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Modelling the Interconnected Synchronous Generators and Its State Estimations

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ABSTRACT In contrast to the traditional centralized power system state estimation approaches, this paper investigates the optimal filtering problem for distributed dynamic systems. Particularly, the interconnected synchronous generators are modeled as a state-space linear equation where sensors are deployed to obtain measurements. As the synchronous generator states are unknown, the estimation is required to know the operating conditions of large-scale power networks. Availability of the system states gives the designer an accurate picture of power networks to avoid blackouts. Basically, the proposed algorithm is based on the minimization of the mean squared estimation error, and the optimal gain is determined by exchanging information with their neighboring estimators. Afterward, the convergence of the developed algorithm is proved so that it can be applied to real-time applications in modern smart grids. Simulation results demonstrate the efficacy of the developed algorithm.

INDEX TERMS Automatic voltage regulator, convergence analysis, distributed estimation, power systems, synchronous generators.

I. INTRODUCTION

Generally speaking, power systems are continuously monitored in order to maintain the normal and secure operating conditions. The state estimation provides an accurate information about the power system operating conditions. Due to the increasing size and complexity of power systems, the traditional centralized estimation technique is not suitable [1]. Furthermore, the implementation of state estimation over a whole interconnected power system is becoming a challenging problem. Interestingly, the deregulation of the power industry leads to appearance of multiple local utilities or independent system operators. In other words, the industrial domain application becomes more and more distributed due to advancement in information and communication technology [2], [3]. As the information is locally processed, it can handle big data with flexible remote multi-service communication, deliver required functionality and services in sustainable and efficient ways.

A. RELATED LITERATURE

There is a wealth of studies related to the power system state estimation. First of all, the distributed weighted least square state estimation method using the additive Schwarz domain decomposition technique is proposed in [4]. This decomposition divides the data set into several subsets to reduce the execution time. The Kalman filter (KF) based state estimation via wireless sensor networks over fading channels is presented in [5]. This kind of centralized estimation technique generally requires massive amount of communication and computation resources, and is vulnerable to the central point failures. To deal with the communication impairments, the distributed fusion based KF algorithm for sensor networks is developed in [6] and [7]. In order to accommodate the effects of random delay in measurements, the extended KF based power system state estimation method is proposed in [8]. All of the aforementioned papers consider the centralized way of estimation approaches.

There have been numerous efforts for the state estimation of the distributed power system. The concept of distributed state estimation is proposed for large-scale power systems in [9]. The two-level distributed state estimation scheme is presented in [10] where the local states are estimated in a distributed way and the higher level coordinates them. Unfortunately, the higher coordination level needs to access all local estimation results and the complicated communication infrastructure is required [1]. Furthermore, the distributed state estimation algorithm based on the synchronized phasor measurement is presented in [11]. In order to estimate the microgrid states, the effective scheme for real-time operation and protection is explored in [12]. Although the distributed state estimations have been extensively explored, there has been little effort to use the dynamic state estimation for interconnected power systems and analyze the consensus. The consensus analysis ensures the consistency of the estimation across the power networks.

In the distributed estimation process, the estimator exchanges information with the neighboring connected nodes to reach a consensus on estimation. The consensus-based distributed state estimation algorithms for sensor networks have been proposed in [13], [14], and [15]. Moreover, the distributed information consensus filter for simultaneous input and state estimation is explored in [16]. However, the calculations of gain and error covariance in all preceding methods are based on the *suboptimal filter*. Therefore, it can be considered that the optimal consensus analysis has not been fully investigated as it does not trace back to the original system in the optimal sense. Motivated by the aforementioned research gaps in the smart grid research community, this paper proposes a distributed dynamic state estimation method with the optimal observation gain, and its convergence is analyzed without approximations. This paper is an extended version of work published in [3].

B. KEY CONTRIBUTIONS

The key contribution of this paper is to propose a distributed state estimation scheme for interconnected power systems that does not need a consensus step. The major contributions are summarized as follows:

- The distribution power systems with interconnected synchronous generators and loads are modeled as a discrete-time state-space equation. After modeling the interconnected power network in a distributed way, the sensors are deployed into the observation points to obtain the system state information. Then information is transmitted to the energy management system (EMS) through a communication network where estimators run in a distributed way.
- Based on the mean squared error principle, the optimal gain is computed to obtain a distributed state estimation. Each estimator communicates with the neighboring nodes for reaching a consensus on estimation.
- The convergence of the proposed algorithm is proved based on the Lyapunov method. Consequently, the estimated states converge to the true states.

PAPER OUTLINE: The remainder of this paper is organized as follows. The problem is formulated in Section II. An interconnected network with multiple synchronous generators and its state-space model are illustrated in Section III. In Section IV, the proposed algorithm is derived and its convergence is analyzed in Section V. Simulation test is carried out in Section VI. This paper ends with conclusion in Section VII.

NOTATIONS: Bold face upper and lower case letters are used to represent matrices and vectors respectively. Superscripts $\boldsymbol{x}'$ denotes the transpose of $\boldsymbol{x}$, diag($\mathbf{x}$) denotes the diagonal matrix, $E(\cdot)$ denotes the expectation operator and I denotes the identity matrix.

II. PROBLEM FORMULATION

There is a strong drive in power industry to design, create and analyze the system in a distributed way considering flexible communication infrastructure [2]. In order to develop a distributed estimation algorithm, consider the following system:

$$x_{k+1} = A_d x_k + B_d u_k + n_k, \tag{1}$$

where x_k is the system state at time instant k, u_k is the control effort and n_k is process noise whose covariance matrix is Q_k. The system measurements are obtained by a set of sensors as follows:

$$y_k^i = Cx_k + w_k^i, \; i = 1, 2, \cdots, n \tag{2}$$

where y_k^i is the observation information by the i-th estimator at time instant k, C is the observation matrix and w_k^i is the measurement noise whose covariance matrix is R_k^i. Similar to [13] and [17], we assume that measurements are same but the observation noises are different from each other. The assumption is probably due to the fact that the system operators deploy a number of similar sensors that are interconnected power systems. Secondly, the designed sensors have similar power and processing capability.

