Lai, M. Y., & Murray, S. (2015). Hong Kong grade six students' performance and mathematical reasoning in decimal tasks: Procedurally based or conceptually based? *International Journal of Science and Mathematics Education*, *13*(1), 123-149.

# Hong Kong Grade Six students' performance and mathematical reasoning in decimal tasks: Procedurally based or conceptually based?

ABSTRACT. Most studies of students' understanding of decimals have been conducted within Western cultural settings. The broad aim of the present research was to gain insight into Chinese Hong Kong grade 6 students' general performance on a variety of decimals tasks. More specifically, the study aimed to explore students' mathematical reasoning for their use of 'rules' and algorithms and to determine whether connections exist between students' conceptual and procedural knowledge when completing decimals tasks. Results indicated that conceptual understanding for rules and procedures were built into the students' knowledge system for most of the items concerned with place value in decimals—ordering decimals, translating fractions into decimals, the representation of place value in decimals, the concept of place value in decimals on number line and the concept of continuous quantity in decimals. However, the students were not able to provide such clear explanations for the use of algorithms for the multiplication and division items. The findings are discussed in the light of Chinese perspectives on procedural and conceptual understanding.

KEY WORDS: conceptual knowledge, decimal numbers, procedural knowledge

## Introduction

Mathematics educators have had a long standing interest in students' understanding of decimal number. Much research has been undertaken about students' misunderstanding of decimal numbers and misconceptions about the meaning of decimal number notation (see for example, Bell, Fischbein & Greer,1984; Graeber & Tirosh, 1990; Moloney & Stacey, 1997; Okazaki & Koyama, 2005; Pierce, Steinle, Stacey & Widjaja, 2008; Resnick, Nesher, Leonard, Magone, Omanson & Peled, 1989; Stacey, Helme & Steinle, 2001; Steinle, 2004, Steinle & Stacey, 2003; and Steinle & Stacey, 1998). Previous studies have consistently identified three erroneous "rules" that many children use in comparing decimal numbers (Desmet, Gregoire & Mussolin, 2010; Nesher & Peled, 1986; Peled, 2003; Sackur-Grisvar &

Leonard, 1985; Stacey & Steinle, 1999; Steinle & Stacey, 2010). In summary, Resnick, Nesher, Leonard, Magone, Omanson, and Peled, (1989) named these rules the whole-number rule, fraction rule and zero rule. The whole-number rule (also known as the Longer-is-Larger rule) is the selection of the number with more decimal places as the larger of two decimals. The fraction rule (also known as the Shorter-is-Larger rule) involves the selection of the number with fewer decimal places as the larger of two decimals. The zero rule is employed when students select the decimal with zero(s) to the immediate right of the decimal point as the smaller decimal. Baturo and Cooper (1995), and Steinle and Stacey (2001) report that the zero rule always produces correct results but for an inappropriate reason.

Some researchers have concluded that many of the misconceptions held by students arise because of students' reliance on memorizing the procedures with little understanding of the associated concepts that underlie them (Hiebert, 1992). Stacey (2005) argues that students who have been classified as 'experts' because of their performance in decimal comparison tests are not actually experts because such students frequently complete the tasks by merely following syntactical 'rules' (Lachance & Confrey, 2002). Stacey (2005) contends that in reality these students have very little understanding of decimal notation. Likewise, many other studies report that decimal computation tasks among elementary and high school students are completed in a superficial way (Baturo, 1997; Bell, Swan & Taylor, 1981; Bonotto, 2005; Graeber & Tirosh, 1990; Hiebert & Wearne, 1985; Lachance & Confrey, 2002; and Okazaki & Koyama, 2005). Like Stacey, these authors argue that what students learn about decimal computation is purely 'syntactic'; that is, merely applying memorised rules to manipulate symbols in a certain sequence (Hiebert & Wearne, 1985). They point out that a common phenomenon in children's learning of operations is the tendency to acquire only mechanistic procedures. They cite as an example the procedure for multiplication of decimals: lining up the most right digit of the decimals, doing the multiplication as whole number, then counting the number of decimal places in both the multiplier and multiplicand, and finally putting the decimal point accordingly in the answer. Hiebert and Wearne (1985) further argue that not only are students relying solely on procedural knowledge (that is, syntax-based rules) but their performance also reveals deficiency in conceptual knowledge, the semantic understanding of decimal notation system and the underlying relationships between whole numbers, fractions and decimals. Simply speaking, Stacey (2005) and other scholars argue that students who complete the tasks by following syntactical rules have very little understanding of decimal notation.

Most studies of students′ understanding of decimals have been conducted within Western cultural settings. In Western mathematics education, there has been tension between procedural knowledge and conceptual knowledge (Lai & Murray, 2012) and animated discussion on their respective roles in student's learning of mathematics (Star, 2005). Studies of students′ understanding of decimal numbers in other countries, including Asian nations, are important for a number of reasons, perhaps most importantly because they can provide insights that may benefit the mathematical learning of all students. Western educators often emphasize the need for students to construct a conceptual understanding of mathematical symbols and rules before they practise the rules (Li, 2006). On the other hand, Chinese learners tend to be oriented towards rote learning and memorization (Marton, Watkins & Tang, 1997). Chinese learners have been criticized for relying solely on procedural knowledge but their performance reveals their proficiency in conceptual knowledge. The present study was conducted in Hong Kong and had two major aims: first, to gain insight into Chinese Hong Kong Grade Six students′ general performance on a variety of decimals tasks and second, to explore students' mathematical reasoning for their selection of "rules" and determine whether connections exist between students' conceptual and procedural knowledge when completing decimal tasks.

**Procedural versus conceptual understanding**

Procedural understanding involves knowledge of the rules and procedures (Hiebert & Wearne, 1986; Skemp, 1976), or the steps taken to complete a mathematics task (Fuchs, Fuchs, Hamlett, Phillips, Karns & Dutka, 1997). Wearne and Hiebert (1988) describe procedural understanding as syntactic processes which involve symbol-manipulation and routinizing the rules for symbols. Conceptual understanding involves knowing the relationship between related concepts (Wearne & Hiebert, 1988), an understanding of why a procedure works (Hiebert & Wearne, 1986) and whether a procedure is legitimate (Bisanz & Lefevre, 1992). Hiebert (1992) concludes that conceptual knowledge is knowledge that is rich in relationships but not rich in techniques for completing tasks, while procedural knowledge is rich in rules and strategies but not rich in relationships.

Conceptual and procedural knowledge tend to be dichotomised in Western mathematics education; that is: conceptual versus procedural knowledge. The need for both types of knowledge has been posited, yet in Western mathematics there is arguably a tendency to devalue the importance of procedural understanding in children's learning. Boss and Bahr (2008) for example, found that the pre-service and in-service teachers in the United States highly valued conceptual understanding in learning and considered that procedural knowledge is tantamount to no understanding at all. However, scholars investigating students' mathematical understanding in Chinese cultures have questioned the dichotomisation of conceptual and procedural knowledge (Lai & Murray, 2012). Whilst some Western scholars are inclined to associate procedural knowledge with a mere exercise of memory and rote practice, and believe that memory cannot lead to conceptual understanding (Bosse & Bahr, 2008), these two types of knowledge are closely linked when conceptualizing learning in the Chinese context.

