^T Q_\delta & \delta_h \widehat{X}_i^m & \left(\widehat{\Gamma}_3^m\right)^T \\ * & -Q_0 & 0 & 0 & 0 \\ * & * & -Q_\delta & 0 & 0 \\ * & * & * & -Q_\delta & 0 \\ * & * & * & * & -I \end{bmatrix},$$

$$\widehat{\Omega}_{mn} = \begin{bmatrix} \widehat{\varphi}_{11} & \widehat{\varphi}_{12} & Q_0 & 0 & \widehat{\varphi}_{15} & \widehat{\varphi}_{16} \\ * & \widehat{\varphi}_{22} & 0 & 0 & \widehat{\varphi}_{25} & \widehat{\varphi}_{26} \\ * & * & R_\tau - R_0 + R_\delta - Q_0 - Q_\delta & 0 & Q_\delta & 0 \\ * & * & * & -R_\delta - Q_\delta - P_\tau & Q_\delta & 0 \\ * & * & * & * & -(1-\dot{\tau}(t))(R_\tau - P_\tau) - 2Q_\delta & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\begin{aligned}
&\widehat{\varphi}_{11} = P_1 A_m + A_m^T P_1 + N_{2n} C_m + C_m^T N_{2n}^T + R_0 - Q_0, \\
&\widehat{\varphi}_{12} = N_{1n} + A_m^T U + C_m^T N_{2n}^T, \quad \widehat{\varphi}_{22} = N_{1n} + N_{1n}^T, \\
&\widehat{\varphi}_{15} = P_1 A_{\tau m} + N_{2n} C_{\tau m}, \quad \widehat{\varphi}_{25} = U A_{\tau m} + N_{2n} C_{\tau m} \\
&\widehat{\varphi}_{16} = P_1 B_m + N_{2n} D_m, \quad \widehat{\varphi}_{26} = U B_m + N_{2n} D_m. \\
&\Gamma_1^m := \begin{bmatrix} A_m & 0 & 0 & 0 & A_{\tau m} & B_m \end{bmatrix}, \\
&\widehat{\Gamma}_3^m := \begin{bmatrix} L_m - N_{4n} C_m & -N_{3n} & 0 & 0 & L_{\tau m} - N_{4n} C_{\tau m} & G_m - N_{4n} D_m \end{bmatrix}.
\end{aligned} \tag{4.3}$$

Moreover, a suitable filter in the form of (2.5) *is given by*

$$A_{fi} = N_{1i} U^{-1}, \quad B_{fi} = N_{2i}, \quad C_{fi} = N_{3i} U^{-1}, \quad D_{fi} = N_{4i} \quad (i = 1, 2, \dots, r). \tag{4.4}$$

Proof. Set

$$N_k(t) := \sum_{i=1}^{r} h_i(\theta(t))[N_{ki}], \quad k = 1, 2, 3, 4, \tag{4.5}$$

$$\widehat{X}_i(t) := \mathrm{col}\left\{ X_{i1}(t) \;\; \widehat{X}_{i2}(t) \;\; X_{i3}(t) \;\; X_{i4}(t) \;\; X_{i5}(t) \;\; X_{i6}(t) \right\}, \quad i = 1, 2, \tag{4.6}$$

where

$$\begin{gathered}
X_{ik}(t) := \sum_{j=1}^{r} h_j(\theta(t)) \left[X_{ik}^j(t)\right], \quad i = 1, 2; \; k = 1, 3, \dots, 6, \\
\widehat{X}_{i2}(t) := \sum_{j=1}^{r} h_j(\theta(t)) \left[\widehat{X}_{i2}^j(t)\right], \\
\Gamma_1(t) := \sum_{m=1}^{r} h_m(\theta(t)) [\Gamma_1^m], \qquad \widehat{\Gamma}_3(t) := \sum_{m=1}^{r} h_m(\theta(t)) \left[\widehat{\Gamma}_3^m\right].
\end{gathered} \tag{4.7}$$