Generally speaking, the distribution power sub-system is interconnected to each other as shown in Fig. 1. When the estimator exchanges information with the connected neighboring nodes, it is called the distributed estimation [14], [6]. In this way, each estimator reaches its consensus estimation, so the estimated state converges to the actual state. In general, the proposed distributed state estimator is written as follows:

$$\hat{x}_{k+1|k}^i = A_d \hat{x}_{k|k}^i + B_d u_k. \tag{3}$$

$$\hat{x}_{k|k}^i = \hat{x}_{k|k-1}^i + K_k^i \sum_{l \in N^i} (y_k^l - C\hat{x}_{k|k-1}^i). \tag{4}$$

Here, $\hat{x}_{k|k}^i$ is the updated state estimation at the i-th estimator, $\hat{x}_{k|k-1}^i$ is the predicted state estimation, K_k^i is the local gain and N^i denotes the set of neighboring estimators including i. The second term in (4) is used to exchange information with the neighboring estimators for reaching a consensus on estimations. In the traditional distributed state estimation methods [18], [14], it requires both of local and consensus steps with their corresponding gain. That is, the derived covariance expression is not scalable in the number of estimators, so it needs to approximate leads to a *suboptimal filter* [14]. Generally, including the consensus step in the filter structure, the computational complexity of the gain and error covariance is significantly increased.

The first problem is how to express the interconnected power systems in a state-space model, which is easy to analyze. Our second problem is to design the optimal observer

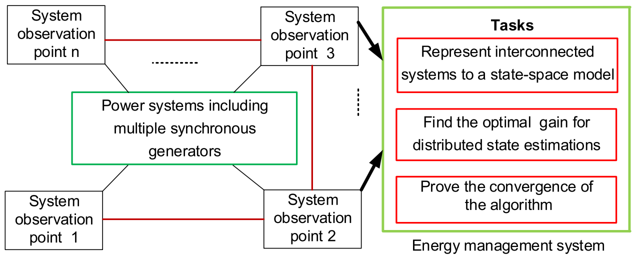

FIGURE 1. A framework for distributed estimations and its research questionnaires.

gain. The next problem is to prove the convergence of the proposed algorithm, so that the developed approach can be applied in power systems. Generally, the consensus analysis confirms the consistency of the estimation over the interconnected networks. The problem statement is summarized as follows:

Develop a distributed state estimation algorithm for complex power systems such that all estimators reach the consensus on estimation.

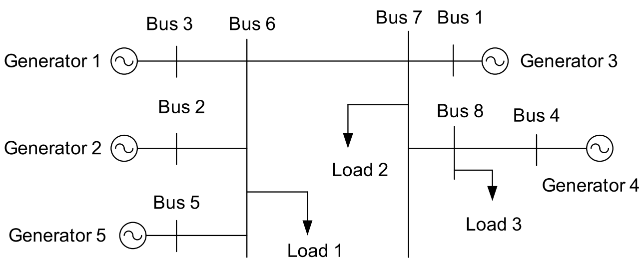

FIGURE 2. Synchronous generators are connected to the complex power systems [20], [21].

III. SYNCHRONOUS GENERATORS STATE-SPACE MODEL

The power power can be represented by diverse stages of complexity which depends on the planned applications of the system model. Generally speaking, there are many synchronous generators and loads are connected to the complex power networks. To illustrate, Fig. 2 shows the typical synchronous generators and loads which are connected to the 8-bus distribution lines [19], [20], [3]. Basically, the ith-synchronous generator can be represented by the following third order differential equations as follows [20], [3], [22]:

$$\Delta\dot{\delta}_i = \Delta\omega_i. \tag{5}$$

$$\Delta\dot{\omega}_i = -\frac{D_i}{H_i}\Delta\omega_i - \frac{\Delta P_{ei}}{H_i}. \tag{6}$$

$$\Delta\dot{E}'_{qi} = -\frac{\Delta E'_{qi}}{T'_{doi}} + \frac{\Delta E_{fi}}{T'_{doi}} + \frac{X_{di}}{T'_{doi}}\Delta i_{di} - \frac{X'_{di}}{T'_{doi}}\Delta i_{di}. \tag{7}$$

Here, δ_i is the rotor angle, ω_i is the rotor speed, H_i is the inertia constant, D_i is the damping constant, P_e is the active power delivered at the terminal, E'_{qi} is the quadrature-axis transient voltage, E_{fi} is the exciter output voltage, T'_{doi} is the direct-axis open-circuit transient time constant, X_{di} is the direct-axis synchronous reactance, X'_{di} is the direct-axis transient reactance and i_{di} is direct-axis current [19].

Usually, the typical automatic voltage regulator (AVR) is used to control the excitation current which leads to control the terminal voltage [20], [21]. A second-order transfer function is used to represent the AVR whose dynamic equations are given by [20]:

$$\Delta E_{fi} = b_{0i}z_{1i} + b_{1i}z_{2i}. \tag{8}$$

$$\dot{z_{1i}} = z_{2i}. \tag{9}$$

$$\dot{z_{2i}} = -c_{1i}z_{2i} - c_{0i}z_{1i} + \Delta v_i. \tag{10}$$

Here, z_{1i} and z_{2i} are the AVR internal states, b_{0i} and b_{1i} are transfer function coefficients of the voltage control, c_{0i} and c_{1i} are the transfer function coefficients of the excitation system and Δv_i is the control input signal.

If there are N generators in the system, the d-axis current I_{di} and electrical power P_{ei} are expressed as follows [21]:

$$I_{di} = \sum_{j=1}^{N} \Delta E'_{qi}[B_{ij}\, cos(\delta_i - \delta_j) - G_{ij}\, sin(\delta_i - \delta_j)]. \tag{11}$$

$$P_{ei} = \Delta E'_{qi}\sum_{j=1}^{N}[B_{ij}\, sin(\delta_i - \delta_j) + G_{ij}\, cos(\delta_i - \delta_j)]\Delta E'_{qj}. \tag{12}$$

Here, $i, j \in \{1, \cdots, N\}$, G_{ij} and B_{ij} are the real and imaginary part of the network admittance matrix Y, which is given in the Appendix A.

After linearizing (11) and (12), the power increment ΔP_{ei} and current increment ΔI_{di} are given by [3] and [19]:

$$\Delta P_{ei} = \begin{bmatrix} \frac{\partial P_{ei}}{\partial \delta} & \frac{\partial P_{ei}}{\partial E'_q} \end{bmatrix}[\Delta\delta\ \Delta E'_q]'. \tag{13}$$

$$\Delta I_{di} = \begin{bmatrix} \frac{\partial I_{di}}{\partial \delta} & \frac{\partial I_{di}}{\partial E'_q} \end{bmatrix}[\Delta\delta\ \Delta E'_q]'. \tag{14}$$

Here, $\Delta\delta$ and $\Delta E'_q$ are the rotor angle deviations and transient voltage deviations. By combining (5)-(10) and (13)-(14), the system dynamics can be written as follows:

$$\dot{x}_i = A_i x_i + B_i u_i + \sum_{j\in N_i} A_{ij}x_j. \tag{15}$$

Here, the ith-generator state $x_i = [\Delta\delta_i\ \Delta\omega_i\ \Delta E'_{qi}\ z_{2i}\ z_{1i}]'$, the input signal $u_i = \Delta v_i$, N_i denotes the set of generators that are physically connected with the i-th generator, the system matrices $A_i \in \mathbb{R}^{5\times 5}$, $B_i \in \mathbb{R}^{5\times 1}$ and $A_{ij} \in \mathbb{R}^{5\times 5}$ are given in Appendix B.

