Preface

Sliding mode control (SMC) commenced in the Soviet Union in the late 1950s, but this new control technique was not published until the publications [70] and [113]. Then, the sliding mode research community expanded quickly and the number of publications on this control framework grew correspondingly. Due to the fact that SMC relies on an infinite switching frequency of the input signal, it is inherently a continuous-time control strategy. However, the infinite switching is not achievable in real applications, especially for discrete-time controllers whose input signal can only be varied at the sampling instances. This fact limits the switching frequency to the discrete-time system's sampling frequency. It is worth noting that in a number of applications the assumption of an infinite switching frequency can be relatively justified. In the case that the sampling rate is much faster than the dynamics of the system under control, the influence of the bounded switching frequency will be confined. It is thus a usual approach to design sliding mode controllers in the continuous-time domain, even if the system is computer-aided-controlled [149], regarded as continuous-time sliding mode controller (CSMC), since it is designed according to a continuous-time model of the system, regardless of the sampling issue. However, the effectiveness of the obtained controller will, in addition to many other parameters, strongly depend on the sampling frequency. It means that the faster sampling is performed, the less the influence of the sampling rate will be. More importantly, for a relatively low sampling frequency, the limited switching frequency may result in undesirable effects on the input signal or even instability of the closedloop system.

Alternatively, the idea of discrete-time sliding mode control (DSMC) has been proposed in literature, which is significantly different from its continuous-time counterpart; see [83] for more information. The results presented in e.g. [83] demonstrate that an appropriate choice of sliding surface, used with the *equivalent control*, can ensure a bounded motion about the surface in the presence of bounded matched uncertainty. Notice also that from this viewpoint, the DSMC problem can be seen as a robust optimal control problem and is related to discrete-time Lyapunov min-max problems [83]. The problem is to select, among all possible feedback controllers, the feedback gain that minimizes the worst case effect of the uncertainty on the Lyapunov difference function [83]. Moreover, the discrete-time equivalent control law can be considered as a solution of the discrete-time linear quadratic regulator (LQR) problem under the assumption of *cheap control*; that is, no penalty is assigned to the control effort in the cost function.

In this book, we explain our recent investigations to improve DSMC and adopt this control strategy to different fields. The first introductory chapter (Chapter 1) discusses the reasons to consider DSMC. Furthermore, for tutorial purposes, a brief review of CSMC is given in the context of a second-order system. Lastly, in this chapter, the well-known regular form-based method for the design of SMC is reviewed in the framework of discrete-time systems.

Chapter 2 first provides an overview of the relevant literature and places the contribution of the book in a proper context. Further in this chapter, two new forms of switching function are proposed which can be more efficient in terms of reducing the ultimate bound on the system state and reducing the chattering created by traditional switching functions. This new switching function basically uses a disturbance estimator which comes from the same idea presented in [133]. The main idea is, with the assumption of continuity of the original continuous-time disturbance signal, to use the previous value of the sampled disturbance for estimating the current one in the control law. However, model uncertainty is not considered in [133]. In Chapter 2, it is also discussed that using the mentioned estimator directly in the controller will increase the order of the system and, in addition, it results in a system involving timedelay. Stability analysis and ultimate boundedness are then investigated for this kind of system. This method greatly reduces the conservatism of the current linear matrix inequality (LMI)-based methods presented in the few existing works that consider the problem of applying DSMC to the systems including unmatched uncertainties. Specifically, this method avoids using inequalities to deal with the uncertain negative signum quadratic terms appearing in the derived Riccati-like inequality, which is not easy to be directly arranged as an LMI problem. Instead, a lossless technique is proposed to convert the mentioned inequality to a form that can be easily written as an LMI. These results were previously published in the paper [13].

While Chapter 2 proposes a state feedback DSMC for uncertain discretetime systems whose whole states' information is available, Chapter 3 proposes an observer-based output feedback DSMC for discrete-time multi-input multi-output (MIMO) systems. This is more practical, as in many real applications, only systems' output is accessible. Furthermore, the disturbance estimator in Chapter 2 has been designed for the cases that the system states are entirely available. By exploiting output information only for discrete-time MIMO systems with unmatched disturbances and without uncertainties, a framework has been proposed in [32]. Chapter 3 uses an integral term of the estimation output error, in addition to the well-known Luenberger observer which observes the system state with a proportional loop, to allow more degrees of freedom. This matter is referred to as proportional integral observer (PIO) in the literature [32]. Nevertheless, the underlying system in [32] does not involve unmatched uncertainties, unlike the system considered in this chapter. The proposed scheme here extends the problem of utilizing disturbance observer in the output feedback DSMC (ODSMC) to uncertain discrete-time systems using an innovative LMI based framework. Many of the results in Chapter 3 were previously published in the conference paper [11].

The main goal of Chapter 4 is to stabilize a networked control system (NCS) involving consecutive data packet dropout with a sliding mode control strategy that can improve the existing approaches. In doing so, a novel sliding function is introduced

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by employing the available communicated system states involving packet losses. This is significantly different from the existing DSMC in the literature [101, 33], and it also provides the possibility to directly build the switching component of the DSMC by exploiting only the available system states. The results in Chapter 4 are based on the papers [6, 15].

The DSMC, given for NCSs in Chapter 4, is derived based on two major assumptions:

- the packet losses occur only in the channel from the sensor to the controller;
- 2. the system states are entirely available.