Thus, from (2.8) and the above definition, we have

$$\Pi_i(t) = \sum_{m=1}^{r} h_m^2(\theta(t))[\Pi_i(m,m)] + \sum_{m<n}^{r} h_m(\theta(t))h_n(\theta(t))[\Pi_i(m,n) + \Pi_i(n,m)], \quad (i=1,2), \tag{4.8}$$

where

$$\Pi_i(t) := \begin{bmatrix} \Psi(t) + \left[-\mathrm{I}_i^T Q_\delta I_i + \delta_h \widehat{X}_i(t)\mathrm{I}_i + \delta_h \mathrm{I}_i^T \widehat{X}_i^T(t)\right] & h_a\Gamma_1^T(t)Q_0 & \delta_h\Gamma_1^T(t)Q_\delta & \delta_h\widehat{X}_i(t) & \widehat{\Gamma}_3^T(t) \\ * & -Q_0 & 0 & 0 & 0 \\ * & * & -Q_\delta & 0 & 0 \\ * & * & * & -Q_\delta & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (i=1,2) \tag{4.9}$$

with

$$\Psi(t) = \begin{bmatrix} \Psi_{11} & \Psi_{12} & Q_0 & 0 & \Psi_{15} & \Psi_{16} \\ * & \Psi_{22} & 0 & 0 & \Psi_{25} & \Psi_{26} \\ * & * & R_\tau - R_0 + R_\delta - Q_0 - Q_\delta & 0 & Q_\delta & 0 \\ * & * & * & -R_\delta - Q_\delta - P_\tau & Q_\delta & 0 \\ * & * & * & * & -(1-\dot{\tau}(t))(R_\tau - P_\tau) - 2Q_\delta & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\begin{gathered} \Psi_{11} = P_1A(t) + A^T(t)P_1 + N_2(t)C(t) + C^T(t)N_2^T(t) + R_0 - Q_0, \\ \Psi_{12} = N_1(t) + A^T(t)U + C^T(t)N_2^T(t), \qquad \Psi_{22} = N_1(t) + N_1^T(t), \\ \Psi_{15} = P_1A_\tau(t) + N_2(t)C_\tau(t), \qquad \Psi_{25} = UA_\tau(t) + N_2(t)C_\tau(t), \\ \Psi_{16} = P_1B(t) + N_2(t)D(t), \qquad \Psi_{26} = UB(t) + N_2(t)D(t). \end{gathered} \tag{4.10}$$

Next, based on Theorem 3.1, we calculate the feasibility of the LMIs $\Pi_i(t) < 0$, $(i=1,2)$.

Due to $U > 0$, there exist a nonsingular real $n \times n$ matrix P_2 and a real $n \times n$ matrix $P_3 > 0$ such that $U = P_2P_3^{-1}P_2^T$.Let us define

$$J := \operatorname{diag}\{I \;\; P_2^{-T}P_3 \;\; I \;\; I \;\; I \;\; I \;\; I \;\; I \;\; I \;\; I\} \tag{4.11}$$

left- and right-multiply $\Pi_i(t)$, $i = 1, 2$ defined in (4.8) by J^T and J, respectively, and take $X_{i2}(t) := P_3 P_2^{-1} \widehat{X}_{i2}(t)$, $i = 1, 2$ and

$$\begin{aligned} \overline{A}(t) &= P_2^{-1} N_1(t) U^{-1} P_2, \qquad \overline{B}(t) = P_2^{-1} N_2(t), \\ \overline{C}(t) &= N_3(t) U^{-1} P_2, \qquad \overline{D}(t) = N_4(t), \end{aligned} \tag{4.12}$$

By replacing $(A_f(t), B_f(t), C_f(t), D_f(t))$ in $\Xi_i(t)$, $i = 1, 2$ defined in (3.1) with $(\overline{A}(t), \overline{B}(t), \overline{C}(t), \overline{D}(t))$, one yields

$$\Xi_i(t) = J^T \Pi_i(t) J, \quad i = 1, 2 \tag{4.13}$$

Note that if LMIs (4.1) and (4.2) hold, from (4.8), we arrive at $\Pi_i(t) < 0$, then $\Xi_i(t) < 0$.

On the other hand, from (4.1), notice that $P_1 - U = P_1 - P_2 P_3^{-1} P_2^T > 0$, applying Schur complement yields $\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0$.

