A Simple Particle Size Distribution Model for Granular Materials

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ABSTRACT: Particle size distribution (PSD) is a fundamental soil property that plays an important role in soil classification and soil hydro-mechanical behaviour. A continuous mathematical model representing the PSD curve facilitates the quantification of particle breakage, which often takes place when granular soils are compressed or sheared. This paper proposes a simple and continuous PSD model for granular soils involving particle breakage. The model has two parameters and is able to represent different types of continuous PSD curves. It is found that one model parameter is closely related to the coefficient of non-uniformity (C_u) and the coefficient of curvature (C_c), while the other represents a characteristic particle diameter. A database of 53 granular soils with 154 varying PSD curves are analyzed to evaluate the performance of the proposed PSD model, as well as three other PSD models in the literature. The results show that the proposed model has improved overall performance and captures the typical trends in PSD evolution during particle breakage. In addition, the proposed model is also used for assessing the internal stability of 27 widely graded soils.

Keywords: granular soil; PSD; mathematical model; particle breakage; internal stability

INTRODUCTION

Particle size distribution (PSD) is a basic soil property and the main basis for soil classification. It is used in analysis of stability of granular filters (Kenney and Lau 1985; Åberg 1993; Indraratna et al. 2007), internal instability and suffusion of granular soils (Wan and Fell, 2008; Indraratna et al. 2015; Moraci et al. 2014, 2015; Ouyang and Takahashi 2016a, 2016b), groutability of soils (Karol 1990; Vipulanandan and Ozgurel 2009; EI Mohtar et al. 2015), soil-water characteristic curves (Fredlund et al. 2002; Gallage and Uchimura 2010), and debris flow (Sanvitale and Bowman 2017). Particle size distribution curves are widely used to represent soil composition in real engineering practice and academic research. Particle size distribution curves can be obtained by sieving test, where several constrained grain sizes are predetermined. At present, indices such as the coefficient of uniformity C_u and the coefficient of curvature C_c are usually used to evaluate the whole gradation of a soil. For example, the standard for engineering classification of soils in China (GB/T50145-2007) suggest that the soil is well graded when C_u>5 and 1≤C_c≤3; otherwise, the soil is poorly graded. However, neither C_u or C_c can describe a PSD curve completely, as no unique relation exists between these coefficients and a PSD curve.

Another important application of studying the PSD lies in studying particle breakage of granular soils. A large number of studies have shown that soil particles, especially coarse-grained soil particles, can break under loading (Lee and Farhoomand 1967; Marsal 1967; Hardin 1985; Zheng and Tannant 2016; Hyodo et al. 2017). Some trends have been highlighted when particles break (Mayoraz et al. 2006; Altuhafi and Coop 2011; Miao and Airey 2013), for example, there seems to be an ultimate fractal PSD according to a large number of studies in the literature (Sammis et al. 1987; McDowell et al. 1996; Einav 2007). The three key elements in studying the behaviour of a soil that involves particle breakage are: (1) a simple and adequate representation of an evolving PSD, (2) the evolution of the PSD under various stresses and strains, and (3) the correlation between the PSD and soil hydro-mechanical properties (Muir Wood and Maeda 2008; Zhang et al. 2015). The first element is the foundation for studying the second and third elements, and is the purpose of this study. The second element is studied in Einav (2007), Zhang et al. (2015) and others, while the third element is an area of future interest. In the literature, studies on PSD representation (first element) and PSD evolution (second element) are usually carried out separately, often by different researchers from different backgrounds. However, it will be shown in this paper that these two are somewhat related for soils subjected to particle breakage.

While there are simple quantitative representations of soil PSDs in the literature (for example, C_c and C_u), an alternative way to describe a PSD curve is perhaps to adopt a suitable mathematical model which covers the full range of particle sizes. Such a mathematical model has several advantages: (1) characteristics of the whole PSD curve (such as d_{10}, d_{60}, C_c, C_u, etc.) can be obtained when the parameters of the model are determined; (2) it is easier to correlate the entire PSD curve with other properties of the soil. A key challenge is in developing a model that has a limited number of parameters while still capturing the widely varying nature of soil PSDs. In the case of particle breakage, the PSD model should ideally be able to predict the evolution of the grading.

A number of studies have attempted to characterize PSD curves using mathematical models, with up to seven input parameters. The most commonly used PSD model is perhaps the Gates-Gaudin-Schuhmann model (GGSM) (Schuhmann 1940), which was previously proposed by Fuller (Fuller and Thompson 1907) and Talbot (Talbot and Richart 1923):

$$P(d)=(\frac{d}{d_{\max}})^m, \qquad 0<d<d_{\max} \tag{1}$$

where parameter m is a fitting parameter, $d_{\max}$ is the diameter of the largest particle; $P(d)$ is the mass percentage of particles passing a particular size d. Equation (1) has the same form with Fractal Models (FM) in the literature (Turcotte 1986; Einav 2007). In a Fractal Model, parameter m equals 3-D, with D being the fractal dimension of the soil specimen. For a uniformly graded soil, the PSD is not fractal, but we can still use an appropriate D value to describe the PSD. In this case, D is not the fractal dimension, but a fitting parameter. However, with particle breakage, the PSD tends to become more and more fractal, and therefore D is usually called the fractal dimension.

Another widely-used one-parameter model is the Gaudin-Melog model (GMM) proposed by Harris (1968):

$$P(d)=1-(1-\frac{d}{d_{\max}})^k, \qquad 0<d<d_{\max} \tag{2}$$

where k is a fitting parameter.