Chinese educators are criticized as not providing a learning environment which is conducive to 'good learning' and using a teaching method which is merely 'passive transmission' (Gu, Huang & Marton, 2004), 'rote drilling' (Gu, Huang & Marton, 2004), and a 'surface approach' (Marton & Saljo, 1976). However, Chinese Hong Kong students consistently outperform their Western counterparts in many international comparative studies on mathematics achievement such as TIMSS (Beaton, Mullis, Martin, Gonzalez, Kelly & Smith; 1997; Mullis, Martin, & Foy; 2008). This includes items within these tests specifically concerned with decimal numbers (see for example, Mullis, Martin & Foy, 2008; p.119 and p.123). This contradictory phenomenon is referred to by Marton, Dall' Alba and Lai (1993) as the 'paradox of the Chinese learner'. Watkins and Biggs (2001) conclude that the 'paradox of the Chinese learner' might be a misconception by Western scholars arising from limited understanding of the philosophies and theories of learning and teaching in the Chinese context.

De Jong and Ferguson-Hessler (1996) and Star (2005) disagree with the Western interpretation of procedural knowledge and conceptual knowledge and argue that these terms suffer from an entanglement of knowledge *type* and knowledge *quality*. Star (2005) discusses the concept of "deeper procedural knowledge, in which knowledge of procedures is associated with comprehension, flexibility, and critical judgment and is distinct from (but possibly related to) knowledge of concepts" (p.408). Likewise, Biggs (1996) and Kember (1996; 2000) argue that 'surface learning' (Marton & Saljo, 1976) and 'deep learning' (Marton & Saljo, 1976) are generic terms and therefore, the interpretation should be dependent on the context, the task, and the individual's encoding of both. They claim that the distinction between surface learning and deep learning lies in the learner's intention, or the absence of intention, to understand. If a student attempts to make sense of what is to be learned, to internalize their learning and relate it to his/her real life by using his/her own methods including memorizing and repetition, this student is using a deep approach to learning. For this learner, rote learning, memorizing and repetition are only a learning strategies which help him/her plan ahead and monitor his/her own learning progress, all of which, Biggs (1996) considers part of a learner's metacognition. Similarly, Haller, Fisher and Gapp (2007) argue that repetitive learning uses repetition as a means of ensuring correct recall and is intentional in some point in the development of the learned material. In this context, repetitive learning is used as a strategy to memorise some learned materials for the learners' further understanding. This argument is further evidenced by the results of many studies that consistently find that Chinese teachers and students do not see memorizing and understanding as separate, but rather as interlocking processes, complementary to each other (Biggs, 1996; Dahlin & Watkins, 2000; Kennedy, 2002; Marton, Dall' Alba & Tse, 1996; Marton, Watkins & Tang, 1997; Waktins & Biggs, 2001; Wang, 2006). Marton, Watkins and Tang (1997) conclude that memorisation and understanding are structurally related in the way that Chinese students switch their learning between an emphasis on memorisation (that is; procedural knowledge) and an emphasis on understanding (that is; conceptual knowledge).

As noted earlier, the broad aim of the current study was to investigate Chinese Hong Kong Grade Six students′ general performance on a variety of decimals tasks. These tasks include comprehending place value after the decimal point; comparing the size of decimals and the proper use of the algorithms for computation of decimals. Further related aims of the study were to explore students' mathematical reasoning for their selection of "rules" and to determine whether connections exist between students' conceptual and procedural knowledge when completing decimal tasks. In summary, we aimed to understand how Chinese Hong Kong Grade six students completed decimals tasks, and in particular how their conceptual understanding for procedures was built into their knowledge system.

**Overview of interpretive framework**

Sumpter (2013) presents a cogent case for the use of theoretical frameworks to understand and characterise students' mathematical reasoning. However, as Sumpter (2013) and Lithner (2008) argue, there are relatively few frameworks that provide sufficiently clear definition to allow for different types of reasoning to be categorized. In this study, an interpretive framework was used to analyse students' written tests and interviews, and to determine whether links between conceptual and procedural knowledge had been built into students' knowledge system. The framework is based largely on Hiebert and Wearne's work (Heibert, 1992; Hiebert & Wearne, 1986) and is supported by other research findings (e.g., Bell, Fischbein & Greer, 1984; Bell, Swan, and Taylor, 1981; Greer, 1987; Hiebert & Wearne, 1985; Lachance & Confrey, 2002; Okazaki & Koyama, 2005; Stacey, 2005). This framework was chosen as it provides categories of knowledge specific to the topic of decimal number and embeds different but related types of "sub-knowledge".

According to Hiebert (1992), three different types of knowledge are distinguished in the decimal knowledge system: *knowledge of the notation*, *knowledge of the symbol rules* and *knowledge of quantities and actions on quantities*. In summary, *knowledge of notation* refers to the "knowledge of the symbols that are used to write decimal fractions and knowledge of the form that constrains how the symbols are positioned on paper" (Hiebert, 1992; p.290). This type of knowledge does not require understanding of what the symbols mean and what quantities they represent. For example, for the task of comparison of decimals, Stacey (2005) comments that some students can easily provide correct answers but provide incorrect mathematical reasons through the use of the three erroneous 'rules' discussed earlier. *Knowledge of the symbol rules* is knowledge of "the rules that prescribe how to manipulate the written symbols to produce correct answers" (Hiebert, 1992; p.290). For example, at the syntactic level, the rule for adding and subtracting decimals can be described in terms of lining up decimal points (Hiebert, 1992). Another example, the multiplication rule for decimals, has been described earlier (Hiebert & Wearne, 1985). These rules will produce correct answers if followed precisely step-wise, even without fully understanding the underlying concepts. *Knowledge of notation* and *the symbol rules* can be categorised as knowledge for procedural understanding.

*Knowledge of quantities and actions on quantities* represents the "knowledge of concrete or visual objects that can be measured by units, tenths of units, hundredths of units, and so on" (Hiebert, 1992; p.291) and "includes knowledge of what happens when the quantities are moved, partitioned, combined, or acted upon in other ways" (Hiebert, 1992; p.291). Hence, this type of knowledge provides the meaning for symbols and notation, and reasons for the rules. It is best understood as knowledge for conceptual understanding. Hiebert (1992) points out that "there is an intended relationship among the three kinds of knowledge" (p.291).

Hiebert (1992) describes place value in the decimal number system in the following way: "the value of a particular position is determined by beginning with the unit and, if moving to the right, dividing the previous value by 10 and, if moving to the left, multiplying the previous value by 10; and the ones position is marked with a decimal point on its immediate right" (p.286). This is the fundamental knowledge of quantities for decimal notation and forms the basis for the procedural rules. Thus, combining the digits with the same positional value is the underlying concept for 'lining up the decimal points' for addition and subtraction of decimals. For multiplication of decimals, the procedural rule is 'counting the decimal places in multiplicand and multiplier for locating the decimal point in the answer'. So, for example, for the problem 0.1×0.02, the notion of hundredths of tenths is the underlying concept for the procedural rule. Similarly, for division of decimals, (for example 9.5÷0.5), obtaining equivalent fractions (i.e., 95/5) is the underlying concept for 'multiplying both the dividend and divisor by 10' before executing division.

## Method

### Participants

Three hundred and eighty-four Hong Kong Grade Six students (mean age of 11 years 7 months, standard deviation of 5.3 months) completed a written test on decimal numbers. The students were from six elementary schools located in different districts with varied socio-economic status. A further sample of 31 students was then interviewed individually to further explore their mathematical reasoning for their answers. These students were chosen on the basis of response on the written test. The sample was randomly selected from those students whose answers revealed typical misconceptions reported in the research literature or unusual misconceptions.