So far, we conclude from Theorem 3.1 that the filter, that is,

$$\begin{aligned} \dot{\overline{x}}(t) &= \overline{A}(t)\overline{x}(t) + \overline{B}(t) y(t), \qquad \overline{x}(0) = 0, \\ \overline{z}(t) &= \overline{C}(t)\overline{x}(t) + \overline{D}(t) y(t), \end{aligned} \tag{4.14}$$

with $(\overline{A}(t), \overline{B}(t), \overline{C}(t), \overline{D}(t))$ defined in (4.12), guarantees that the $H\infty$ filter error system (2.7) is asymptotically stable and has a prescribed $H\infty$ performance level γ.

And, performing an irreducible linear transformation $\widehat{x}(t) = P_2 \overline{x}(t)$ in (4.14) yields

$$\begin{aligned} \dot{\widehat{x}}(t) &= N_1(t) U^{-1} \widehat{x}(t) + N_2(t) y(t), \qquad \widehat{x}(0) = 0, \\ \widehat{z}(t) &= N_3(t) U^{-1} \widehat{x}(t) + N_4(t) y(t). \end{aligned} \tag{4.15}$$

Therefore, the desired filter (2.5) with the filter matrices in (4.4) is readily obtained from (4.15). This completes the proof. □

Similar to Corollary 3.2, when d_1 is unknown, by substituting $\dot{\tau}(t) = d_2$ into (4.2), the following result is then obtained.

Corollary 4.2. *Given scalars $0 \le h_a \le h_b, d_2$ and $\gamma > 0$, the fuzzy $H\infty$ filter design problem, for all differentiable delay $\tau(t) \in [h_a, h_b]$ with $\dot{\tau}(t) \le d_2$, is solvable if there exist matrices $R_0 > 0$, $R_\delta > 0$, $Q_0 > 0$, $Q_\delta > 0$, $P_1 > 0$, $U > 0$, $R_\tau \ge 0$, $P_\tau \ge 0$, and real matrices $N_{1i}, N_{2i}, N_{3i}, N_{4i}$, $(i = 1, 2, \ldots, r)$, $\widehat{X}_i^k :=$ col $\{X_{i1}^k \ \widehat{X}_{i2}^k \ X_{i3}^k \ X_{i4}^k \ X_{i5}^k \ X_{i6}^k\}$, $i = 1, 2$; $k = 1, 2, \ldots, r$ with appropriate dimensions such that the LMIs:* (4.1) *and* (4.2) *where $\dot{\tau}(t) = d_2$, are feasible. Meanwhile, a desired filter in the form of* (2.5) *is given by the filter matrices in* (4.4).

Moreover, if the above LMIs are feasible with $R_\tau = 0$, $P_\tau = 0$, then the fuzzy $H\infty$ filter design problem, for all fast-varying delay $\tau(t) \in [h_a, h_b]$, is solvable in which a desired filter in (2.5) *is given by the filter matrices in* (4.4).

Remark 4.3. Notice that for any scalar σ, if $(\sigma Z - P)Z^{-1}(\sigma Z - P) \geq 0$, then $-PZ^{-1}P \leq -2\sigma P + \sigma^2 Z$. The fact played a key role in the existing results in [4, 5, Lemma 1], respectively. But there existed some coupled matrix variables in the LMIs in [4, 5].Therefore, to solve filter design problem, [4, 5] must use decoupling technique similar to [42] to convert the conditions in [4, 5, Lemma 1] into another form, respectively. These decoupling approaches were shown as [4, 5, Lemma 2], respectively. Furthermore, because of a scalar being predescribed, the constraint may lead to considerable conservativeness of these results. Examples below show that for different δ yields different $\gamma_{\min}$. From simulation results in Table 2, we can see that if $\delta = 0.7$ or $\delta = 20$, the conditions in [4, 5] are unsolvable when $h_b = 1.0$, while our result works. Meanwhile, the scalar is not needed in this paper. Examples 5.1 and 5.2 below show that our approach yields less conservative results.

5. Numerical Examples

In this section, three examples are given to show the effectiveness of the proposed method in this paper.