Moreover, the interconnected power system is expressed as a linearised continuous-time state-space framework as follows:

$$\dot{x} = Ax + Bu + n. \tag{16}$$

Here, $x \in \mathbb{R}^{5N\times 1}$ and $u \in \mathbb{R}^{N\times 1}$ are the states and input signals of all N generators, $n \in \mathbb{R}^{5N\times 1}$ is the process noise with covariance matrix $Q \in \mathbb{R}^{5N\times 5N}$, the system state matrix $A \in \mathbb{R}^{5N\times 5N}$ and input matrix $B \in \mathbb{R}^{5N\times N}$ are given by:

$A = \begin{bmatrix} A_1 & A_{12} & \cdots A_{1N} \\ A_{21} & A_2 & \cdots A_{2N} \\ \vdots & \vdots & \vdots \\ A_{N1} & A_{N2} & \cdots A_N \end{bmatrix}$ and $B = diag(B_1 \cdots B_N)$. Now, the above system is expressed as a discrete-time state-space linear form as follows:

$$x_{k+1} = A_d x_k + B_d u_k + n_k, \quad (17)$$

where $A_d = I + A\Delta t$, I is the identity matrix, Δt is the sampling period and $B_d = B\Delta t$.

In order to sense and monitor the distribution power systems, the system operators deploy a set of sensors around the grid. The system measurements are described by (2).

IV. SOLUTION TO THE OPTIMAL DISTRIBUTED ESTIMATOR

The following theorem gives the optimal gain K_k^i in (4).

Theorem 1: For the given system (1), and observation (2), the minimization of mean squared error $E[(x_k - \hat{x}^i_{k|k})(x_k - \hat{x}^i_{k|k})']$ can be achieved, if one can obtain the optimal gain as follows:

$$K_k^i = n^i P^i_{k|k-1} C' [(n^i)^2 C P^i_{k|k-1} C' + \textstyle\sum_{l \in N^i} R_k^l]^{-1}. \quad (18)$$

The error covariance $P^i_{k|k} = E[(x_k - \hat{x}^i_{k|k})(x_k - \hat{x}^i_{k|k})']$ is the solution to the following expression:

$$P^i_{k|k} = (I - n^i K_k^i C) P^i_{k|k-1} (I - n^i K_k^i C)' + K_k^i \textstyle\sum_{l \in N^i} R_k^l K_k'^i. \quad (19)$$

Here, $n^i = n^i(N^i)$ represents the cardinality of N^i, $P^i_{k|k-1} = A_d P^i_{k-1|k-1} A'_d + Q_{k-1}$ is the predicted error covariance matrix and $P^i_{k-1|k-1}$ is the error covariance matrix of the previous step [3]. The proof is derived in Appendix C.

Analytically, finding an optimal estimator does not guarantee to reach a consensus on estimation. Driven by this motivation, our next problem is to confirm the consensus of the proposed algorithm so that it can be utilized for monitoring power systems.

V. CONVERGENCE ANALYSIS

Let e^i denote the estimation error between the actual state and estimated state of the i-th estimator, which can be expressed as follows [3]:

$$e^i_{k|k} = x_k - \hat{x}^i_{k|k}. \quad (20)$$

$$e^i_{k|k-1} = x_k - \hat{x}^i_{k|k-1}. \quad (21)$$

Let's take the Lyapunov function as follows:

$$V(e_{k|k}) = \sum_{i=1}^{N} e'^i_{k|k} (P^i_{k|k})^{-1} e^i_{k|k}. \quad (22)$$

The first difference of the Lyapunov function can be expressed as follows:

$$\begin{aligned} E[\Delta V(e_{k|k})] &= E[V(e_{k+1|k+1}) - V(e_{k|k})] \\ &= E[\sum_{i=1}^{N} \{e'^i_{k+1|k+1} (P^i_{k+1|k+1})^{-1} e^i_{k+1|k+1} \\ &\quad - e'^i_{k|k} (P^i_{k|k})^{-1} e^i_{k|k}\}]. \end{aligned} \quad (23)$$

The following lemmas are used to simplify the above expression.

Lemma 1: Defining the information matrix $S_k^i = (n^i)^2 C' (\sum_{l \in N^i} R_k^l)^{-1} C$, then $P^i_{k|k} = [(P^i_{k|k-1})^{-1} + S_k^i]^{-1}$.

Proof: See Appendix D.

Lemma 2: The following statement holds: $P^i_{k+1|k+1} = F^i_{k+1} G^i_{k+1} F'^i_{k+1}$ with the simplified terms $G^i_{k+1} = A_d P^i_{k|k} A'_d + W^i_{k+1}$, $W^i_{k+1} = Q_k + P^i_{k+1|k} S^i_{k+1} P^i_{k+1|k}$ and $F^i_{k+1} = [I - n^i K^i_{k+1} C]$.

Proof: See Appendix E.

Now $e^i_{k+1|k+1}$ can be expressed as follows:

$$\begin{aligned} e^i_{k+1|k+1} &= x_{k+1} - \hat{x}^i_{k+1|k+1} \\ &= x_{k+1} - \hat{x}^i_{k+1|k} - K^i_{k+1} \sum_{l \in N^i} (y^l_{k+1} - C\hat{x}^i_{k+1|k}) \\ &= [I - n^i K^i_{k+1} C][A_d(x_k - \hat{x}^i_{k|k}) + n_k] \\ &\quad - K^i_{k+1} \sum_{l \in N^i} w^l_{k+1} \\ &= (I - n^i K^i_{k+1} C)(A_d e^i_{k|k} + n_k) - K_{k+1} \sum_{l \in N^i} w^l_{k+1}. \end{aligned} \quad (24)$$