### Instruments

A written test was designed to assess students' conceptions of decimal numbers in general and their procedural knowledge in particular. A follow-up interview was designed to further elicit and diagnose students' ways of thinking so that their mathematical reasoning could be more fully understood. The interview also aimed to explore students' conceptual knowledge of decimal numbers. Overall, a close examination of the responses to both the written test and follow-up interview enabled a comprehensive understanding of students' use of procedural and conceptual knowledge, and any link between them.

**Instrument one –Written test**

A written test (refer to Table 1 for sample test items) on decimal numbers was constructed with reference to the Hong Kong primary mathematics curriculum (The Education Department, 2000).

**<insert Table 1: Sample test items here>**

A content validity panel was set up, which included three Hong Kong elementary mathematics educators, ten experienced elementary mathematics teachers and two pre-service teachers undertaking a university course in elementary school mathematics education. The test items were modified in response to advice provided by the panel. A pilot study was undertaken to establish test-retest reliability and check for clarity of language. A class of grade 7 students (in their first year of secondary school) was given the test on two occasions, with a period of time in between of three months. Cronbach's alpha was 0.86, indicating a high degree of reliability.

**Instrument two – Interview**

The main focus of the interview was to explore students' mathematical reasoning for their answers and their conceptual knowledge of decimal numbers. Students re-completed each item in the test, in the presence of the interviewer. The items were the same as the written test items, except that the numbers were changed for each question. When students finished each item they were asked to describe the procedures they had used. Follow-up questions were directed towards understanding the mathematical reasons for students' selection of a 'rule'. In each instance, direct questions about the procedure used were followed with questions about the particular 'logic' that made the identified rules work in that instance. The aim of these questions was to elicit the procedural and conceptual knowledge of the participants and whether there were links between these two types of knowledge.

## Procedure

The written test was in Chinese, the students' first language. It was administered collectively to the students in their classrooms one month before the end of the semester. Each student worked individually, in silence and without time constraints. All students completed the test within 35 minutes. The test was marked and scored immediately after students′ works were collected. Thirty-one students whose written test evidenced typical and/or atypical misconceptions were selected for the interview. Of the 31 students who were interviewed, seven students scored nearly 98% in their written tests, five scored less than 40% and the remainder scored between 50% and 80%. Students were asked questions about decimal numbers such as number identification, place value, counting, comparing of decimals; and algorithms for addition, subtraction, multiplication and division of decimals.

# Results and discussion

## The written test

The results of the written test are summarised in Table 2. As noted earlier, Chinese Hong Kong students score highly in many international mathematics written tests, and most of the findings of the current study are consistent with this trend.

**<insert Table 2: Percentage of scores for each of the questions (N=384) here>**

### A. Comparison of decimals

For the six questions dealing with comparison of decimals (Type A), students were required to choose a larger decimal from a pair of incongruent length of decimals for six pairs. Nearly 78% students made no errors and only 2% got all six questions wrong. An analysis of the students' responses was undertaken, looking for the application of the three types of 'rules' outlined earlier: the longer-is-larger rule, shorter-is-larger rule and the zero rule. About 5% students revealed a shorter-is-larger misconception, about 4% applied the longer-is-larger rule and the zero rule in conjunction. The last group displayed error types of various kinds throughout this task.

**B. Convert fraction with denominators of either 10 or 100 to decimals**

For the two questions of type B, students were required to translate fractions with a denominator expressed as power of 10 to decimal numbers. About 75% of the students provided correct answers for both items. About 5% provided incorrect answers for both questions. The most frequent errors on the task were 0.05 or 0.005 for $\frac{500}{1000}$ and 0.9 or 0.009 for $\frac{9}{100}$. The results indicate that about one quarter of students did not have well-established understanding of the relationship between decimal symbols and fraction quantities.

**C. The representation of place value in decimals**

For the three questions of type C, students were required to indicate what a digit in a certain position of a decimal represents; for example, what does 7 mean in 0.723?. Over 85% of the sample provided correct answers for all of the items of this type and over 8% got all of the questions wrong. Not surprisingly, the most frequent errors (60% of the incorrect responses) were "seven 100s, two 10s and three ones" and the second most frequent errors (40 % of the incorrect responses) were "seven 10s, two 100s and three 1000s" for 0.723. The results reflect the fact that some students did not have a well-developed understanding of the notation of decimals - they might have interpreted place value of digits after the decimal point by "mirroring" the concept of whole number (Hiebert & Wearne, 1986).

**D. The concept of place value in decimals on a number line**

For questions in category D, students were required to indicate the number next to 0.9 on a number line. Over 88% of students provided correct answers. It is not surprising that the most frequent error was 0.10. The results reflect the fact that some students treated the number after the decimal point as a whole number – 10, but not 1, comes next to 9 in the whole number system.

**E. The concept of continuous quantity in decimals**

For question E, students were required to give a decimal between 0.75 and 0.8. Eighty-three percent of students provided the correct answer. Among the students who provided incorrect answers, two-thirds answered with a decimal smaller than 0.75 and one-third gave a decimal bigger than 0.8.

**F. The addition and subtraction of decimals**

The questions in set F were addition and subtraction computation tasks. Over 83% of students provided correct answers for all of the items. Most of the incorrect responses were due to general arithmetic errors in whole number computation such as 're-grouping up and down' mistakes. One student did not line up decimal points for these items, and was therefore unable to provide correct answers. The results illustrate the general finding that some students had mastered the basic concept of place value in decimals and addition of digits of the same place values but failed to regroup correctly.

**G. Multiplication of decimals**

Question G was a multiplication computation task. Over 80% of students provided the correct answer for this item. Almost all the students who provided correct answers used the multiplication rule (that is, lined up the most right digit) as discussed earlier. Among those students who scored no marks, nearly 50% used the multiplication rule correctly but made some general arithmetic errors in calculation. In general, these responses contained the correct location of decimal points in incorrect answers. Another 50% undertook the arithmetic correctly but located the decimal points in the wrong place in their answers. This type of error revealed that some students had developed very good multiplication techniques but believed that ‘multiplication always produces a bigger number’, a common misconception in decimal number multiplication.

**H. Division of decimals**

Question H was a division computation. Over 84% of students provided correct answers for this item. Over 90% completed the question by applying the division rule for decimals as discussed earlier. Of those students who got this question wrong, 60% students located the decimal points in the wrong places, and provided either 4 or 0.4 as their answers. A few students made general whole number division errors in algorithms.