Example 5.1. Consider the following fuzzy system borrowed from [4, 5]:

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{2} h_i(\theta(t))[A_i x(t) + A_{\tau i} x(t-\tau(t)) + B_i w(t)], \\
y(t) &= \sum_{i=1}^{2} h_i(\theta(t))[C_i x(t) + C_{\tau i} x(t-\tau(t)) + D_i w(t)], \\
z(t) &= \sum_{i=1}^{2} h_i(\theta(t))[L_i x(t) + L_{\tau i} x(t-\tau(t)) + G_i w(t)],
\end{aligned} \tag{5.1}$$

where

$$\begin{gathered}
A_1 = \begin{bmatrix} -2.1 & 0.1 \\ 1 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}, \quad A_{\tau 1} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, \quad A_{\tau 2} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}, \\
B_1 = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \\
C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.5 & -0.6 \end{bmatrix}, \quad C_{\tau 1} = \begin{bmatrix} -0.8 & 0.6 \end{bmatrix}, \quad C_{\tau 2} = \begin{bmatrix} -0.2 & 1 \end{bmatrix} \\
D_1 = 0.3, \quad D_2 = -0.6, \\
L_1 = \begin{bmatrix} 1 & -0.5 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}, \quad L_{\tau 1} = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \quad L_{\tau 2} = \begin{bmatrix} 0 & 0.2 \end{bmatrix}, \\
G_1 = 0, \quad G_2 = 0.
\end{gathered} \tag{5.2}$$

For $d_2 = 0.3$ and $\gamma = 0.5$, choosing d_1 and h_a in Table 1 and applying Theorems 4.1, the results are d_1-dependent (see Table 1). Moreover, for unknown d_1 and d_2, that is, fast-varying delay

Table 1: Maximum values of h_b for $d_2 = 0.3$.

$h_a \setminus d_1$	0	−0.1	−0.3	−0.5	−0.7	−1
$h_a = 1$	2.358	2.357	2.355	2.353	2.349	2.351
$h_a = 0$	2.011	2.012	2.012	2.011	2.011	2.012

Table 2: Minimum index γ for $d_2 = 0.2$ (d_1 unknown and $h_a = 0$).

Method	$\delta = 0.7$		$\delta = 1$		$\delta = 2$		$\delta = 10$		$\delta = 20$		Any δ
	[4]	[5]	[4]	[5]	[4]	[5]	[4]	[5]	[4]	[5]	Our results
$h_b = 0.5$	0.59	0.42	0.38	0.27	0.35	0.25	0.34	0.24	0.37	0.26	0.2054
$h_b = 0.6$	1.03	0.74	0.43	0.31	0.36	0.25	0.35	0.25	0.45	0.32	0.2100
$h_b = 0.8$	11.98	8.54	0.83	0.59	0.38	0.27	0.37	0.26	1.01	0.70	0.2204
$h_b = 1$	—	—	2.22	1.57	0.41	0.29	0.45	0.32	—	—	0.2324

Table 3: Minimum index γ for different cases (d_1 unknown and $h_b = 1.25$).

h_a	method	$d_2 = 0.4$	$d_2 = 0.6$	$d_2 = 0.8$	$d_2 \geq 1$
0	[4]	0.44	2.77	∞	∞
	[37]	0.42	1.41	∞	∞
	[36]	0.32	0.49	0.84	1.14
	Our results	0.29	0.41	0.79	1.03
0.8	[37]	0.40	0.89	1.06	1.06
	[36]	0.32	0.40	0.40	0.40
	Our results	0.24	0.24	0.24	0.24
1.0	[37]	0.37	0.38	0.38	0.38
	[36]	0.28	0.28	0.28	0.28
	Our results	0.20	0.20	0.20	0.20

Table 4: Minimum performance level γ.

Method	$\delta = 0.7$		$\delta = 1$		$\delta = 2$		$\delta = 4$		Any δ
	[4]	[5]	[4]	[5]	[4]	[5]	[4]	[5]	Our results
$h_b = 0.5$	0.37	0.26	0.35	0.24	0.36	0.24	0.38	0.26	0.218
$h_b = 0.6$	0.44	0.31	0.38	0.27	0.38	0.27	0.41	0.29	0.241
$h_b = 0.8$	0.63	0.45	0.49	0.34	0.44	0.31	0.55	0.39	0.300

case, according to Corollary 4.2, by setting $R_\tau = 0, P_\tau = 0, h_a = 0,$ and $h_b = 0.5$, we get the optimal attenuation level $\gamma_{\text{opt}} = 0.230$ after 38 iterations.