For simplicity the convergence analysis, it is assumed that there are no noisy terms in (24), so, it can be rewritten as follows:

$$\begin{aligned} e^i_{k+1|k+1} &= (I - n^i K^i_{k+1} C) A_d e^i_{k|k} \\ &= F^i_{k+1} A_d e^i_{k|k}. \end{aligned} \quad (25)$$

Here, $F^i_{k+1} = [I - n^i K^i_{k+1} C]$. For $E[\Delta V(e_{k|k})]$ expression, (23) is used together with (25) to yield:

$$\begin{aligned} E[\Delta V(e_{k|k})] = \sum_{i=1}^{N} e'^i_{k|k} [A'_d F'^i_{k+1} (P^i_{k+1|k+1})^{-1} F^i_{k+1} A_d \\ - (P^i_{k|k})^{-1}] e^i_{k|k}. \end{aligned} \quad (26)$$

Using Lemma 2, the Lyapunov function (26) can be written as follows:

$$\begin{aligned} E[\Delta V(e_{k|k})] &= \sum_{i=1}^{N} e'^i_{k|k} [A'_d (G^i_{k+1})^{-1} A_d - (P^i_{k|k})^{-1}] e^i_{k|k} \\ &= -\sum_{i=1}^{N} e'^i_{k|k} [(P^i_{k|k})^{-1} - A'_d (G^i_{k+1})^{-1} A_d] e^i_{k|k} \\ \Rightarrow E[\Delta V(e_{k|k})] &= -\sum_{i=1}^{N} e'^i_{k|k} \mathbf{\Lambda}^i_{k+1} e^i_{k|k}, \end{aligned} \quad (27)$$

where $\mathbf{\Lambda}^i_{k+1}$ is defined as follows:

$$\mathbf{\Lambda}^i_{k+1} = (P^i_{k|k})^{-1} - A'_d (G^i_{k+1})^{-1} A_d. \quad (28)$$

TABLE 1. Five generators G1-G5 parameters [20], [21].

Parameters	G1	G2	G3	G4	G5
H_i	4.6	4.75	4.53	4.04	5
D_i	3.14	3.77	3.45	4.08	3.5
X_{di}	0.1026	0.1026	1.0260	0.1026	1.0260
X'_{di}	0.0339	0.0339	0.3390	0.0339	0.3390
T'_{doi}	5.67	5.67	5.67	5.67	5.67
b_{1i}	1332	1332	1332	1332	1332
b_{oi}	666	666	666	666	666
c_{1i}	33.3	33.3	33.3	33.3	33.3
c_{oi}	3.33	3.33	3.33	3.33	3.33
V	1.05	1.03	1.025	1.05	1.025
θ	0	0.1051	0.0943	0.0361	0.0907
P	3.1621	4.1026	0.4708	4.0678	0.1647
Q	2.9241	1.3921	0.4197	2.1902	0.3479

TABLE 2. Transmission line parameters [20], [21].

Node i	Node j	R_{ij}	X_{ij}	B_{ij}
1	7	0.00435	0.01067	0.01536
2	6	0.00213	0.00468	0.00404
3	6	0.02004	0.06244	0.06406
4	8	0.00524	0.01184	0.01756
5	6	0.00711	0.02331	0.02732
6	7	0.04032	0.12785	0.15858
7	8	0.01724	0.04153	0.06014

In order to apply the well-known matrix inversion Lemma, $(A+BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}$ [14], pre and post-multiplying to (28) by $P^i_{k|k}$ yields:

$$P^i_{k|k}\boldsymbol{\Lambda}^i_{k+1}P^i_{k|k} = P^i_{k|k} - P^i_{k|k}A'_d(W^i_{k+1} + A_dP^i_{k|k}A'_d)^{-1}A_dP^i_{k|k} = [(P^i_{k|k})^{-1} + A'_d(W^i_{k+1})^{-1}A_d]^{-1}. \quad (29)$$

Now pre and post-multiplying to (29) by $(P^i_{k|k})^{-1}$ leads to:

$$\boldsymbol{\Lambda}^i_{k+1} = (P^i_{k|k})^{-1}[(P^i_{k|k})^{-1} + A'_d(W^i_{k+1})^{-1}A_d]^{-1}(P^i_{k|k})^{-1}. \quad (30)$$

It shows that $\boldsymbol{\Lambda}^i_{k|k}$ is a symmetric and positive definite matrix. This ensures that the Lyapunov function (27) becomes:

$$E[\Delta V(e_{k|k})] = -\sum_{i=1}^{N} e'^i_{k|k}\boldsymbol{\Lambda}^i_{k+1}e^i_{k|k} < 0. \quad (31)$$

It shows that the Lyapunov function is less than zero, so the estimation error dynamic is asymptotically stable [3]. This concludes that the proposed algorithm is stable. The performance of the aforementioned algorithm is demonstrated by performing numerical simulations in the next section.

VI. SIMULATION RESULTS AND DISCUSSIONS

In order to simplify our discussion, here, it is assumed there are n=4 observation stations and N=5 synchronous generators in the distribution power networks as shown in Figs. 1-2. The proposed work can be easily extended to the generic case. The simulation is implemented in Matlab where the parameters are summarized in Tables 1 and 2 [20], [21], [3]. Moreover, the considered process and measurement noise covariances are diagonal matrices [19], [23], [24] and their values are $Q = 000001I$ and $R^1 = 0.003I$, $R^2 = 0.004I$ $R^3 = 0.005I$, $R^4 = 0.006I$. The sampling period for discretization is 0.0015 *sec*.

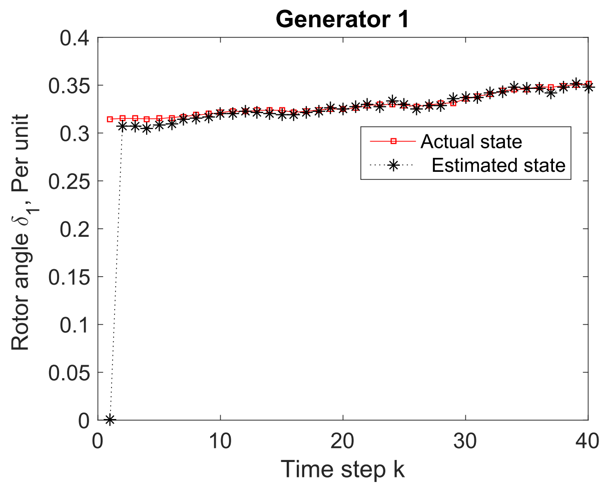

FIGURE 3. Rotor angle δ_1 and its estimation.