**Summary of the written test results using interpretive framework**

Overall, the results of tasks A to C indicated that over 70% of students exhibited good *knowledge of notation*. As mentioned earlier, this knowledge does not require understanding of the meaning for numerical symbols and its connection with quantitative referents. Similarly, for other tasks such as converting fractions to decimals and the representation of place value in decimals, it could be argued that students who provided correct answers may not necessarily be able to connect written symbols to the actual quantities they represent. However, the results of tasks D and E revealed that over 85% of students exhibited a fairly good understanding of *knowledge of quantities and actions on quantities*. In the task concerned with concept of place value in decimals on a number line, students showed their understanding of the quantities of 0.7, 0.8 and 0.9, and were able to report the ‘decimal’ after 0.9. Hiebert (1992) points out that the key feature of decimal notation is that there are infinite decimals in between two decimals. Thus, examining students’ understanding of the concept of continuous quantity of decimals (Hiebert, 1992) is crucial for assessing whether students fully comprehend decimal numerical symbols. Task E revealed that the great majority did understand the continuous nature of decimals. If students mirror the whole number concept onto the digits to the right of the decimal points, they cannot provide correct answers for this task. The results of task F indicated that over 83% of students had mastered good understanding of the *knowledge of the symbol rules* for addition and subtraction of decimals. Almost all students followed the syntactic rules: lining up decimal points for all the items in this task. Comparatively, students did not perform well in tasks G and H, multiplication and division of decimals. About 35% of students made different types of procedural errors, such as wrongly locating the decimals point in the answers, making arithmetic errors, or using the wrong algorithms. The results revealed that some students did not have a firm understanding of *knowledge of the symbol rules* for multiplication and division of decimals.

**The interview**

As noted earlier, the focus of the interview was to explore whether connections exist between students' conceptual and procedural knowledge when completing decimal tasks. The interview protocol provided an examination of the kinds of individual rules and procedures that students use and the kinds of justifications they give for their answers. Pseudonyms are used in the interview protocols.

**i. The concept of place value and visible zero in decimals**

In order to investigate how students interpret the digit zero in critical positions in decimals (in the tenths column, on the far right and interspersed throughout other digits), students were asked to explain why or why not one can 'take away' the zeros from decimals. A simple syntax convention for whole number is that a zero to the right of a whole number increases the number by a factor of 10, whereas placing a zero on the left has no effect (Hiebert & Wearne, 1986). An opposite type of syntax applies to decimal numbers. For example, there is no effect on the value of a decimal if a zero is placed on the far right, but the value is decreased by a factor of ten if a zero is placed immediately to the right of the decimal point. All of the 31 students provided the correct answer that the zero in 1.035 could not be taken away while the zero in 2.340 could. The typical explanation given by students is illustrated in the following interview protocols:

> *Ah Lai: The 3 in 1.035 is the hundredth. You can't take away the 0. But you can do it to 2.340.*
>
> *Tai Wai: The 3 in 1.035 is the hundredths. You can't take away the zero as there are still some digits after the zero. But you can do it to 2.340 as the zero is already the last digit.*
>
> *Ching Yee: The 3 in 1.035 is the hundredth. You can't take away the 0 in 1.035 and you can only do it when it appears at the back. I am talking about decimals only. If you take the zero away from 1.035, it would be funny as something was missing.*

The explanations in the preceding protocols exhibit strong procedural knowledge through students' *knowledge of notation* but do not clearly reveal whether or not students had an accompanying conceptual understanding. However, nearly half of the interviewees could further elaborate upon their explanations. The following protocols exemplify the mathematical reasoning the students used:

> *Mei Fan: If you remove the zero in 1.035, the actual value will become bigger. It is because 35 will move to the left and therefore, the decimal will become bigger.*
>
> *Chi On: The 3 in 1.035 is in the hundredths. You can't take the zero away because it is in the tenths. If you do so, 3 will change from hundredths to tenths and 5 from thousandths to hundredths and therefore, the decimal will become bigger. You can do it to 2.340 because there isn't any digit after the zero.*

These students are clearly exhibiting an understanding of the *knowledge of quantities and actions on quantities*. Another three students mentioned that the zero at the back of a decimal number was meaningless and therefore, could be taken away. One student who provided the same mathematical reasoning as Chi On emphasised that his explanation was only applicable to decimal numbers. Another student commented that the 'back zero' was redundant. The findings indicate that students understand that the 'back zero' has no effect on the value of decimals but the interspersed zero is significant as it represents an actual value of zero in the place where it is positioned. In general, the interview responses revealed that many of the students had acquired and internalised the *knowledge of quantities* for decimal notation as they could describe the change of place value of digits from 'hundredths to tenths' and from 'thousandths to hundredths' when the zero was removed. More importantly, they recognised the change in quantity of digits from smaller to bigger when 'moving' from the right to the left. This showed that students had established a connection between the numerical symbols and the quantitative meaning for those symbols. The students' explanations were not as explicit as Hiebert's summary (presented earlier), but certainly indicate that many of the Hong Kong Grade Six interview participants had a very good understanding of place value and the significance of a visible zero in decimal number numeration. Thus their *knowledge of notation* and *knowledge of quantities and action on quantities* were intrinsically linked.

Five students who provided incorrect responses to an item on representation of place value (for example, what does 7 means in 0.732?) were questioned to elicit their reasoning about the concept of place value and the mathematical reasons for their misconceptions. They were asked to give meanings to the digits in a decimal. The following protocols illustrate some of their responses.

*Yuen Ling: In 1.256, 5 is five tenths and 6 is six ones.*

*Tsz Lan: In 1.256, 1 represents one 0.001, 2 represents two 0.01s, 5 represents five 0.1s and 6 represent six 1s because 6 is at the back.*

Both Yuen Ling and Tsz Lan incorrectly interpreted the last digit of decimals as a unit of one and when the digit moves to the left, thought that the value is divided the previous value by ten. Their misconception may arise from their incomplete understanding of decimal numbers; and the quantitative meaning of place value. They assumed that the major difference between whole number and decimal number was the direction of progression of place value, when moving to the left, multiplying the previous value by 10 for whole numbers but dividing the previous value by 10 in decimals.

**ii. Comparison of decimal numbers**

All 31 students were asked to compare the size of three pairs of decimals and to arrange 0.503, 0.53, 0.053 and 0.0503 from smallest to largest. This task was aimed at investigating mathematical reasoning use to make decisions about the size of decimals and how students order decimals in comparison with zero (Steinle & Stacey, 2001). Twenty-seven of the students provided correct answers to all of the questions in this set. The following quotes illustrate students' typical mathematical reasoning for deciding the size of decimals.

> *Kai Chung: My answer is 0.0503<0.053<0.503<0.53. You just compare the digits of each place one by one. In 0.0503 and 0.053, there are two zeros in the front, so you compare the 5 but they are the same. You then compare 0 and 3 and therefore 0.053 is bigger than 0.0503. You can use the same method to compare 0.503 and 0.53.*

The students were further asked to generalise the rule they were applying. Many students illustrated their sound procedural knowledge (*knowledge of notation*) for comparing the size of decimals.

As noted earlier some scholars (for example, Steinle & Stacey, 2001) argue that those students who provide correct answers are not necessarily true experts because they memorise the syntactic rules without understanding the underlying concepts for the rules. In fact, in the present study, many of these students were able to explain that they firstly compared the tenths of the pair of decimals, then hundredths and thousandths and so on until one digit was bigger than the other of the same place value in the pair. The following quotes illustrate this point:

> *Mei Fan: When comparing 0.0503, 0.053, 0.53, 0.503, you firstly compare the tenth, then the hundredth and so on.*
>
> *Mei Fan: In 0.457 and 0.4, there is no digit in hundredth in 0.4, you can add zero in, you then compare their hundredth, 5 is bigger than 0, therefore, 0.457 is bigger than 0.4.*
>
> *Ho Yan: 0.384 is bigger than 0.32. Firstly, you compare their tenths but they are the same (i.e., 3). Then you have to compare their hundredths, because 8 is bigger than 2, therefore, 0.384 is bigger than 0.32.*

These responses indicate that the students knew and comprehended the numerical symbols of decimal as an integrated conceptual system; that is, the actual quantities that the symbols represent.