For $h_a = 0$, $d_1$unknown and $d_2 = 0.2$, to compare with the recently developed fuzzy $H\infty$ filter, it is worthwhile to point out that a given scalar δ is needed in [4, 5] while the scalar δ is any value in our results. Thus, we consider different h_{b} and δ to find the minimum index γ. The results obtained by various methods in the literature and in this paper are listed in Table 2. Moreover, for the case of no additional prescribed scalar, in order to demonstrate the advantages of the proposed approach over the existing results, a detailed comparison between the minimum $H\infty$ performance levels obtained by the methods in [4, 36, 37] and in this paper for different cases is summarized in Table 3. From Tables 2 and 3, it can be seen that stability conditions obtained in this paper are less conservative than the existing ones.

As an example, for given $h_a = 0, h_b = 0.5, d_1 = 0,$ ans $d_2 = 0.3$, according to Theorem 4.1, solve LMIs in (4.1) and (4.2), and get the minimum performance level $\gamma_{\text{opt}} = 0.206$ after 32 iterations, and then compute the fuzzy $H\infty$ filter matrices from (4.4) as follows

$$\begin{aligned}
A_{f1} &= \begin{bmatrix} -7.1207 & -5.3463 \\ -0.7273 & -4.5289 \end{bmatrix}, \qquad B_{f1} = \begin{bmatrix} -0.1932 \\ 0.2146 \end{bmatrix}, \\
C_{f1} &= \begin{bmatrix} -6.3345 & -2.6742 \end{bmatrix}, \qquad D_{f1} = 0.2486, \\
A_{f2} &= \begin{bmatrix} -3.5662 & -1.3711 \\ -7.4183 & -10.6811 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} -0.1765 \\ 0.1956 \end{bmatrix}, \\
C_{f2} &= \begin{bmatrix} -2.3625 & -5.2980 \end{bmatrix}, \qquad D_{f2} = 0.2498.
\end{aligned} \tag{5.3}$$

In order to further show the merit of our method, let us consider the following numerical example.

Example 5.2. Consider the following fuzzy system with interval time-varying delay:

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{2} h_i(\theta(t))[A_i x(t) + A_{\tau i} x(t-\tau(t)) + Bw(t)], \\
y(t) &= \sum_{i=1}^{2} h_i(\theta(t))[C_i x(t) + C_{\tau i} x(t-\tau(t)) + Dw(t)], \\
z(t) &= \sum_{i=1}^{2} h_i(\theta(t))[L_i x(t) + L_{\tau i} x(t-\tau(t)) + G_i w(t)],
\end{aligned} \tag{5.4}$$

where

$$\begin{gathered}
A_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -0.9 & 0 \\ 0 & -0.5 & -1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.9 & 0.2 & 0 \\ -0.2 & -0.5 & 0 \\ 0 & -0.1 & -0.8 \end{bmatrix}, \qquad A_{\tau 1} = \begin{bmatrix} -0.8 & 0.2 & -0.1 \\ 0.1 & -0.8 & 0 \\ -0.4 & 0.25 & -1 \end{bmatrix}, \\
A_{\tau 2} = \begin{bmatrix} -1 & 0.5 & 0.1 \\ 0.5 & -1 & 0 \\ -0.8 & 0.9 & -0.25 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0.5 & 0.4 & 0 \end{bmatrix}, \\
C_2 = \begin{bmatrix} 0.5 & -1 & 0 \end{bmatrix}, \qquad C_{\tau 1} = \begin{bmatrix} 1 & -0.5 & 0.5 \end{bmatrix}, \qquad C_{\tau 2} = \begin{bmatrix} 1 & 0.1 & -0.5 \end{bmatrix}, \\
D = 0.25, \qquad L_1 = \begin{bmatrix} 0.5 & 0 & 0 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 1 & -0.5 & 0 \end{bmatrix}, \\
L_{\tau 1} = \begin{bmatrix} 0.1 & 0.5 & 0.5 \end{bmatrix}, \qquad L_{\tau 2} = \begin{bmatrix} 0.1 & 0 & 0.5 \end{bmatrix}, \qquad G_1 = 0, \quad G_2 = 0.
\end{gathered} \tag{5.5}$$

Table 5: Maximum values of h_b.