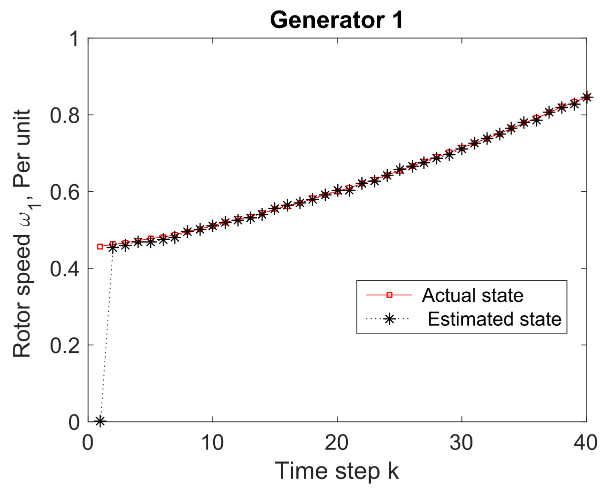

FIGURE 4. Rotor speed ω_1 and its estimation.

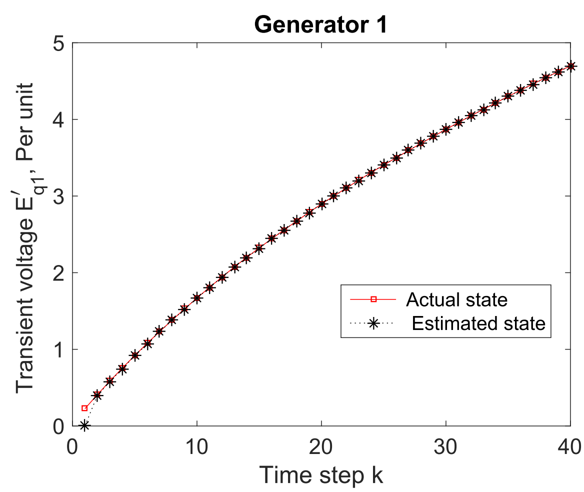

FIGURE 5. Transient voltage E'_{q1} and its estimation.

Figures 3-17 show the system's true and estimated states versus time step. It can be seen that the proposed method can estimate the system states with reasonable accuracy. This is because the developed approach can effectively solve the