Equations (1) and (2) are perhaps the simplest mathematical representations of a PSD. They have one fitting parameter and one specific particle size (d_{max}). Other models in the literature can have as many as two to seven fitting parameters (Vipulanandan and Ozgurel 2009; Fredlund et al. 2000). A widely used model for well-graded soils is the Fredlund unimodal model (FUM) (Fredlund et al. 2000):

$$P(d)=\frac{1}{\left\{\ln[\exp(1)+\left(\frac{a_{gr}}{d}\right)^{n_{gr}}]\right\}^{m_{gr}}}\left\{1-\left[\frac{\ln\left(1+\frac{d_{rgr}}{d}\right)}{\ln\left(1+\frac{d_{rgr}}{d_m}\right)}\right]^7\right\} \tag{3}$$

where d_m is the minimum size particle and a_{gr}, n_{gr}, m_{gr} and d_{rgr} are the four fitting parameters: a_{gr} defines the inflection point, n_{gr} the uniformity of the PSD (i.e. steepness of the PSD), m_{gr} the shape of the curve at small particle sizes and d_{rgr} is related to the amount of fines.

The performance of the different models have been previously compared against experimental data (Hwang et.al. 2002; Merkus 2009; Vipulanandan and Ozgurel 2009; Luo et.al. 2014; Bayat et.al. 2015; Zhou et.al. 2016). In general, a model with more parameters leads to better fitting of the experimental results. Models currently in the literature are used to fit specific PSD curves, not necessarily an evolving PSD curve due to particle breakage. The evolution of PSD during particle breakage follows certain trends, which are more identifiable for an initially uniformly graded soil specimen (Zhang et al. 2015), and the capacity of existing models in predicting an evolving PSD curve remains unclear.

In this paper, a simple two-parameter PSD model for granular soils is proposed based on the studies of particle breakage. The performance of the proposed model and two other simple one-parameter models (GGSM and GMM listed above) and the four-parameter model (FUM) are compared against experimental data obtained for soil specimens involving particle breakage and . The evolution of model parameters during particle breakage is studied. The proposed PSD model is also applied to assess the internal stability of widely graded granular soils.

A SIMPLE PSD MODEL AND DETERMINATION OF ITS PARAMETERS

In our previous studies (Zhang et.al. 2015; Tong et.al. 2015), we considered particle breakage as a probabilistic event, and defined a breakage probability to measure the degree of particle breakage of a uniformly graded soil sample. A two-parameter Weibull distribution was proposed to describe the distribution of new particles generated from the breakage (Figure 1). As shown in Figure 1, the initially uniformly graded soil sample (with particle sizes between $d_{\max-1}$ and $d_{\max}$) will break by a percentage (p) of the original mass, leading to a Weibull distribution of new particles (with particle size of d_1, d_2,...,$d_{\max-1}$) as Equation 4:

$$P^{*}=1-e^{-\left[\frac{x_i}{\lambda(1-x_i)}\right]^{\kappa}} \tag{4}$$

where P^* is the distribution of new particles generated from the particle breakage of an initially uniformly graded sample, $x_i=d_i/d_{\max-1}$ is the particle size ratio, $d_{\max}$ is the diameter of the maximum size particle, and $d_{\max-1}$ is the second largest particle diameter (second largest sieve size); λ is a scale parameter and κ is a shape parameter. As shown in Zhang et al. (2015), the main advantage of the proposed Weibull distribution is twofold: (1) it captures particle breakage of different patterns such as asperity breakage, surface grinding and particle splitting; and (2) it can be integrated into a Markov chain model to describe the breakage process of a non-uniformly graded soil sample.

Equation (4) defines the distribution of new particles, with sizes less than $\mathrm{d}_{\max-1}$, generated as a result of breakage, for example. The PSD of the whole specimen after breakage, is then based on the breakage probability or the percentage of broken mass (p). The percentages of particles in different size groups can then be calculated as follows:

$$\begin{cases} P(d)=\left\{1-e^{-\left(\frac{\frac{d}{d_{\max-1}}}{\lambda\left(1-\frac{d}{d_{\max-1}}\right)}\right)^{\kappa}}\right\}\times p, & 0<d\le d_{\max-1} \\ P(d)=p+\dfrac{1-p}{d_{\max}-d_{\max-1}}\times(d-d_{\max-1}), & d_{\max-1}<d\le d_{\max} \end{cases} \tag{5}$$

Equation (5) is a two-part function that is not continuously differentiable. It can be treated as a PSD model to some extent. When the size of particles is between 0 and $d_{\max-1}$, $P(d)$ can be calculated from the first part of Equation (5). When the particle size is between $d_{\max-1}$ and $d_{\max}$, the $P(d)$ can be obtained by linear interpolation as used in the second part of Equation (5). Equation (5) is supposed to describe the PSD of a granular soil of an arbitrary breakage probability (p). For a uniformly graded sample and a zero breakage probability, the PSD of the soil is $P(d)=(d-d_{\max-1})/(d_{\max}-d_{\max-1})$. It represents a line in the $P(d)$-d space, as shown in Figure 1. It is important to note that for real samples, the second largest particle may not be determined, as the PSD is defined at distinct points. Here, for a uniformly graded sample, the second largest particle size $d_{\max-1}$ is the same as $d_{\min}$.

The PSD of an initially uniformly graded granular soil after breakage tends to be continuous, or well graded, after breakage (Nakata et al. 2001; Zhang and Baudet 2013). Here, we consider the PSD curve of a granular soil after breakage as a continuous curve, with the second largest particle size $d_{\max-1}$ infinitely approaching the maximum particle size $d_{\max}$. In this case, the particle breakage probability p approaches 100% as shown in Figure 1. Equation (5) is then reduced to

$$P(d)=\lim_{d_{\max-1}\to d_{\max}}\left\{1-e^{-\left(\frac{\frac{d}{d_{\max-1}}}{\lambda\left(1-\frac{d}{d_{\max-1}}\right)}\right)^{\kappa}}\right\}=1-e^{-\left(\frac{d}{\lambda(d_{\max}-d)}\right)^{\kappa}} \tag{6}$$

Equation (6) is a new PSD model for non-uniformly graded granular soils broken from uniformly graded sample. It is also a modified Weibull distribution model, with two parameters: a scale parameter λ and shape parameter κ. This model reflects the fact that the mass percentage of particles $P(d)$ has a limit value of 1 when passing a particular size $d_{\max}$. The values of the two parameters can be calculated if two PSD points, such as: (d_{10}, $P(d_{10})$) and (d_{60}, $P(d_{60})$) are known. Once the values of the two parameters are determined, the PSD of a granular soil can be determined uniquely.