Irwin and Britt (2004) have also questioned students' apparent 'expertise'. They argue that sometimes, students' answers are outcomes of mechanical memorisation of some syntax rules and comment that some students operate with decimals "by rounding or truncating them to the nearest number that they can easily understand" (p.312). The following students' mathematical reasoning could be seen to support Irwin and Britt's point, but instead we argue that their explanations are not merely the outcomes of mechanical memorisation but also reveal their sound number sense and understanding of place value:

> *Man Lun: In 3.06 and 3.053, 3.06 is bigger because 0.06 is closer to 1.*

*Wai Hung: When comparing 0.384 and 0.32, you firstly round off 0.384 to the nearest hundredth and it is now 0.38. Then, as 38 and 32 are the hundredth, 38 is closer to 100 than 32. So, 0.384 is bigger than 0.32.*

*Li Li: Ok, when comparing 0.384 and 0.32, the 4 in 0.384 is less than 5 and so you can round off 0.384 to 0.38. Now, 8 is bigger than 2 and therefore, 0.384 is bigger than 0.32.*

*Pak Yin: In 0.025 and 0.0025, 0.025 rounds off to 0.03 and 0.0025 rounds off to 0.003. 0.03 is bigger than 0.003, so 0.025 is bigger than 0.0025.*

The students' responses exhibited good understanding of decimal quantities and actions on quantities.

As mentioned earlier, four of the thirty-one students provided partly correct responses. They were identified having Shorter-is-Larger misconceptions:

*Wai Man: 0.32 is bigger than 0.384. The reason is, 0.384 means a whole is divided by 384 equal parts and therefore each share is smaller than those on 0.32 which means a whole is divided into 32 equal parts.*

Wai Man was then asked to arrange decimals from smallest to largest and to explain his answer:

*Wai Man: My answer is 0.0503<0.053<0.503<0.53. It is because 0.0503 has the longest decimals so it is the smallest. In 0.053 and 0.503, there is a zero after the decimal point in 0.053 so it is smaller than 0.503. 0.53 has the shortest decimal and therefore it is the largest.*

Wai Man's responses are surprising in that he seemed to have good understanding of the notation of fractions but confused the relationship between fractions and decimals.

The following protocol illustrates a student who generally presented the right answers but did not have consistently strong conceptual understanding. The student arrived at correct and incorrect conclusions through her use of the ‘shorter is larger rule’, and the interview reveals that she sometimes arrived at correct answers for the wrong reasons. However, when she was unable to apply the rule, she did show conceptual understanding through her comprehension of decimal quantity:

*Siu Fung: 0.32 > 0.384 because the more the decimal places, the smaller the value.*

*Interviewer: How about 0.56 and 0.25?*

*Siu Fung: 0.56 > 0.25 because 5 is the tenths and bigger than the 2 in 0.25.*

*Interviewer: OK, this time 3.06 and 3.053, which one is bigger?*

*Siu Fung: 3.06 > 3.053 because more the decimal places, smaller the value.*

*Interviewer: Which one is bigger, 0.025 or 0.0025?*

*Siu Fung: 0.025 > 0.0025 because more the decimal places, smaller the value.*

*Interviewer: Can you re-arrange 0.0503, 0.053, 0.53, 0.503 from the smallest to the biggest?*

*Siu Fung: 0.0503 < 0.053 < 0.503 < 0.53 because there are 4 decimal places in 0.0503 so it is the smallest. Comparing 0.053 and 0.503, there is a 0 after the decimal point in 0.053 and therefore smaller than 0.503. 0.53 is the shortest and therefore is the biggest.*

Another student used the Shorter-is-Larger rule, and also provided correct answers for most of the comparison tasks. The mathematical reasoning was only partly correct but again revealed some conceptual understanding:

*Suk Ping: When comparing “3.06 and 3.053”; “0.023 and 0.0023”, less decimal places, the decimal is bigger because they are closer to 1.*

*Interviewer: Then how about the size of 0.0503, 0.053, 0.53, 0.503?*

*Suk Ping: When comparing 0.0503, 0.053, 0.53, 0.503, more decimal places, the decimal is smaller; less decimal places, the decimal is bigger. In 0.503 and 0.053, they have three decimal places, so you have to compare their tenth, 5 is bigger than 0, therefore, 0.503 > 0.053. So, my answer is 0.0503 < 0.053 < 0.503 < 0.53.*

None of the interviewed students used the Longer-is-Larger rule to determine the size of decimals.

Generally speaking, many of the students who participated in the interview provided fairly clear explanations for their selection of rules when completing items concerned with place value of decimal, a visible zero in decimals, and ordering decimals. On further questioning many students were able to provide conceptual explanations for the procedures they used. The findings are consistent with previous studies (such as Steinle & Stacey, 1998) that some of the students are genuine 'experts', with a solid conceptual understanding of decimal notation.

**iii. The addition and subtraction of decimals**

Students were asked about the rationale for lining up decimal points for computing addition and subtraction and to explain why it works. Most students could not explain explicitly that you need to add 'the digits with the same positional value' (units together, tenths together, hundredths together and so on) but were able to verbalise that we could not add two digits together if they were at different *positions* (i.e., place values). The following protocol is typical of students' responses:

*Interviewer: How can you get the answer for 11.24 - 3.07?*

*Tai Wai: You firstly line up the decimal point. Then you treat them as whole numbers such that you subtract 307 from 1124.*

*Interviewer: How about 5.02 + 1.99?*

*Tai Wai: Same as subtraction. Line up the decimal point first.*

*Interviewer: Why do you need to align the decimal point? How about if you do not align it?*

*Tai Wai: You cannot do that.*

*Interviewer: Why not?*

*Tai Wai: They are not the same.*

*Interviewer: What are not the same?*

*Tai Wai: Their positions (pointing at the digits) are not the same*.

Certainly, the student's responses exhibited sound *knowledge of the symbol rules* for addition and subtraction of decimals. We argue that they also display some conceptual understanding, in that they reveal some *knowledge of quantities and actions on quantities*. Though the student did not state verbally that digits of different positional values could not be combined, he pointed at the digits and indicated that he could not add the digits because 'their positions' were different. It is possible that students did in fact have an understanding of what they were doing – 'combining the digits with the same positional value', but were unable (or chose not) to provide a more mathematically-based explanation. It is also likely that some students were limited by their capacity to express abstract mathematical ideas, and this obscured some of their genuine understanding of the concepts. This possibility will be explored later.

Two students who scored very low marks in the written test, believed that if the decimal points were not aligned, then the decimal points would appear in two different places in the answer. The following interaction exemplifies this point:

*Interviewer: Why do you have to line up the decimal point?*

*Wai Man: If not, you would have two decimal points in two different places and you do not know which place is for correct answer.*

*Interviewer: Can both be correct?*

*Wai Man: No.*

*Interviewer: Why not?*

*Wai Man: They are different.*

*Interview: What are different?*

*Wai Man: The answers are different. So, I do not know which one is correct and which is incorrect.*

Apparently, these students understood that this would violate the syntactic conventions and consequently would not be correct if the decimal points were not aligned. Again, the results displayed students' strong procedural understanding of *the symbol rules,* but without accompanying conceptual understanding.