Method	$\delta = 0.7$		$\delta = 1$		$\delta = 2$		$\delta = 5$		Any δ
	[4]	[5]	[4]	[5]	[4]	[5]	[4]	[5]	Our results
$\gamma = 0.3$	0.33	0.59	0.33	0.69	0.25	0.78	0.12	0.59	0.80
$\gamma = 0.4$	0.55	0.76	0.64	0.91	0.70	1.08	0.52	0.70	1.16
$\gamma = 0.5$	0.69	0.84	0.82	1.02	0.97	1.13	0.67	0.70	1.19

Table 6: Maximum values of h_b for $h_a = 1.0$ (d_1 unknown).

d_2 \ Method	[38]	[37]	[39]	Our results
unknown	1.50	1.5187	1.6169	1.7001
0.3	—	2.2125	2.2474	2.3076

To compare with the ones existing in [4, 5], we assumed that $d_1$is unknown and $h_a = 0$. According to Corollary 4.2, choose $d_2 = 0.2$ and the simulations are run for two cases. In the first case, we compute the minimum index γ for the given different h_b and δ in [4, 5] or any δ in this paper. In the second case, we compute the maximum values of h_b for the given different γ and δ in [4, 5] or any δ in this paper. The simulation results are shown by Tables 4 and 5, respectively. It can also be clearly seen that our approach has less conservative results than the results in the literatures.

As an example, we assume that $h_a = 0, h_b = 1.0, d_1 = 0, d_2 = 0.2, \gamma = 0.5$, the solutions can be obtained after 20 iterations in which the fuzzy $H\infty$ filter in the form of (2.5) is given by the following filter matrices as

$$\begin{gathered} A_{f1} = \begin{bmatrix} -3.3579 & 3.2252 & -0.3286 \\ 5.2755 & -12.3863 & 12.6979 \\ -0.4419 & -0.1659 & -7.8342 \end{bmatrix}, \qquad B_{f1} = \begin{bmatrix} 4.1741 \\ 2.2921 \\ -0.2958 \end{bmatrix}, \\ C_{f1} = \begin{bmatrix} 0.2597 & 0.0547 & 2.2850 \end{bmatrix}, \qquad D_{f1} = 0.4963, \\ A_{f2} = \begin{bmatrix} -1.6047 & -1.0030 & -4.2861 \\ 1.0643 & -4.9469 & 0.2980 \\ -0.2451 & 0.1274 & -3.3907 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} 5.1135 \\ 2.0674 \\ -0.4810 \end{bmatrix}, \\ C_{f2} = \begin{bmatrix} -0.7162 & 0.2062 & -6.4570 \end{bmatrix}, \qquad D_{f2} = -0.2080. \end{gathered} \tag{5.6}$$

Next, we will give another example to illustrate that our methods are reduced more conservative than the existing results.

Example 5.3. Consider the linear system (3.24) with the following parameters

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \qquad A_d = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}. \tag{5.7}$$

To compare with those results in the previous literatures, assume that d_1 is unknown. For $h_a = 1.0$, d_2 unknown or $d_2 = 0.3$, the result of Corollary 3.3 coincides with the one in [41] (the latter are less conservative than those of [39]). Comparison with various existing methods in the literature for the admissible upper-bound h_b, which guarantee the stability of the system (3.24) is listed in Table 6. It is clear that our results are much less conservative than those in [37–39].

6. Conclusion

This paper deals with the problem of fuzzy $H\infty$ filter design for T-S fuzzy systems with interval time-varying delay through T-S fuzzy models. By constructing a novel Lyapunov-Krasovskii functional and estimating the time derivative of the Lyapunov-Krasovskii functional less conservatively, an improved $H\infty$ filter design scheme is proposed. Three numerical examples are used to illustrate the design procedure and the merit of the proposed method.

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References

[1] D. S. Bernstein and W. M. Haddad, "Steady-state Kalman filtering with an H_∞ error bound," *Systems & Control Letters*, vol. 12, no. 1, pp. 9–16, 1989.

[2] D. Simon, "Kalman filtering for fuzzy discrete time dynamic systems," *Applied Soft Computing Journal*, vol. 3, no. 3, pp. 197–207, 2003.

[3] Fuwen Yang, Z. Wang, and Y. S. Hung, "Robust Kalman filtering for discrete time-varying uncertain systems with multiplicative noises," *IEEE Transactions on Automatic Control*, vol. 47, no. 7, pp. 1179–1183, 2002.