As shown in Equation (6), when $d=\lambda*d_{max}/(1+\lambda)$, the value of $P(d)\equiv 1-1/e\approx 0.632$, irrespective of κ value. Parameter λ is then determined as

$$\lambda = \frac{d_{63.2}}{d_{\max} - d_{63.2}} \tag{7}$$

where $d_{63.2}$ is the characteristic particle diameter at which 63.2% of the sample by mass is smaller. Equation (7) is the theoretical solution of parameter λ. It is a non-dimensional parameter and is only related to characteristic particle diameters $d_{63.2}$ and $d_{\max}$. Substituting Equation (7) into Equation (6) leads to the final form of the PSD model:

$$P(d) = 1 - e^{-\left(\frac{d(d_{\max} - d_{63.2})}{d_{63.2}(d_{\max} - d)}\right)^{\kappa}} \tag{8}$$

Equation (8) is an exponential function. If the values of $d_{\max}$ and parameter λ are known, the parameter κ can easily be obtained by using MATLAB fitting toolbox (cftool) (Matlab R2016b). The performance of the proposed PSD model can be evaluated according to the coefficient of determination R^2, defined as following:

$$R^2 = 1 - \frac{\sum_{1}^{N} (Y_i - Y_j)^2}{\sum_{1}^{N} (Y_i - \overline{Y}_i)^2} \tag{9}$$

where Y_i and Y_j are actual and calculated cumulative mass of particles finer than d, respectively. $\overline{Y}_i$ is mean of actual value.

The flow chart for obtaining and assessing parameter λ and κ is shown in Figure 2. The fitting process can be summarised as follows:

(1) Experimental characteristic diameter $d^*_{63.2}$ is determined by linear interpolation of the sieving test data.

(2) Parameter λ is calculated using Equation (7).

(3) Parameter κ is found by a nonlinear least square fitting of the experimental PSD data, based on the trust-region algorithm method in Matlab.

(4) The predicted characteristic diameter $d_{63.2}^{\#}$ is determined from Equation (6), and is compared with the experimental characteristic diameter $d_{63.2}^{*}$.

(5) Optimal values of λ and κ are obtained only when the coefficient of determination R^2 obtained in step 3 is sufficiently large ($R^2 \geq 0.95$) and the difference between the predicted and experimental values of $d_{63.2}$ is sufficiently small ($\Delta d_{63.2} = \left| d_{63.2}^{*} - d_{63.2}^{\#} \right| / d_{63.2}^{*} \leq 0.01$). Otherwise, the experimental characteristic diameter $d_{63.2}$ is reset to the predicted value and the above steps are repeated.

(6) The maximum iteration is set to 5. If either $R^2<0.95$ or $\Delta d_{63.2}>0.01$ is satisfied, exit the iteration with the latest values of λ and κ.

The iterative process in Figure 2 converges to a unique solution, typically within 1-2 iterations. The number of sieves used in the experimental data affects the convergence rate, and the more sieves lead to a faster convergence.

PARAMETRIC STUDY AND VALIDATION OF MODEL

In this section, we focus on the influences of model parameters on the shape of PSD and the relationship between the model parameters and classification systems commonly used in geotechnical engineering, such as the coefficient of uniformity C_u and the coefficient of curvature C_c. Besides, the proposed PSD model is verified and compared with other three PSD models (GGSM, GMM, FUM) based on a database of 154 continuous PSD curves (with 127 PSD curves broken from initial uniformly graded or non-uniformly graded samples and 27 PSD curves mixed by different group sizes, see the details in section 4 and section 5, respectively).

The most frequent particle size can directly be obtained by a particle size probability density function plotted in a log(d) scale (Fredlund et al. 2000). The differentiation of the proposed PSD model in a logarithm form is given by:

$$p(d)=\frac{dP(d)}{d\log(d)}=\frac{\ln(10)\kappa d_{\max}}{\lambda^{\kappa}}\times\frac{d^{\kappa}}{(d_{\max}-d)^{\kappa+1}}\times e^{-[\frac{d}{\lambda(d_{\max}-d)}]^{\kappa}} \tag{10}$$

There are three main types of continuous PSD curves in $P(d)$-log(d) space: hyperbolic (Type 1), S shaped (Type 2), and nearly linear (Type 3) (Zhu et.al. 2015). In order to verify the performance of the proposed model, we fix $d_{\max}$ at 50mm, and change the values of parameter λ and κ. The results are shown in Figure 3 and Figure 4.

As shown in Figure 3(b), the PSD of Type 1 is a hyperbolic shaped curve in the $P(d)$-log(d) space, and the PDF first increases and then decreases with increasing particle size. Type 2 in Figure 3(c) is an S shaped curve in the $P(d)$-log(d) space. The value of $p(d)$ shares the similar tendency with that of Type 1: first increases and then decreases with increasing particle size. Both of Type 1 and Type 2 PSD are unimodal distribution. However, a soil with Type 1 PSD has much more larger particles than Type 2 PSD. PSD of Type 3 as shown in Figure 3(a) is a nearly linear shaped curve in the P(d)-log(d) space, and the logarithmic density function increases with increasing particle size, which means a larger particle size has a larger mass percentage. General speaking, a soil with Type 3 PSD has the largest amount of large particles.