**vi. Multiplication and Division of decimals**

Students were asked about the rationale for the algorithmic procedure for multiplication and division of decimal numbers. Only three out of thirty-one students were able to make the connection between fractions and decimal numbers for division of decimals. One student gave a sophisticated explanation:

*Interviewer: Why do you need to multiply the numbers by 10 before doing* $9.5 \div 0.5$ *?*

*Pak Yin: To multiply the numbers is to expand the fraction to its equivalent form.*

The student meant that you could rewrite $9.5 \div 0.5$ as a fraction, $\frac{9.5}{0.5}$. To multiply both numbers is to make an equivalent fraction, that is $\frac{95}{5}$ - because $9.5 \div 0.5$ is best considered as $95 \div 5$ and therefore, division of whole number can be applied. However, none of the students could provide a conceptual explanation for the rationale for multiplication of decimals. The following is a typical answer, offered by most of the students:

> *Tai Wai: You have to firstly line up the most right of the decimal numbers. Then, you may just ignore the decimal points and apply the rule of multiplication for whole number. Finally, you have to count the decimal places in multiplicand and multiplier for locating the decimal point in the answer.*
>
> *Interviewer: Do you line up the decimal point?*
>
> *Tai Wai: No need. You only line up the most right digits.*
>
> *Interviewer: Do you think you still can get a correct answer if you line up the decimal point?*
>
> *Tai Wai: Why? You don't need to.*
>
> *Interviewer: OK. Then what is the reason to count the decimal places in multiplicand and multiplier for locating the decimal point in the answer?*
>
> *Tai Wai: If not, you do not know where you locate the decimal point.*
>
> *Interviewer: But why the rule, I mean, to count the decimal places in multiplicand and multiplier, works. What is logic for this rule?*
>
> *Tai Wai: I think I was taught but can't remember.*

The student's response indicates that he did not understand the notion of tenths of tenths, or tenths of hundredths. Thus the interviews revealed students' strong procedural understanding of the *knowledge of the symbols rules* for multiplication and division of decimals but more limited conceptual *understanding of quantities and actions on quantities.*

## Conclusion

Overall, the students performed well on the written test. They performed particularly well on the items concerned with place value in decimals - Tasks A to F. For these types of questions, about 75% of students provided the correct answers for all items. Some students found the tasks involving multiplication and division of decimals more challenging, but approximately two-thirds of all students were able to answer all of these questions correctly. The overall results revealed that the students had mastered good procedural understanding of decimal numbers; they had a sound *knowledge of notation* and were able to express clear *knowledge of the symbol rules*.

However, one may question how much of students' genuine understanding - other than some aspects of their procedural knowledge - can be gleaned from the written test. As noted earlier, some scholars have argued that knowledge which only involves written symbols in the syntactic system and sets of rules and algorithms; and which is rich in techniques for completing tasks does not qualify as genuine understanding. Hiebert et al. (1986) noted, for example, that being able to provide correct labels for the place value of each position in a decimal or to provide correct answers for the value of each position could be isolated pieces of procedural knowledge which in themselves do not imply knowledge of the embedded relationships.

The question of whether connections existed between students' conceptual and procedural knowledge could not be answered unequivocally for the written test items but the interview provided some valuable insights. The most striking finding provided by the individual interviews was how closely the students' procedural knowledge linked with their conceptual knowledge for some of the questions, particularly the items concerned with the concept of place value and the visible zero and, the comparison of decimals. Interview protocols for these types of questions reveal that many of the students provided unambiguous description of rules and clear explanation for the underlying procedures for most of the items. It appeared that conceptual understanding for *rules* and *procedures* were built into the students' knowledge system for these items and that the students had achieved an integration of procedural and conceptual understanding.

The students were not able to provide such clear explanations for the use of algorithms for the multiplication and division items. However, these algorithms are difficult to describe and more difficult to justify mathematically. This point raises a more general question about what is reasonable to expect from Grade Six students in terms of verbal explanations which indicate conceptual understanding of multiplication and division of decimal numbers. Multiplication of decimals can be understood as recursive partitioning; that is, taking a partition of a partition (Izsak, 2008). As discussed earlier, for the problem 0.1×0.02, the notion of hundredths of tenths is the underlying concept for the procedural rule: counting the decimal places in multiplicand and multiplier for locating the decimal point in the answer. Similarly, for division of decimals, (for example 9.5÷0.5), obtaining equivalent fractions (i.e., 95/5) is the underlying concept for "multiplying both the dividend and divisor by 10" before executing division.

These complex rationales difficulty highlight a limitation of the interview process as a way of capturing students' conceptual understanding: students' ability to explain concepts and reason 'out loud' may be restricted by their capacity to express themselves in abstract, mathematical ways and by their capacity to explain lengthy complex procedures. In some cases it may also have been limited by students' reluctance to speak or shyness. It may also be the case that not all students are provided with explanations of why the decimal multiplication and division algorithms work. In fact, such explanations are not part of the mandated instruction in the Hong Kong primary mathematics curriculum. Mathematics teaching in Hong Kong is driven by the subject matter knowledge that students are supposed to learn – that is, by the official curriculum guide (The Education Department, 2000) in which learning objectives of each topic are provided. According to the curricular document, ‘multiply decimal by decimal’ (The Education Department, 2000; p.40) and ‘divide decimal by decimal’ (The Education Department, 2002; p.42) should be taught in Grade five and Grade six respectively. However, the curricular document does not discuss how this subject content knowledge should be covered. Thus, enactment of the curriculum depends heavily on teachers’ mathematical knowledge and specific instructional practices in individual schools. For a better understanding of why only a few students provided explanations for decimal multiplication and division algorithms, further investigation is needed. Overall however, the interview provided insights into students′ reasoning that were not available from the written test. The findings highlight the value of fine-grained questioning for uncovering students′ reasoning, including their misconceptions, both typical and idiosyncratic.

Studies of students′ understanding of decimal numbers in other countries can provide insights that benefit the mathematical learning of all students. As discussed earlier, conceptual and procedural knowledge tend to be dichotomised in Western mathematics education; while Chinese scholars have emphasised the connections between the two (Lai & Murray, 2012). Students in Hong Kong, and indeed elsewhere, may become proficient at procedures through repetition, rote learning and memorizing. These learning strategies are not viewed by Chinese educators as inherently superficial or counter to the development of ‘expertise’. Li (2006) argues that routine practice should not be simply interpreted as mechanically memorizing and imitating rules and skills.  Instead, it provides students with a necessary condition of concept formation and is the first step of mathematics comprehension (Li, 2000). Similarly, Wong (2006) points out that repetition until understanding is internalised could be a general strategy employed to bring deeper understanding. Thus, as well as providing a convenient mechanism for computation, mathematical algorithms can help to make mathematical sense. As pointed out earlier, Chinese teachers and students do not see memorizing and understanding as separate, but rather as interlocking processes, complementary to each other.

The current study has provided some qualitative insights into the links that Grade Six Hong Kong Chinese students make between procedural and conceptual knowledge. Despite some limitations, the interview data provide evidence that for some question types the students′ selection of appropriate rules, procedures and algorithms was not arbitrary but conceptually based.  For these questions, students′ procedural and conceptual understanding were closely related. Further research exploring the nature and extent of connections that students make between procedural and conceptual understanding of decimals would be valuable.