[4] C. Lin, Q.-G. Wang, T. H. Lee, and B. Chen, "H_∞ filter design for nonlinear systems with time-delay through T-S fuzzy model approach," *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 3, pp. 739–746, 2008.

[5] Yakun Su, B. Chen, C. Lin, and H. Zhang, "A new fuzzy H_∞ filter design for nonlinear continuous-time dynamic systems with time-varying delays," *Fuzzy Sets and Systems*, vol. 160, no. 24, pp. 3539–3549, 2009.

[6] C. Lin, Q.-G. Wang, T. H. Lee, and Y. He, "Fuzzy weighting-dependent approach to H_∞ filter design for time-delay fuzzy systems," *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 2746–2751, 2007.

[7] E. Fridman and U. Shaked, "An improved delay-dependent H_∞ filtering of linear neutral systems," *IEEE Transactions on Signal Processing*, vol. 52, no. 3, pp. 668–673, 2004.

[8] G. Feng, "A survey on analysis and design of model-based fuzzy control systems," *IEEE Transactions on Fuzzy Systems*, vol. 14, no. 5, pp. 676–697, 2006.

[9] Y.-C. Lin and J.-C. Lo, "Robust mixed H_2/H_∞ filtering for time-delay fuzzy systems," *IEEE Transactions on Signal Processing*, vol. 54, no. 8, pp. 2897–2909, 2006.

[10] G. Feng, "Robust H_∞ filtering of fuzzy dynamic systems," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 41, no. 2, pp. 658–670, 2005.

[11] S. K. Nguang and P. Shi, "Delay-dependent $\mathscr{H}_\infty$ filtering for uncertain time delay nonlinear systems: an LMI approach," *IET Control Theory & Applications*, vol. 1, no. 1, pp. 133–140, 2007.

[12] Y.-C. Lin and J.-C. Lo, "Robust mixed H_2/H_∞ filtering for discrete-time delay fuzzy systems," *International Journal of Systems Science*, vol. 36, no. 15, pp. 993–1006, 2005.

[13] C.-S. Tseng and B.-S. Chen, "H_∞ fuzzy estimation for a class of nonlinear discrete-time dynamic systems," *IEEE Transactions on Signal Processing*, vol. 49, no. 11, pp. 2605–2619, 2001.

[14] H. Zhang, S. Lun, and D. Liu, "Fuzzy H_∞ filter design for a class of nonlinear discrete-time systems with multiple time delays," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 3, pp. 453–469, 2007.

[15] S. Xu and J. Lam, "Exponential H_∞ filter design for uncertain Takagi-Sugeno fuzzy systems with time delay," *Engineering Applications of Artificial Intelligence*, vol. 17, no. 6, pp. 645–659, 2004.

[16] J. Yoneyama, "H_∞ filtering for fuzzy systems with immeasurable premise variables: an uncertain system approach," *Fuzzy Sets and Systems*, vol. 160, no. 12, pp. 1738–1748, 2009.

[17] J. Yang, S. Zhong, G. Li, and W. Luo, "Robust H_∞ filter design for uncertain fuzzy neutral systems," *Information Sciences*, vol. 179, no. 20, pp. 3697–3710, 2009.

[18] H. Gao and C. Wang, "Delay-dependent robust H_∞ and L_2-L_∞ filtering for a class of uncertain nonlinear time-delay systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 9, pp. 1661–1666, 2003.

[19] K. M. Grigoriadis and J. T. Watson Jr., "Reduced-order H_∞ and L_2-L_∞ filtering via linear matrix inequalities," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 33, no. 4, pp. 1326–1338, 1997.

[20] Z. Li and S. Xu, "Fuzzy weighting-dependent approach to robust L_2-L_∞ filter design for delayed fuzzy systems," *Signal Processing*, vol. 89, no. 4, pp. 463–471, 2009.

[21] Y.-Y. Cao and P. M. Frank, "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 2, pp. 200–211, 2000.

[22] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 15, no. 1, pp. 116–132, 1985.

[23] E. Fridman and U. Shaked, "Delay-dependent H_∞ control of uncertain discrete delay systems," *European Journal of Control*, vol. 11, no. 1, pp. 29–39, 2005.

[24] C. Lin, Q.-G. Wang, T. H. Lee, and Y. He, "Design of observer-based H_∞ control for fuzzy time-delay systems," *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 2, pp. 534–543, 2008.