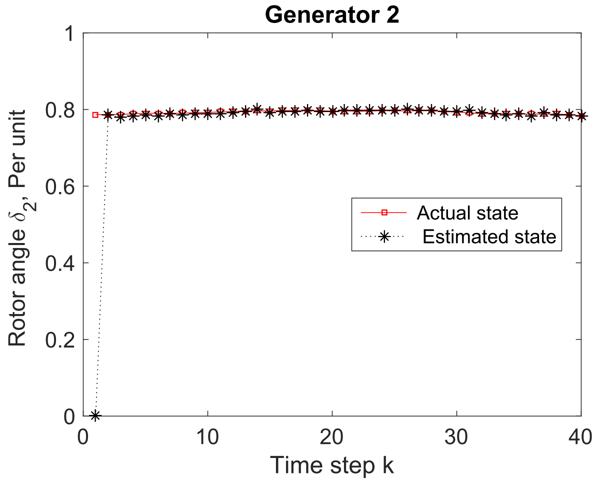

FIGURE 6. **Rotor angle δ_2 and its estimation.**

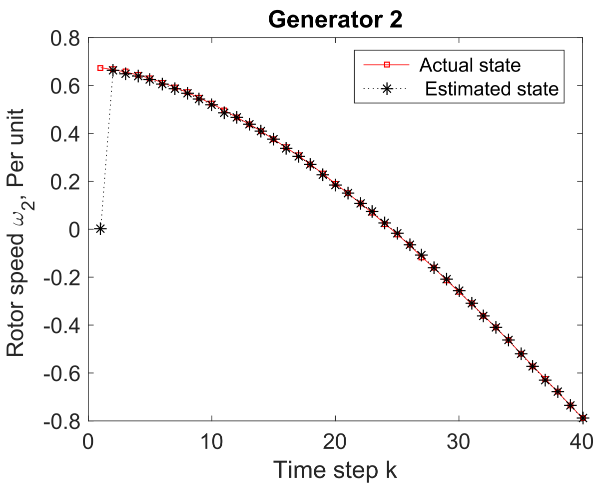

FIGURE 7. **Rotor speed ω_2 and its estimation.**

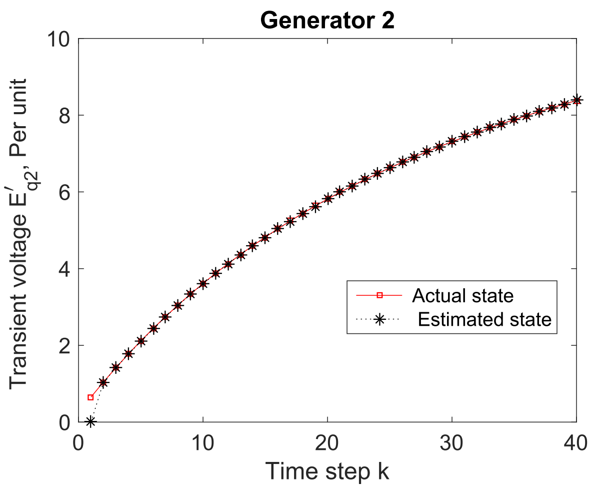

FIGURE 8. **Transient voltage E'_{q2} and its estimation.**

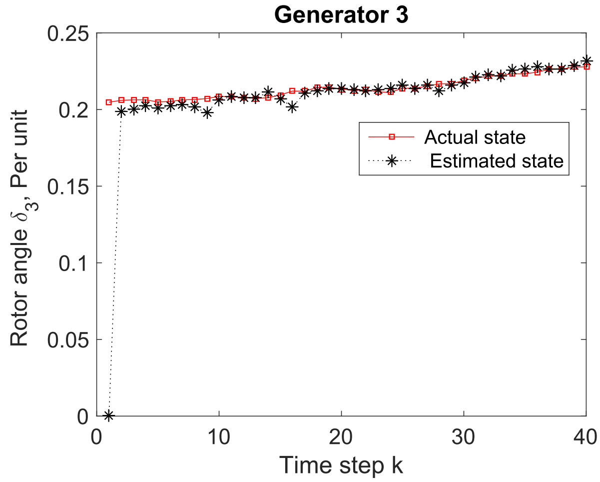

FIGURE 9. **Rotor angle δ_3 and its estimation.**

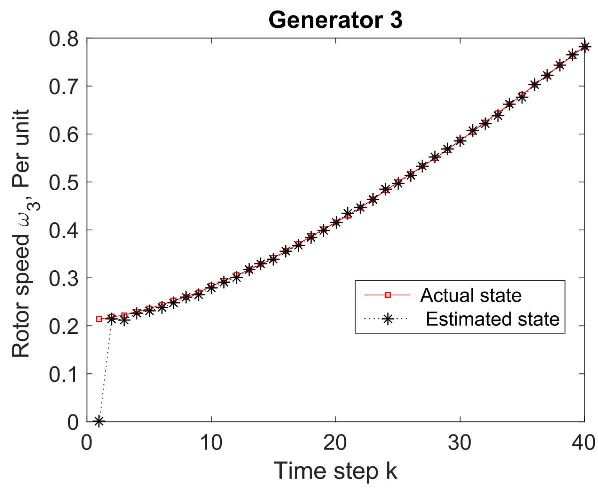

FIGURE 10. **Rotor speed ω_3 and its estimation.**

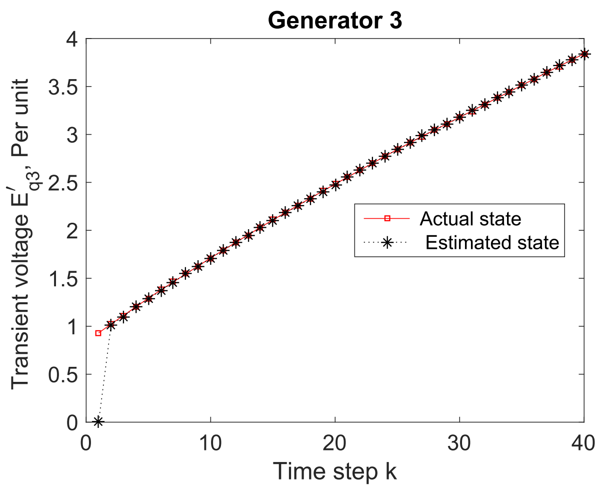

FIGURE 11. **Transient voltage E'_{q3} and its estimation.**

distributed estimation problem to find the optimal solution. So, the estimated states reflect the true state within few steps. For instance, it can be seen from the Fig. 3 that the explored method requires 0.0150 seconds ($k \times \Delta t = 10 \times 0.0015$) to estimate the rotor angle of generator 1. Similar kind of estimation performance is obtained for other states. From the

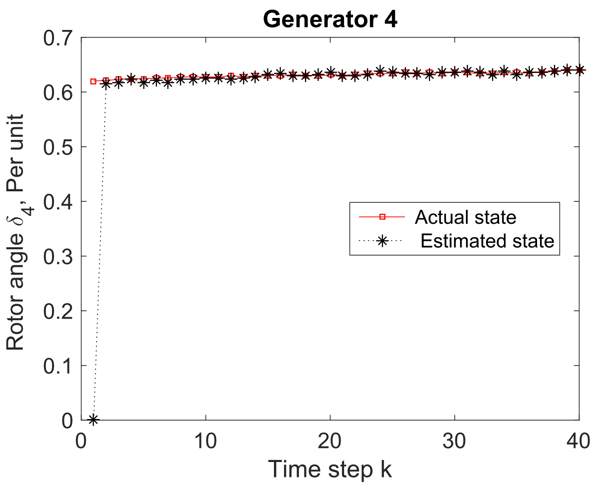

FIGURE 12. Rotor angle δ_4 and its estimation.

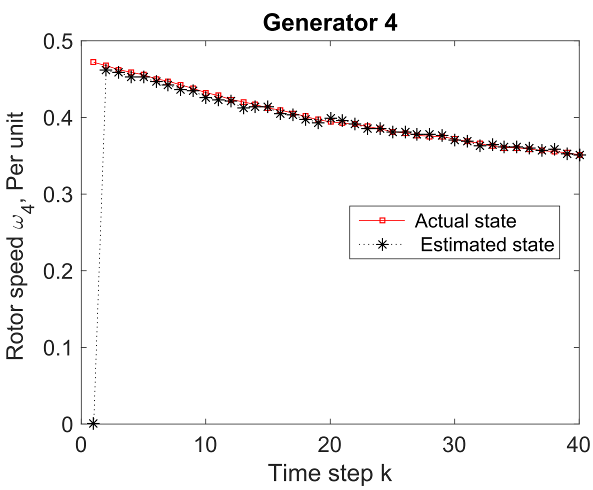

FIGURE 13. Rotor speed ω_4 and its estimation.

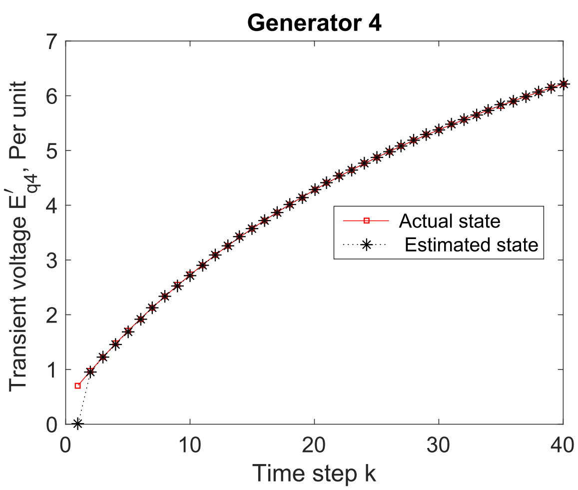

FIGURE 14. Transient voltage E'_{q4} and its estimation.

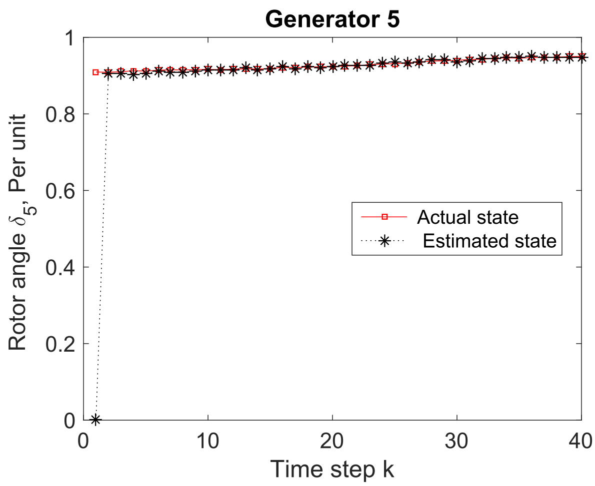

FIGURE 15. Rotor angle δ_5 and its estimation.