Figure 3 shows the influence of λ on PSD. For a constant κ (0.8) in Figure 3(b), the shape of the PSD curves are hyperbolic and of Type 1. As λ increases, the PSD becomes steeper. Figure 3(c) shows a plot of PSD with a constant κ (1.5) and varying λ. Particle sizes become smaller with an decreasing λ, but the shape of the PSD curves remains almost unchanged (Type 2). For a constant κ at 0.2, the PSD curves tend to be more linear (Type3, Figure 3(a)). In general, λ does not affect the shape of the PSD curves much if the value of κ is fixed, but it affects the characteristic particle sizes, for example d_{10} or d_{50}.

Figure 4 shows the influence of κ on PSD. In this figure, for a constant λ, the shape of PSD changes with parameter κ and all the PSD curves intersect at one point ($d_{63.2}$, 0.632). For example, for κ=0.2, the PSD curves are more or less in a linear shape (Type 3, also see Figure 3 (c)), irrespective of λ. As κ increases and λ is kept constant value (Figures 4(a)-4(c)), the shape of the PSD curves changes from a linear shape (Type 3) to a hyperbolic one (Type 1) and then to an S shaped (Type 2).

Figure 5 shows the influence of κ on logarithmic PDF. Again, the PSD curve type will change from Type 3 to Type 1 and then Type 2 with the increasing κ for a fixed λ. According to Figure 5, the most frequent particle size (the size corresponding to the peck value of PDF) will decreases with increasing κ.

In summary, the proposed PSD model is able to describe continuous PSD curves of the three main types. Moreover, the shape of a PSD curve is mainly affected by parameter κ, with parameter λ affecting characteristic particle diameters.

Parameters such as the coefficient of uniformity C_u and the coefficient of curvature C_c are commonly used as basic properties of soil in engineering field. Parameter λ has a theoretical solution as shown by Equation (7), and it is an index similar with C_u. The relationship between parameter κ and the coefficient of uniformity C_u, the coefficient of curvature C_c were investigated based on 154 PSD curves (see details in section 4 and section 5).

Figure 6 and Figure 7 show the correlation between parameter κ and C_c or C_u. Both C_c and C_u decrease with increasing κ and show an asymptote around κ=0.35. The relationship between κ and C_c or C_u can be expressed as power functions as shown in Figures 6-7. These relationships seem to be independent of the tested material or the testing method, as the experimental data listed in Table 1 include different soils in different tests. The correlations shown in Figure 6-7 indicate that parameter κ can be estimated with confidence from commonly used soil grading parameters. For example. as the solid square points shown in Figures 6-7, parameter κ=0.927, 0.372, 3.00 when C_u=5, C_c=1 and C_c=3, respectively, which means the parameter κ should be within the range of 0.372 to 0.927 if the soil sample is expected to be well graded based on the standard for engineering classification of soils in China (GB/T50145-2007).

To verify the proposed PSD model, fifty-three (53) sets of granular materials with 154 PSD curves are used to evaluate the applicability of proposed model. Those PSD curves are all non-uniformly and continuous graded, some of them break from uniformly graded samples (see the details in section 4) and others are an arbitrary mixture of particles from different group sizes (see the details in section 5). Moreover, other three PSD models (GGSM, GMM, and FUM) are also used for comparison. The results are shown in Figure 8.

Figure 8 shows the variation of the correlation coefficient R^2 versus the particle diameter $d_{63.2}$. The reason that we choose $d_{63.2}$ is that $d_{63.2}$ is an important particle size and determines the value of λ in this study. The values of $d_{63.2}$ are obtained by setting $P(d_{63.2})$=0.632 to Equation (6). An R^2 value closer to 1 indicates a better fitting. As shown in Figure 8, the prediction of the proposed model is relatively good across different values of $d_{63.2}$. The overall performance of the GGSM model is better than the GMM model, although some values of R^2 of the GMM are larger than those of GGSM's in certain cases. The model proposed in this paper and FUM are superior to the previous two models. In general, the model proposed in this paper is able to capture a wide variety of PSDs from the literature and it performs better than the FUM model while having less fitting parameters and simple mathematical form.

EVOLVING PARTICLE SIZE DISTRIBUTIONS DUE TO BREAKAGE

In this section, twenty-six (26) sets of granular materials with 127 PSD curves are used to evaluate the applicability of the proposed model involving particle breakage. The selected experimental data covers different material properties and loading types and most of the curves are obtained from tests designed to induce particle crushing tests (Bard 1993; Hagerty et al. 1993; Luzzani and Coop 2002; Coop et al. 2004; Russell and Khalili 2004; Okada et al. 2004; Mayoraz et al. 2006; Guimares et al. 2007; Kikumoto et al. 2010; Xiao et al. 2014, 2016; Zhang et al. 2017). Some typical detailed fitting results are shown in Table 1. The fitting of experimental data in Table 1 was done individually for each PSD curve, which allows us to examine the general capacity of the proposed model in predicting evolving PSD curves.

Table 1 shows the fitting results of the experimental data of the four PSD models. The performances of different PSD models can be evaluated by the correlation coefficient R^2. In Table 1, there is a consistent and monotonic evolution of the 2 fitting parameters (λ and κ) of the proposed model in most cases, except for the data from Coop et al. (2004) at very large strains. The reason for this inconsistent and non-monotonic evolution of 2 parameters is either an experimental error or particle aggregation. The test results in Coop et al. (2004) showed that the number of fine particles first increased with increasing strain and then dropped at very high strains, which is not possible unless particle aggregation occurs at large strains. The proposed model does not consider particle aggregation. It is noted that the performance of FUM model is only verified by those PSD data with more than eight sieving points in Figure 8 and Table 1, because the fitting results may be unreliable when the sieving points are too few to fit for the four fitting parameters.

According to the data in Table 1, the GGSM model and GMM model have a relatively good performance for describing PSD curves for specimens at relatively low stresses or strains (with less particle breakage). However, at large stress or strain, the predictions of the proposed model and FUM model become significantly better than the GGSM model and GMM model, implying that the proposed model captures the particle breakage better than the GGSM and the GMM models.