[25] C. Lin, Q.-G. Wang, T. H. Lee, Y. He, and B. Chen, "Observer-based H_∞ control for T-S fuzzy systems with time delay: delay-dependent design method," *IEEE Transactions on Systems, Man, and Cybernetics, Part B*, vol. 37, no. 4, pp. 1030–1038, 2007.

[26] C. Lin, Q.-G. Wang, and T. H. Lee, "H_∞ output tracking control for nonlinear systems via T-S fuzzy model approach," *IEEE Transactions on Systems, Man, and Cybernetics, Part B*, vol. 36, no. 2, pp. 450–457, 2006.

[27] X. Song, S. Xu, and H. Shen, "Robust H_∞ control for uncertain fuzzy systems with distributed delays via output feedback controllers," *Information Sciences*, vol. 178, no. 22, pp. 4341–4356, 2008.

[28] S. K. Nguang and P. Shi, "$\mathscr{H}_\infty$ output feedback control design for uncertain fuzzy systems with multiple time scales: an LMI approach," *European Journal of Control*, vol. 11, no. 2, pp. 157–170, 2005, With discussion.

[29] B. Chen, X. Liu, and S. Tong, "New delay-dependent stabilization conditions of T-S fuzzy systems with constant delay," *Fuzzy Sets and Systems*, vol. 158, no. 20, pp. 2209–2224, 2007.

[30] C. Lin, Q.-G. Wang, and T. H. Lee, "Delay-dependent LMI conditions for stability and stabilization of T-S fuzzy systems with bounded time-delay," *Fuzzy Sets and Systems*, vol. 157, no. 9, pp. 1229–1247, 2006.

[31] B. Chen and X. Liu, "Delay-dependent robust H_∞ control for T-S fuzzy systems with time delay," *IEEE Transactions on Fuzzy Systems*, vol. 13, no. 4, pp. 544–556, 2005.

[32] S. Zhou, J. Lam, and A. Xue, "H_∞ filtering of discrete-time fuzzy systems via basis-dependent Lyapunov function approach," *Fuzzy Sets and Systems*, vol. 158, no. 2, pp. 180–193, 2007.

[33] L. Wu and Z. Wang, "Fuzzy filtering of nonlinear fuzzy stochastic systems with time-varying delay," *Signal Processing*, vol. 89, no. 9, pp. 1739–1753, 2009.

[34] H. Huang and D. W. C. Ho, "Delay-dependent robust control of uncertain stochastic fuzzy systems with time-varying delay," *IET Control Theory & Applications*, vol. 1, no. 4, pp. 1075–1085, 2007.

[35] J. Yoneyama, "Robust stability and stabilizing controller design of fuzzy systems with discrete and distributed delays," *Information Sciences*, vol. 178, no. 8, pp. 1935–1947, 2008.

[36] J. Qiu, G. Feng, J. Yang, and Y. Sun, "H_∞ filtering design for continuous-time nonlinear systems with interval time-varying delay via T-S fuzzy models," in *Proceedings of the 7th Asian Control Conference (ASCC '09)*, pp. 1006–1011, Hong Kong, August 2009.

[37] Y. He, Q.-G. Wang, C. Lin, and M. Wu, "Delay-range-dependent stability for systems with time-varying delay," *Automatica*, vol. 43, no. 2, pp. 371–376, 2007.

[38] X. Jiang and Q.-L. Han, “On H_∞ control for linear systems with interval time-varying delay,” *Automatica*, vol. 41, no. 12, pp. 2099–2106, 2005.
[39] H. Shao, “New delay-dependent stability criteria for systems with interval delay,” *Automatica*, vol. 45, no. 3, pp. 744–749, 2009.
[40] K. Gu, V. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*, Birkhäuser, Boston, Mass, USA, 2003.
[41] J. Sun, G. P. Liu, J. Chen, and D. Rees, “Improved delay-range-dependent stability criteria for linear systems with time-varying delays,” *Automatica*, vol. 46, no. 2, pp. 466–470, 2010.
[42] H. Gao, J. Lam, P. Shi, and C. Wang, “Parameter-dependent filter design with guaranteed $\mathscr{H}_\infty$ performance,” *IEE Proceedings of Control Theory and Application*, vol. 152, pp. 531–537, 2005.