However, these assumptions may be unrealistic for many practical problems. Thus Chapter 5 intends to design sliding mode controllers for NCSs involving both measurement and actuation consecutive packet losses (or long-term random delays), which exploit only output information. This ODSMC can distinguish itself from the existing literature on the SMCs applied to the NCSs, in the sense that both the measurement and actuation delays are viewed as the Bernoulli distributed white sequence. The results in Chapter 5 were previously published in the paper [7].

Decentralized SMC has previously been developed in the literature for large-scale interconnected systems [144, 145, 112, 92]. However, distributed SMC has received less attention and hence it requires more investigation. Chapter 6 first explores the problem of designing a sparse DSMC network for a given plant network with arbitrary topology. To do so, this chapter considers a priori the control network topology which is a subset of the underlying dynamics network and provides a methodology to stabilize the underlying dynamics utilizing a (sparse) distributed observer and controller network. We will show that the proposed observer-based DSMC has the ability to cover all the cases such as decentralized, distributed, and sparsely distributed topologies. In Chapter 6, as the second step, we will search for a sparse control/observer network structure with the least possible number of links that can satisfy the given stability condition. To this end, a heuristic iterative algorithm will be proposed, distinguishing itself from a trial-and-error process which requires checking of all the possible structures. These results were previously published in the conference paper [14].

Although the SMC is now a well-known strategy, from the standpoint of constraining the available control action, all the traditional methods considered in the literature have shortcomings. This drawback basically comes from the nature of the SMC design process which contains two separate stages. During the synthesize of the sliding function, there is no sense of the control action level that is required to induce and retain sliding. This issue is more crucial in Chapter 7 when it comes to sparsifying the control network structure, as without limits on the available control actions, it may result in the high level of control efforts that each subsystem's controller requires to apply, which is not a practical case. Chapter 7 develops an approach by which we can deal with an \mathcal{H}_2 based optimal structured SMC problem. In this chapter in order to address the problem of designing a sparse SMC controller, a specific form of fictitious system, whose matrices contain the control network struc-

ture, is derived. This makes the well-developed weighted ℓ_1 algorithm infeasible to apply to our problem. Alternatively, Chapter 7 proposes a heuristic scheme to obtain the sparse sliding mode controller. The results in Chapter 7 were published in the papers [12, 8].

According to the so-called 1D quasi-sliding mode, SMC design has been extended for 2D systems in the Roesser Model (RM). In addition, the conditions to ensure the remaining horizontal and vertical states in RM on the switching surfaces and also the reaching condition using a 2D Lyapunov function are investigated in [3]. Another strategy to work with 2D systems is to transfer them to a 1D form. Wave advance model (WAM) is a 1D form of 2D systems established in [111]. From the view point of WAM model, 2D systems are considered as advanced waves and consequently the original stationary 2D system is converted to a time-varying 1D system. Moreover, the system matrices are in rectangular form rather than square form. As a result, the major drawback of this 1D form of 2D systems is the varying dimensions of the defined state vectors. This means that the results developed using this framework are most likely computationally unattractive in terms of possible applications. Motivated by this issue and by the use of stacking vectors, a new approach to converting 2D systems to a 1D form is proposed in Chapter 8. Consequently, the states, inputs and outputs of the obtained 1D system are in the vector form, and more importantly their dimensions are invariant. This framework is basically useful for a class of 2D linear systems in which information propagation in one of the two distinct directions only occurs over a finite horizon. This can be the case of a repetitive process [50] or any inherently 2D system, for instance, the Darboux equation [73]. The suggested 1D vectorial form in Chapter 8 unlike the WAM form has invariable dimension and consequently can be converted to regular form in SMC. In this chapter, first the Fornasini and Marchesini (FM) model of 2D systems which is a second order recursive form is considered. The results in Chapter 8 for 2D systems were published in the paper [5].

In Chapter 9, first, the controllability analysis of the WAM model of the first FM model is studied, and a necessary condition for the controllability of this 1D model is given. On the other hand, during the procedure of designing the sliding surface in Chapter 8, it is assumed that the obtained 1D system is controllable. But, the controllability of the obtained 1D form and its relation to the original 2D system is an unanswered problem in Chapter 8. Hence, motivated by these issues, in this chapter, we focus on the controllability analysis of the proposed 1D form of the underlying 2D systems. Based on the controllability analysis, a new notion, *directional controllability*, for the underlying 2D systems is introduced and studied. More importantly, a necessary and sufficient condition for the directional controllability of 2D systems is presented in this chapter. The controllability analyses of 2D systems here were published in the papers [9, 10].

Finally, Chapter 10 is devoted to the problem of heart rate regulation during cycle-ergometer exercise using both a non-model-based as well as a model-based control strategy along with a real-time damped parameter estimation scheme. The model-based control strategy is a time-varying integral sliding mode controller. A recursive damped parameter estimation method is also developed, by incorporation

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of a weighting upon the one-step parameter variation, which in contrast to the conventional parameter estimation schemes can avoid the occurrence of the so-called blowup phenomena. The calculated control signals are transmitted to the subjects employing a synchronized biofeedback mechanism. Indeed, delivering a feedback signal when the pedals are not in a suitable position to efficiently exert force may be ineffective and this may, in turn, lead to the cognitive disengagement of the user from the feedback controller. Chapter 10 examines a novel form of control system which has been designed for this project. The system is called an "actuator-based event-driven control system". The proposed control and estimation scheme were experimentally verified using several healthy male participants and the results demonstrated that the designed scheme is able to regulate the HR of the exercising subjects to a predetermined HR profile preventing overshooting in the HR responses. The results in this chapter are based on the published papers [16, 17, 18, 19, 20, 21].

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