As mentioned above, the evolution of PSD curves during particle breakage exhibits certain trends, which are easily identifiable for initially uniformly graded samples. Ideally these trends should be captured in the PSD model. Figure 9 shows the evolution of the two model parameters (λ and κ) with stresses or strains for a range of tests and materials. Both parameters follow clear trends during breakage, decreasing with increasing stresses or strains (or increasing extent of breakage) and approaching stationary values at high degrees of breakage. The following equation provides a relatively good prediction of the evolution of the two parameters (λ and κ):

$$\lambda = a_\lambda + b_\lambda e^{c_\lambda \sigma(\gamma,p)}, \qquad \kappa = a_\kappa + b_\kappa e^{c_\kappa \sigma(\gamma,p)} \tag{11}$$

where a, b, c are fitting parameters, $\sigma(\gamma,p)$ is stress (strain) in the test. With Equation (11), only two sets of parameters (a, b, c) or total six parameters are needed to predict the PSD curve at an arbitrary degree of breakage, which is an important advantage of the proposed model.

ASSESSING INTERNAL STABILITY OF WIDELY GRADED GRANULAR SOILS

In additional to particle breakage, the PSD model proposed in this paper can also be applied to assess internal stability of granular filters. One of the most commonly used geometric criteria is the criterion by Kenney and Lau (1985 and 1986). A geometric index ratio of H/F was proposed and applied in the analysis of internal stability of granular soils. A granular sample would be considered as unstable if

$$(H/F)_{\min}<1 \tag{12}$$

where H is the mass fraction of particles with size from d to $4d$, F is the mass fraction of particles with size finer than d as shown in Figure 10. For a widely graded and uniformly graded sample, the search for the minimum value of H/F will end at F=20% and F=30% respectively.

For a widely graded granular soil (with minimum particle size 0.063mm), the whole PSD curve can be represented by the proposed PSD model as shown in Equation (6). Substituting Equation (6) into Equation (12) leads to:

$$\left\{\frac{e^{-\left(\frac{d}{\lambda(d_{\max}-d)}\right)^{\kappa}}-e^{-\left(\frac{4d}{\lambda(d_{\max}-4d)}\right)^{\kappa}}}{1-e^{-\left(\frac{d}{\lambda(d_{\max}-d)}\right)^{\kappa}}}\right\}_{\min}<1, \qquad 0.063<d\le\frac{d_{\max}}{4} \tag{13}$$

Equation (13) means that for a given particle size d (from 0.063 to $d_{\max}/4$), the value of H/F is always less than 1. It is a linear programming problem to some extent. The maximum value of $d_{\max}\le$100mm for most granular soils. Letting $d_{\max}/d=y$, the range of y values should be from 4 to 100/0.063(≈1600). Equation (13) can then be expressed as

$$\begin{cases}(2e^{-\left(\frac{1}{\lambda(y-1)}\right)^{\kappa}}-e^{-\left(\frac{1}{\lambda(\frac{1}{4}y-1)}\right)^{\kappa}}-1)/_{4\le y\le1600}<0\\ \lambda>0\\ \kappa>0\end{cases} \tag{14}$$

Letting $f\left(y\right)=2e^{-\left(\frac{1}{\lambda(y-1)}\right)^{\kappa}}-e^{-\left(\frac{1}{\lambda(\frac{1}{4}y-1)}\right)^{\kappa}}-1$, we can plot $f(y)=0$ in λ-κ space for any $4\leq y\leq1600$, as shown in Figure 11.

Figure 11 shows some typical curves of $f(y)=0$ in λ-κ space for given y. In general, the curve of $f(y)=0$ tends to be flat with increasing y. In the case of $y=4$, the relationship between λ and κ is be plotted as curve 1, while, in the case of $y=1600$, the relationship between λ and κ is plotted as curve 8. Since for any y ($4\leq y\leq1600$), $f(y)$ needs to satisfy $f(y)<0$, that is to say, the range of λ and κ needs to below all the curves of $f(y)=0$ for any y ($4\leq y\leq1600$). In other words, if the granular sample would be considered unstable, the range of λ and κ should fall within the area below curves AB and BC (Area 1, the intersection of all the range of λ and κ for any y) as shown in Figure 11. Point B is the intersection of curve 1 and curve 8.

The same method for determining the range of λ and κ for the consideration of stable of granular soils since the criterion should be rewritten as:

$$(H/F)_{\min}\geq1 \tag{15}$$

Similar conclusion can be obtained that the granular sample would be considered stable if the range of λ and κ falls in the area above curves DB and BE (Area 2). It is worth noting that this area is not, but very close to, the real area because the intersections of all the curves of λ and κ for any given y are close to, but not exactly at point B. Here, for simplicity, we use point B to distinguish the area of λ and κ when assessing the internal stability for soil samples.

Data of internal stability tests on 27 widely graded granular soils from the literature (Kenney and Lau 1985; Aberg 1993; Indraratna et al. 2015; Skempton and Brogan 1994) are collected and used for verifying the stable and unstable areas proposed in the λ-κ space. PSD parameters λ and κ are first obtained using fitting process as shown in Figure 2, and their values are then plotted in the λ-κ space. The results are shown in Figure 12.

Figure 12 shows that more than 50% (10/18) of the stable grading fall into the proposed stable area, 7 stable grading fall into area 4, while, and only 1 stable grading falls into the unstable area.

Unstable gradings tested in the laboratory fall into stable area, unstable area and other two areas with the same proportion (3/9). For areas 3 and 4, Equation (12) and Equation (15) are not always satisfied for any y. If both Equation (12) (or Equation (15)) and $F \leq 20\%$ are met, the granular soil sample can be regarded as unstable (or stable). That is to say, for the PSD parameters λ and κ falling into area 3 and area 4, the internal stability of the granular soil cannot be determined and needs further assessment.