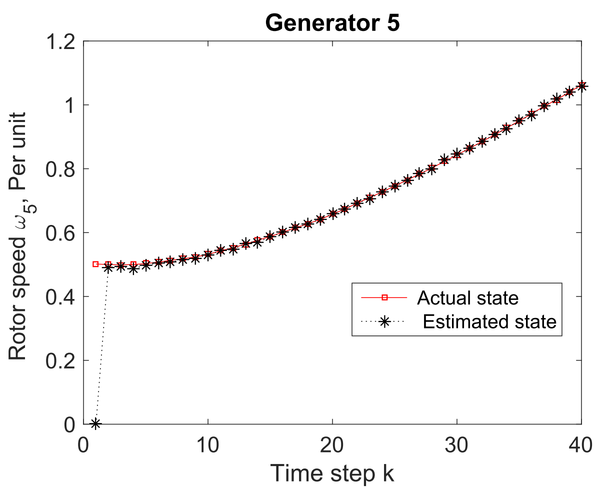

FIGURE 16. Rotor speed ω_5 and its estimation.

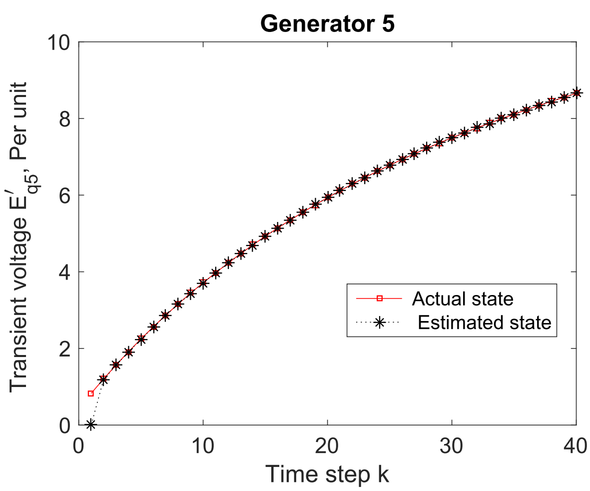

FIGURE 17. Transient voltage E'_{q5} and its estimation.

technical point of view, it means that the explored algorithm requires much less time compared with the standard estimation time of 1 second [25]. As the five generators have different specifications so their true states are different which are also well estimated by the proposed approach. Note that the small fluctuations come from system noise, but it does not affect the estimation accuracy.

VII. CONCLUSION AND FUTURE WORK

This paper presents a distributed algorithm for estimating the synchronous generator states for complex and large-scale power systems. Different from the distributed estimation in the literature, a key feature of the proposed method is that it does not require consensus step, which incurs the computational burdens. The simulation results demonstrate the validity of the analytical approach. The results point out the applicability of the proposed scheme for estimating the multiple synchronous generator states. Finally, the consensus of the explored algorithm is also proved so that it can be applied to practical applications in modern smart grids. Consequently, these contributions are valuable for designing the distributed smart energy management system as it provides precise estimation performance and requires less computational resources in the estimation process. Further investigations include the following aspects:

- Designing a suitable distributed feedback control framework to stabilize the system.
- Testing the proposed estimation algorithm considering the bad data in the measurements.

APPENDIX A (NETWORK ADMITTANCE MATRIX)

The network admittance matrix is given by [3] and [20]:

$$Y = Y_{rr} - Y_{re}Y_{ee}^{-1}Y_{re}'. \tag{32}$$

Here, the simplified term $Y_{rr} = diag[Y_{17} + jB_{17}, Y_{26} + jB_{26}, Y_{36} + jB_{36}, Y_{46} + jB_{46}, Y_{56} + jB_{56}]$, where the mutual admittance is computed as follows as an example: $Y_{17} = 1/(R_{17} + jX_{17})$, R_{17} is the resistance between node 1 and 7, X_{17} and B_{17} are it's reactance and susceptance, respectively. The second simplified term Y_{re} is given by:

$$Y_{re} = \begin{bmatrix} 0 & -Y_{17} & 0 \\ -Y_{26} & 0 & 0 \\ -Y_{36} & 0 & 0 \\ 0 & 0 & -Y_{48} \\ -Y_{56} & 0 & 0 \end{bmatrix}. \tag{33}$$

The last simplified term Y_{ee} is given by:

$$Y_{ee} = \begin{bmatrix} Y_{66} & -Y_{67} & 0 \\ -Y_{67} & Y_{77} & -Y_{78} \\ 0 & -Y_{78} & Y_{88} \end{bmatrix}, \tag{34}$$

where Y_{ii} is the self-admittance which is the sum of admittances connected to it in the network.

APPENDIX B (SYSTEM MATRICES)

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{H_i}\frac{\partial P_{ei}}{\partial \delta i} & -\frac{D_i}{H_i} & -\frac{1}{H_i}\frac{\partial P_{ei}}{\partial E_{qi}'} & 0 & 0 \\ X_i\frac{\partial I_{di}}{\partial \delta_i} & 0 & -\frac{1}{T_{doi}'} + X_i\frac{\partial I_{di}}{\partial E_{qi}'} & \frac{b_{1i}}{T_{doi}'} & \frac{b_{oi}}{T_{doi}'} \\ 0 & 0 & 0 & -c_{1i} & -c_{0i} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$A_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{H_i}\frac{\partial P_{ei}}{\partial \delta j} & 0 & -\frac{1}{H_i}\frac{\partial P_{ei}}{\partial E_{qj}'} & 0 & 0 \\ X_i\frac{\partial I_{di}}{\partial \delta_j} & 0 & -\frac{1}{T_{doi}'} + X_i\frac{\partial I_{di}}{\partial E_{qj}'} & \frac{b_{1i}}{T_{doj}'} & \frac{b_{oi}}{T_{doj}'} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = [0\,0\,0\,1\,0]' \text{ and } X_i = \frac{X_{di} - X_{di}'}{T_{doi}'}.$$

APPENDIX C (PROOF OF THEOREM 1)

Let $n^i = n^i(N^i)$ represents the cardinality of N^i. Now substituting (4) into (20), and using (2) one can obtain the following error expression [3]:

$$\begin{aligned} e_{k|k}^i &= x_k - \hat{x}_{k|k-1}^i - K_k^i \sum_{l\in N^i}(y_k^l - C\hat{x}_{k|k-1}^i) \\ &= (I - n^iK_k^iC)(x_k - \hat{x}_{k|k-1}^i) - K_k^i\sum_{l\in N^i} w_k^l \\ &= (I - n^iK_k^iC)e_{k|k-1}^i - K_k^i\sum_{l\in N^i} w_k^l. \end{aligned} \tag{35}$$