**Reference**


Bell, A.W., Fischbein, E., & Greer, B. (1984). Choice of operation in verbal arithmetic problems: The effects of number size, problem structure and content. *Educational Studies in Mathematics, 15*(2), 129-147.

Bell, A., Swan, M., & Taylor, G. (1981). Choice of operation in verbal problems with decimal numbers. *Educational Studies in Mathematics, 12*(4), 339-420.

Baturo, A. R. (1997). The implication of multiplicative structure for students’ understanding of decimal-number numeration. In Biddulph, F. and Carr, K., (Eds.) *Proceedings People in Mathematics Education: 20th Annual conference of the Mathematics Education Research Group of Australasia (Vol. 1*, pp.89-95). Rotorua, New Zealand: MERGA.

Baturo, A. R., & Cooper, T. J. (1995). Strategies for comparing decimal numbers with the same whole-number part. In S. Flavel et al. (Eds.). GALTHA: *Proceedings of the 18th Annual Conference of the Mathematics Education Research Group of Australasia* (pp.73-79). Darwin, Australia: MEGA.

Beaton, A. E., Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Kelly, D. L., & Smith, T. A. (1997). *Mathematics Achievement in the Middle School Years: IEA's Third International Mathematics and Science Report.* Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

Biggs, J. (1996). Western misconceptions of the Confucian-heritage learning culture. In D. Watkins & J. Biggs (Eds.), *The Chinese learners: Cultural, psychological and contextual influences* (pp.45-67). Melbourne, Australia: ACER.

Bisanz, J., & LeFevre, J. (1992). Understanding elementary mathematics. In J. I. D. Campbell (Ed.), *The nature and origins of mathematical skills* (pp. 113–136). Amsterdam: North Holland.

Bonotto, C. (2005). How informal out-of-school mathematics can help students sense of formal in-school mathematics: The case of multiplying by decimal numbers. *Mathematical Thinking and Learning, 7*(4), 313-344.

Bosse, M. J., & Bahr, D. L. (2008). The state of balance between procedural knowledge and conceptual understanding in mathematics teacher education. *International Journal for Mathematics Teaching and Learning,* November.

Dahlin, B. & Watkins, D. (2000). The role of repetition in the processes of memorizing and understanding: A comparison of the views of German and Chinese secondary school students in Hong Kong. *British Journal of Educational Psychology, 70*(1), 65-84.

De Jong, T., & Ferguson-Hessler, M. (1996). Types and qualities of knowledge. *Educational psychologist, 31*(2), 105-113.

Desmet, L., Gregoire, J., & Mussolin, C. (2010). Developmental changes in the comparison of decimal fractions. *Learning and Instruction, 20*(6), 521-532.

Fuchs, L. S., Fuchs, D., Hamlett, C. L., Phillips, N. B., Karns, K., & Dutka, S. (1997). Enhancing Students' Helping Behavior during Peer-Mediated Instruction with Conceptual Mathematical Explanations. *The Elementary School Journal, 97*(3), 223-249.

Graeber, A. O., Tirosh, D. (1990). Insight fourth and fifth grades brings to multiplication and division with decimals. *Educational Studies in Mathematics, 21*(6), 565-588.

Greer, B. (1987). Nonconservation of multiplication and division involving decimals. *Journal for Research in Mathematics Education, 18*(1), 37-45.

Gu, L., Huang, R. & Marton, F. (2004). Teaching with variation: A Chinese way of promoting effective Mathematics learning. In L. Fan, N. Y. Wong, J. Cai & S. Li (Eds.), *How Chinese learn Mathematics: Perspectives from insiders* (pp.309-347). Singapore: World Scientific Publishing.

Haller, C., Fisher, R., & Gapp, R. (2007). Reflection as a means of understanding: Ways in which Confucian heritage students learn and understand organisational behaviour. *Multicultural Education and Technology Journal, 1*(1), 6-24.

Hiebert, J. (1992). Mathematical, cognitive, and instructional analyses of decimal fraction. In Leinhardt, G., Putnam, R. & Hattrup, R. A. (Eds), *Analysis of arithmetic for Mathematics teaching* (pp.283-321). Hillsdale, NJ : L. Erlbaum Associates.

Hiebert, J. & Wearne D. (1985). A model of students' decimal computation procedures. *Cognition and Instruction, 2*(3 & 4), 175-205.

Hiebert, J. & Wearne, D. (1986). Procedures over concepts: The acquisition of decimal number knowledge. In J, Heibert (Eds), *Conceptual and procedural knowledge: The case of Mathematics* (pp.199-223). Hillsdale, NJ : L. Erlbaum Associates.

Irwin, K. C. & Britt, M. (2004). Operating with decimal fractions as a part-whole concept. In I. Putt, R. Faragher & M. McLean (Eds.), *Mathematics education for the third millennium: towards 2010. Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia* (*Vol. 2*, pp.312-319). Queensland, Australia: MEGRA.

Izsak, A. (2008). Mathematical knowledge for teaching fraction multiplication. *Cognition and Instruction, 26*, 95-143.

Kember, D. (1996). The intention to both memorise and understand: Another approach to learning? *Higher Education, 31*(3), 341-354.

Kember, D. (2000). Misconceptions about the learning approaches, motivation and study practices of Asian students. *Higher Education, 40*(1), 99-121.

Kennedy, P. (2002). Learning cultures and learning styles: myth-understandings about adult (Hong Kong) Chinese learners. *International Journal of Lifelong Education, 21*(5), 430-445.

Lachance, A. & Confrey, J. (2002). Helping students build a path of understanding from ratio and proportion to decimal notation. *Journal of Mathematical Behaviour, 20*(4), 503-526.

Lai, M. Y. & Murray, S. (2012). Teaching with procedural variation: A Chinese way of promoting deep understanding of mathematics. *International Journal of Mathematics Teaching and Learning*, April.

Li, S. (2006). Practice makes perfect: A key belief in China. In F. K. S. Leung; K. Graf & F. J. Lopez-Real (Eds.), *Mathematics education in different cultural traditions: A comparative study of East Asia and the West* (pp.129-138). New York, USA: Springer.

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics, 67*(3), 255-276.

Marton, F., Dall' Alba, G. & Lai, K. T. (1993). *The paradox of the Chinese learner (Occasional Paper 93.1)*. Melbourne, Australia: Educational Research and Development Unit, Royal Melbourne Institute of Technology.

Marton, F., Dall' Alba, G. & Tse, L. K. (1996). Memorizing and understanding: The keys to the paradox? In D. Watkins & J. Biggs (Eds.), *The Chinese learner: Cultural, psychological and contextual influences* (pp.69-83). Melbourne, Australia: ACER.

Marton, F. & Saljo, R. (1976). Approach to learning. In F. Marton, D. Hounsell & N. Entwistle (Eds.), *The experiences of learning* (pp.36-55). Edinburgh, UK: Scottish Academy Press.

Marton, F., Watkins, D. & Tang, C. (1997). Discontinuities and continuities in the experience of learning: An interview study of high-school students in Hong Kong. *Learning and Instruction, 7*(1), 21-48.

Moloney, K. & Stacey, K. (1997). Changes with Age in Students' Conceptions of Decimal Notation. *Mathematics Education Research Journal, 9*(1), 25-38.

Mullis, I.V.S., Martin, M.O., & Foy, P. (with Olson, J.F., Preuschoff, C., Erberber, E., Arora, A., & Galia, J.). (2008). *TIMSS 2007 International Mathematics Report: Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades.* Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

Nesher, P., & Peled, I. (1986). Shifts in reasoning. *Educational Studies in Mathematics, 17*(1), 67-79.