The stable area and unstable area proposed in this paper are based on Kenney and Lau's criterion. It is a more straightforward and simpler way for assessing internal stability of widely graded granular soils, compared against other methods in the literature. As shown in Figure 12, the stable and unstable area defined in the λ-κ space are in relatively good agreement with experimental results.

CONCLUSIONS

A simple particle size distribution model for granular materials is proposed in this paper. The model contains two parameters, one parameter (λ) representing a characteristics particle diameter, and the other parameter κ closely correlated to the coefficient of uniformity (C_u) or the coefficient of curvature (C_c). Parameter κ mainly affects the shape of the PSD curve, while parameter λ affects characteristic particle sizes of the soil sample.

The proposed PSD model can capture the main types of continuous PSD curves. Its performance is compared against the Gates-Gaudin-Schuhmann model, Gaudin-Melog model and Fredlund unimodal model by analysing 53 soil specimens with 154 PSD curves. It is shown that the proposed PSD model performs better than the Gates-Gaudin-Schuhmann model and the Gaudin-Melog model, particularly for PSD curves obtained at high degrees of particle breakage. The proposed two-parameter PSD model displays a similar performance to the four-parameter Fredlund unimodal model. It is shown that the two parameters in the proposed model follow clear trends identifiable during particle breakage of initially uniformly graded soil samples. Equations are proposed for these trends, with which the evolution of PSD curves during particle breakage of one soil sample can be predicted with two sets of model parameters.

A continuous particle size distribution model provides a quantitative method for estimating other soil properties and is an important element in studying problems such as particle breakage and assessment of internal stability. An initially non-uniformly graded sample can be treated either as the product of a uniformly graded sample due to particle breakage, or an arbitrary mixture of particles from different group sizes (Zhang et al. 2015). For initially non-uniformly graded samples, the situation can be more complex. The proposed model can be extended to capture more complex PSDs (e.g. bimodal distributions representative of gap-graded soils) through superposition of two or more unimodal PSDs.

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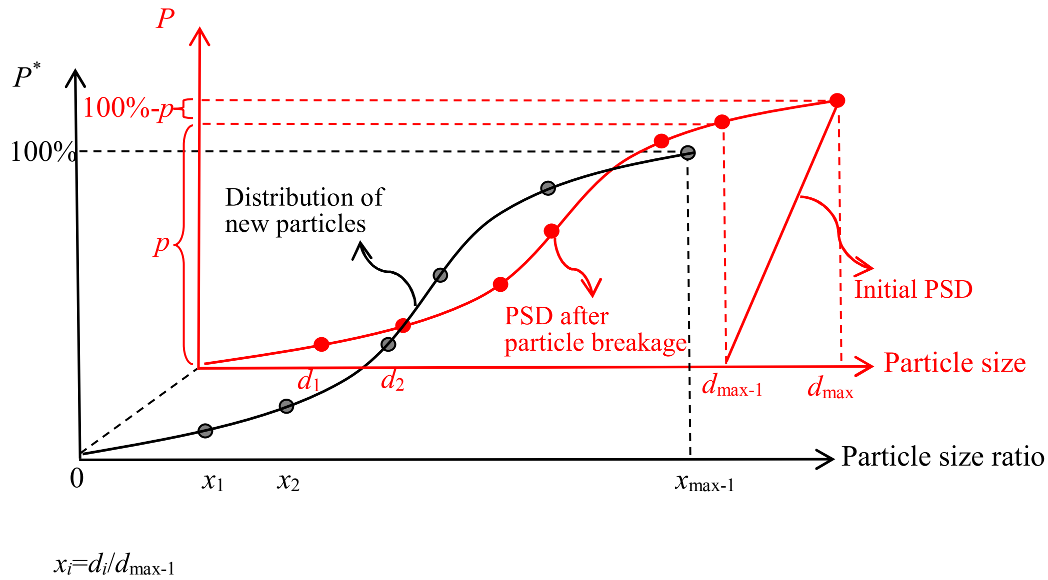
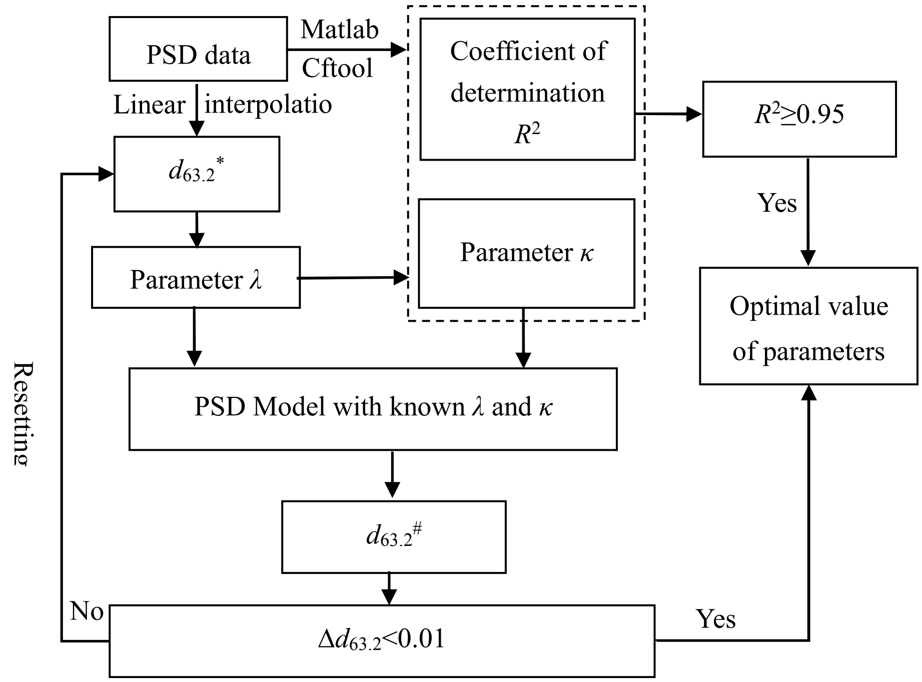