Now the state estimation error covariance $P_{k|k}^i$ is defined by:

$$P_{k|k}^i = E(e_{k|k}^i e_{k|k}'^i). \tag{36}$$

Substituting (35) into (36), one can obtain:

$$P_{k|k}^i = (I - n^iK_k^iC)P_{k|k-1}^i(I - n^iK_k^iC)' + K_k^i\sum_{l\in N^i} R_k^l K_k'^i. \tag{37}$$

Here, the error covariance $P_{k|k-1}^i = E(e_{k|k-1}^i e_{k|k-1}'^i)$. The following partial derivatives are used to obtain the optimal expression of the gain K_k^i. For any two compatible matrices X and Y, the following partial derivatives holds:

$$\frac{\partial tr(YX)}{\partial X} = Y'. \tag{38}$$

$$\frac{\partial tr(XYX')}{\partial X} = X(Y + Y'). \tag{39}$$

In order to find the optimal gain K_k^i, taking the partial derivative of $P_{k|k}^i$ in (37) with respect to K_k^i and applying (38) and (39) yields:

$$\frac{\partial[trP_{k|k}^i]}{\partial K_k^i} = -2n^iP_{k|k-1}^iC' + 2K_k^i[(n^i)^2CP_{k|k-1}^iC' + \sum_{l\in N^i} R_k^l]. \tag{40}$$

Now putting $\frac{\partial[trP^i_{k|k}]}{\partial K^i_k} = 0$ in (40), the optimal gain K^i_k is given by:

$$K^i_k = n^i_k P^i_{k|k-1} C'[(n^i)^2 C P^i_{k|k-1} C' + \sum_{l \in N^i} R^l_k]^{-1}. \quad (41)$$

This finishes the proof of the Theorem 1.

APPENDIX D (PROOF OF LEMMA 1)

Substituting K^i_k (41) into $P^i_{k|k}$ (37) and after simplifying matrix manipulations, we have [3]:

$$\begin{aligned} P^i_{k|k} &= P^i_{k|k-1} - n^i K^i_k C P^i_{k|k-1} \\ &= P^i_{k|k-1} - n^i\{n^i P^i_{k|k-1} C'[(n^i)^2 C P^i_{k|k-1} C' \\ &\quad + \sum_{l \in N^i} R^l_k]^{-1}\} C P^i_{k|k-1}. \end{aligned} \quad (42)$$

Using the matrix inversion Lemma, $A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1} = (A + BCD)^{-1}$, right hand side of (42) can be written as follows:

$$\begin{aligned} P^i_{k|k} &= [(P^i_{k|k-1})^{-1} + (n^i)^2 C'(\sum_{l \in N^i} R^l_k)^{-1} C]^{-1} \\ &= [(P^i_{k|k-1})^{-1} + S^i_k]^{-1}. \end{aligned} \quad (43)$$

Here, $S^i_k = (n^i)^2 C'(\sum_{l \in N^i} R^l_k)^{-1} C$.

APPENDIX E (PROOF OF LEMMA 2)

Generally speaking, the stability and convergence study deals with an infinite time horizon. So, throughout this proof without loss of generality, we adopt a notation that is free of the time index k and the updated variable x_{k+1} is denoted by x_+. Inspired by [14], the optimal gain (18) can be written in the *information form* as follows:

$$\begin{aligned} K^i &= n^i P^i C'[(n^i)^2 C P^i C' + \sum_{l \in N^i} R^l]^{-1} \\ &= [(P^i)^{-1} + (n^i)^2 C'(\sum_{l \in N^i} R^l)^{-1} C]^{-1} n^i C'(\sum_{l \in N^i} R^l)^{-1} \\ &= n^i M^i C'(\sum_{l \in N^i} R^l)^{-1}. \end{aligned} \quad (44)$$

Here, from Lemma 1 the error covariance matrix $M^i = P^i_{k|k}$ in the information form is given by:

$$M^i = [(P^i)^{-1} + S^i]^{-1}. \quad (45)$$

The information matrix S^i is described as follows:

$$S^i = (n^i)^2 C'(\sum_{l \in N^i} R^l)^{-1} C. \quad (46)$$

Utilizing (44) and (46), the simplified term F^i becomes:

$$\begin{aligned} F^i &= I - n^i K^i C = I - (n^i)^2 M^i C'(\sum_{l \in N^i} R^l)^{-1} C \\ &= I - M^i S^i. \end{aligned} \quad (47)$$

Motivated by (45), the identity matrix can be expressed as follows:

$$\begin{aligned} &[(P^i)^{-1} + S^i]^{-1}[(P^i)^{-1} + S^i] = I \\ &\Rightarrow M^i[(P^i)^{-1} + S^i] = I \\ &\Rightarrow M^i(P^i)^{-1} = I - M^i S^i = F^i. \end{aligned} \quad (48)$$

Using (35), (44), (46), and (48), the error covariance matrix (19) can be rewritten as follows:

$$\begin{aligned} M^i_+ &= F^i_+ P^i_+ F'^i_+ + K^i_+ \sum_{l \in N^i} R^l_+ K'^i_+ \\ &= F^i_+(A_d M^i A'_d + Q)F'^i_+ + [n^i M^i_+ C'(\sum_{l \in N^i} R^l_+)^{-1}] \\ &\quad (\sum_{l \in N^i} R^l_+)[n^i M^i_+ C'(\sum_{l \in N^i} R^l_+)^{-1}]' \\ &= F^i_+(A_d M^i A'_d + Q)F'^i_+ + (n^i)^2 M^i_+ C'(\sum_{l \in N^i} R^l_+)^{-1} C M^i_+ \\ &= F^i_+(A_d M^i A'_d + Q)F'^i_+ + M^i_+ S^i_+ M^i_+ \\ &= F^i_+(A_d M^i A'_d + Q)F'^i_+ + F^i_+ P^i_+ S^i_+ P^i_+ F'^i_+ \\ &= F^i_+(A_d M^i A'_d + Q + P^i_+ S^i_+ P^i_+)F'^i_+ \\ \Rightarrow M^i_+ &= F^i_+ G^i_+ F'^i_+. \end{aligned} \quad (49)$$

Here, the simplified terms are: $G^i_+ = A_d M^i A'_d + W^i_+$ and $W^i_+ = Q + P^i_+ S^i_+ P^i_+$. The proof is completed.

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