Okazaki, M. & Koyama, M. (2005). Characteristics of 5th graders' logical development through learning division with decimals. *Educational Studies in Mathematics, 60*(2), 217-251.

Peled, I. (2003) Improving decimal number conception by transfer from fractions to decimals. *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp.1-6). Hawaii: PME.

Pierce, R. U., Steinle, V. A.; Stacey, K. A. & Widjaja, W. (2008). Understanding decimal numbers: A foundation for correct calculations. *International Journal of Nursing Education Scholarship, 5* (1), 1-15.

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S., & Peled, I. (1989). Conceptual bases of arithmetic errors: The case of decimal fractions. *Journal for Research in Mathematics Education, 20*(1), 8-27.

Sackur-Grisvard, C., & Leonard, F. (1985). Intermediate cognitive organization in the process of learning a mathematical concept: The order of positive decimal number. *Cognitive and Instruction, 2*(2), 157-174.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching,* 77, 20-26.

Stacey, K. (2005). Travelling the road to expertise: A longitudinal study to learning. In H. L. Chick & J. K. Vincent (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, (Vol. 1,* pp. 19-36). Melbourne, Australia: PME.

Stacey, K., Helme, S. & Steinle, V. (2001). Confusions between decimals, fractions and negative numbers: A consequence of the mirror as a conceptual metaphors in three different ways. *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education (Vol. 4,* pp.217-224*)*. Utrecht, The Netherlands: PME.

Stacey, K. & Steinle, V. (1999). A longitudinal study of children's thinking about decimals: A preliminary analysis. *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education (Vol. 4,* pp.233-240*)*. Haifa, Israel: PME

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education, 36*(5), 404-411.

Steinle, V. (2004*). Changes with age in students' misconceptions of decimal numbers*. Unpublished doctoral thesis, University of Melbourne. http://eprints.unimelb.edu.au/archive/00001531/

Steinle, V., & Stacey, K. (1998). The incidence of misconceptions of decimal notation amongst students in Grades 5 to 10. In C. Kanes, M. Goos & E. Warren (Eds.). *Teaching mathematics in new times. Proceedings of the 21st Annual Conference of the Mathematics Education Research Group of Australasia (Vol. 2,* pp.548-555). Gold Coast, Australia: MERGA.

Steinle, V., & Stacey, K. (2001). Visible and invisible zeros: Sources of confusion in decimal notation. In J. Bobis, B. Perry & M. Mitchelmore (Eds.), *Numeracy and Beyond. Proccedings of the 24th Annual Conference of the Mathematics Education Research Group of Australasia (Vol.2,* pp.434-441). Sydney, Australia: MERGA.

Steinle, V. & Stacey, K. (2003). Graded-related trends in the prevalence and persistence of decimal misconceptions. *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education (Vol. 4,* pp.259-266). Geelong, Australia: MERGA

Steinle, V. & Stacey, K. (2010). A longitudinal study of students' understanding of decimal notation: An overview and refined results. *Proceedings of the 27th annual conference of the Mathematics*

*Education Research Group of Australasia: Mathematics education for the third millennium: Towards 2010* (*Vol. 2*, pp.541–548). Townsville, Australia: MERGA.

Sumpter, L. (2013). Themes and interplay of beliefs in mathematical reasoning. *International Journal of Science and Mathematical Education*, *11*(5), 1115-1135.

The Education Department (2000). *Mathematics Curriculum Guide*. Hong Kong: HKSAR.

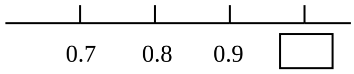
Wang, T. (2006). Understanding Chinese culture and learning. *The Australian Educational Researcher*. http://www.aare.edu.au/06pap/wan06122.pdf.

Watkins, D. A. & Biggs, J. B. (2001). The paradox of the Chinese learner and beyond. In D. A. Watkins & J. B. Biggs (Eds.), *Teaching the Chinese learner: Psychological and pedagogical perspectives* (pp.3-23). Melbourne, Australia: ACER.

Wearne, D., & Hiebert, J. (1988). A cognitive approach to meaningful mathematics instruction: Testing a local theory using decimal numbers. *Journal for research in mathematics education*, *19* (5), 371-384.

Wong, N. Y. (2006). From “entering the way” to “exiting the way”: In search of a bridge to span “basic skills” and “process abilities”. In F. K. S. Leung; K. Graf & F. J. Lopez-Real (Eds.), *Mathematics education in different cultural traditions: A comparative study of East Asia and the West* (pp.129-138). New York, USA: Springer.

**Table 1: Sample test items**

| **Examined content area** | **Number of items of this type** | **Illustrative test item** |
|---|---|---|
| A. Comparison of decimals | 6 | Circle the larger decimal in the pair of six pairs. For example, 4.8 and 4.63. |
| B. Convert fractions with denominators either 10 or 100 to decimals | 2 | Convert $3\frac{500}{1000}$ to decimal. |
| C. The representation of place value in decimals | 3 | In 0.723, the digit “7” represents seven lots of a) 1000 b)100 c)10 d)0.1 e)0.01 or f)0.001 |
| D. The concept of place value in decimals (number line) | 1 | What is next to 0.9 in the following number line? Write it into the box as shown.   |
| E. The concept of continuous quantity in decimals | 1 | Give a decimal between 0.75 and 0.8 |
| F. The addition and subtraction of decimals | 2 | 11.05 – 3.8 |
| G. Multiplication of decimals | 2 | $23 \times 0.12$ |
| H. Division of decimals | 3 | $0.12 \div 3$ |

* Items of Questions F, G and H were presented in horizontal form and were free-response items.

**Table 2: Percentage of scores for each of the questions (N=384)**

| | | Score | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Type of question (number of items of that type)** | **Score if all items correctly answered** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **Missing data** |
| A. Comparison of decimals (6) | 6 | 2.3 | 1.8 | 1.8 | 2.1 | 7.3 | 6.5 | **77.6** | 0.5 |
| B. Convert fractions with denominators either 10 or 100 to decimals (2) | 2 | 4.9 | 19.5 | **75.0** | *N/A* | *N/A* | *N/A* | *N/A* | 0.5 |
| C. The representation of place value in decimals (3) | 3 | 8.1 | 4.2 | 2.6 | **85.2** | *N/A* | *N/A* | *N/A* | 0 |
| D. The concept of place value in decimals on number line (1) | 1 | 11.5 | **88.5** | *N/A* | *N/A* | *N/A* | *N/A* | *N/A* | 0 |
| E. The concept of continuous quantity in decimals (1) | 1 | 17.7 | **82.3** | *N/A* | *N/A* | *N/A* | *N/A* | *N/A* | 0 |
| F. The addition and subtraction of decimals (2) | 2 | 1.3 | 14.6 | **83.6** | *N/A* | *N/A* | *N/A* | *N/A* | 0.5 |
| G. Multiplication of decimals (2) | 2 | 8.6 | 26.6 | **64.8** | *N/A* | *N/A* | *N/A* | *N/A* | 0 |
| H. Division of decimals (3) | 3 | 2.9 | 9.2 | 20.2 | **67.4** | *N/A* | *N/A* | *N/A* | 0.5 |

*missing data: responses did not relate to the tasks