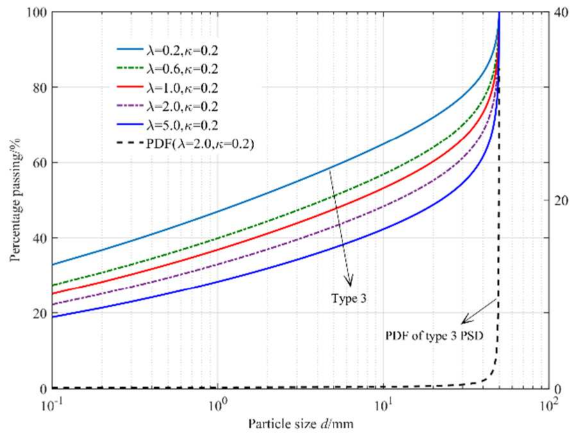
1

2 Fig. 1. Schematic diagram of PSD of uniformly graded sample after particle breakage

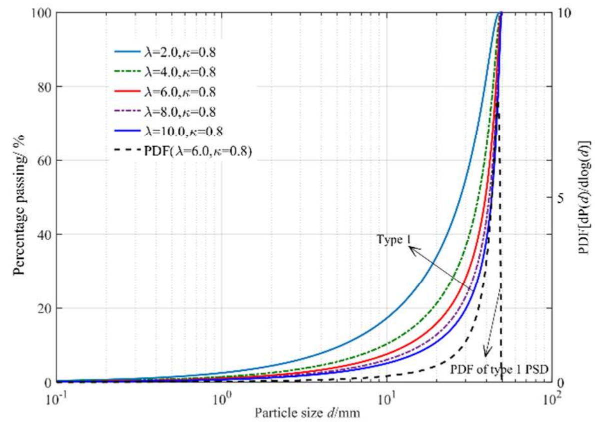


3

4 Fig. 2. Flow chart for obtaining and assessing parameter λ and κ



(a)



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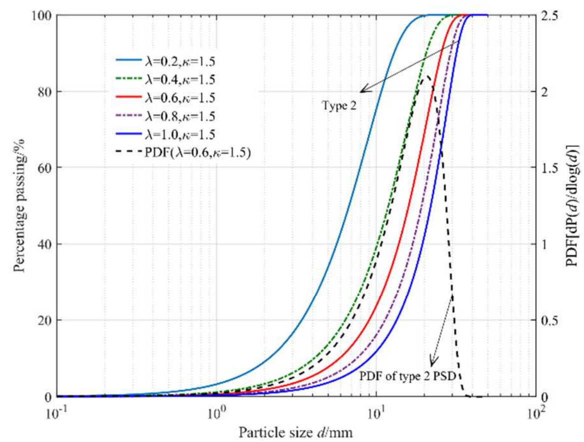
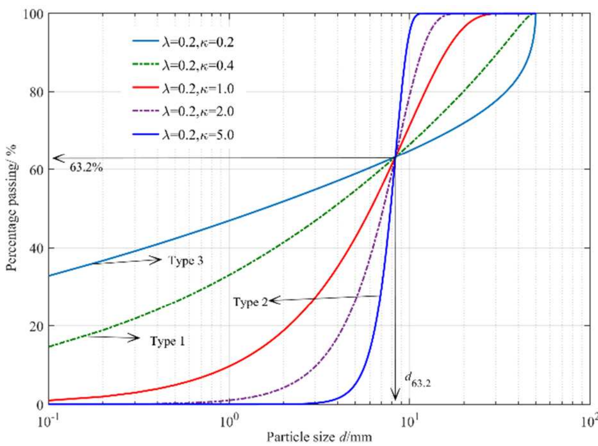


(c)

Fig. 3. Influence of parameter λ on particle size distribution: (a) varying of λ with a fixed κ=0.2; (b) varying of λ with a fixed κ=0.8; (c) varying of λ with a fixed κ=1.5.



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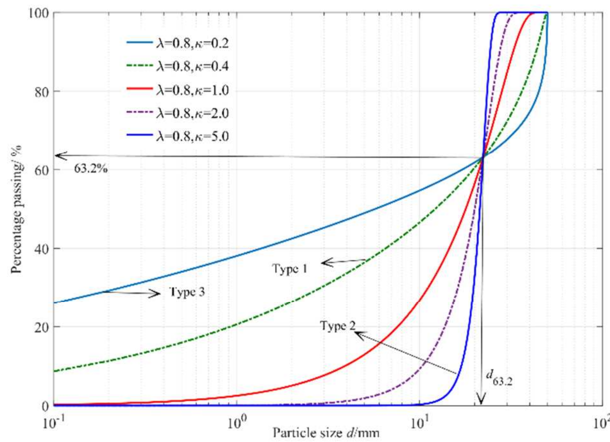

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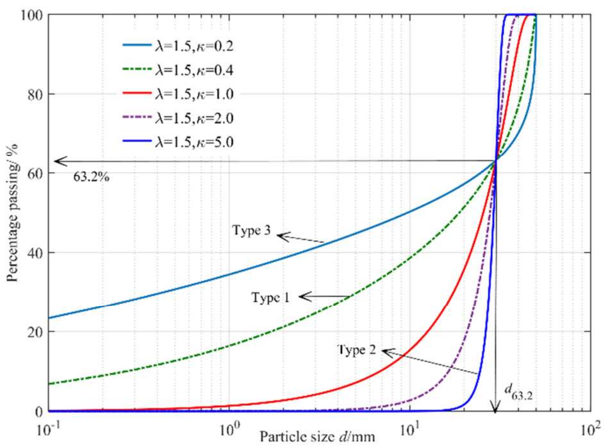
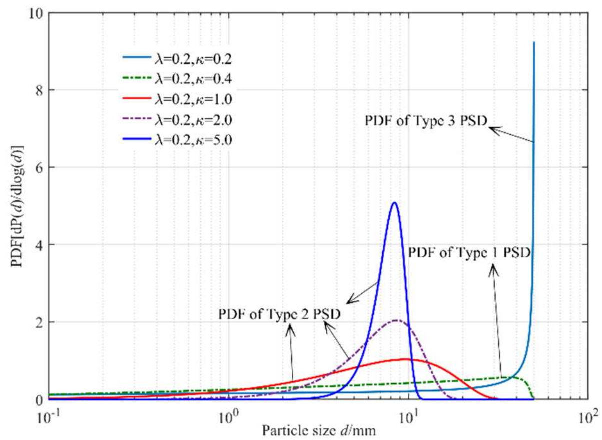
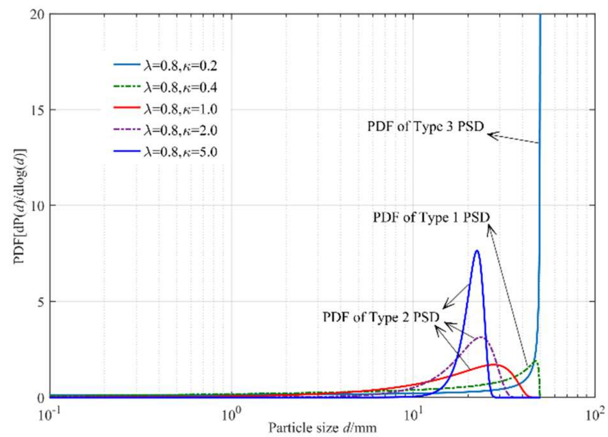


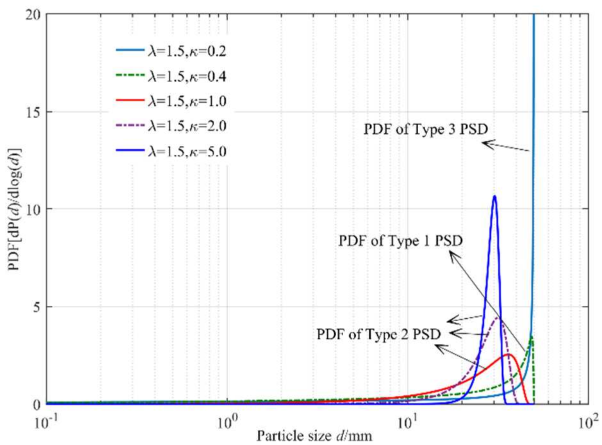
(c)

Fig. 4. Influence of parameter κ on particle size distribution: (a) varying of κ with a fixed λ=0.2; (b) varying of κ with a fixed λ =0.8; (c) varying of κ with a fixed λ =1.5.





(a) (b)



(c)

Fig. 5. Influence of parameter κ on logarithmic PDF: (a) varying of κ with a fixed λ=0.2; (b) varying of κ with a fixed λ =0.8; (c) varying of κ with a fixed λ =1.5.

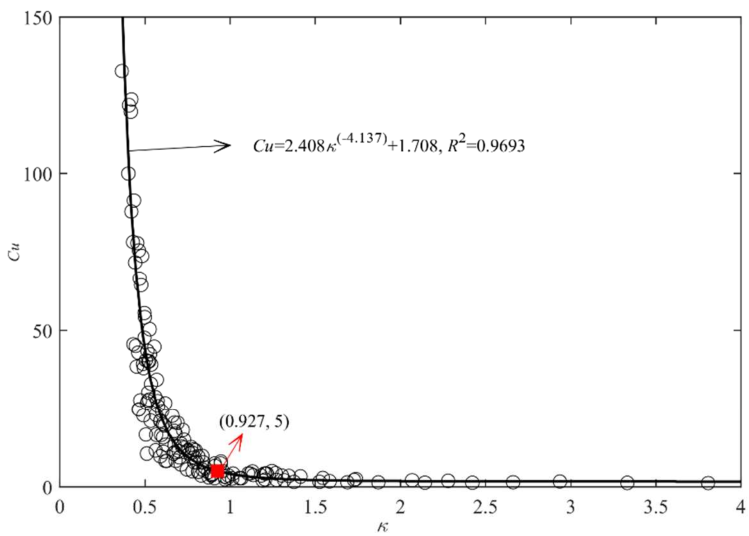


Fig. 6. Correlation between parameter κ and coefficient of non-uniformity C_u

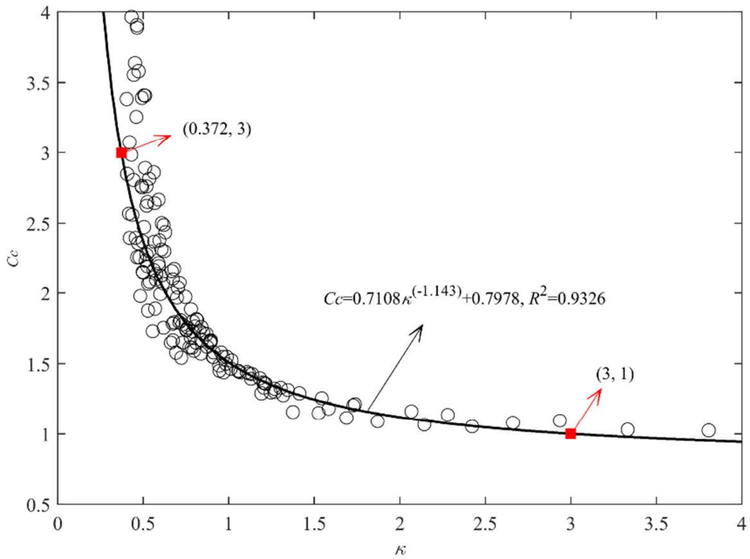


Fig. 7. Correlation between parameter κ and coefficient of curvature C_c

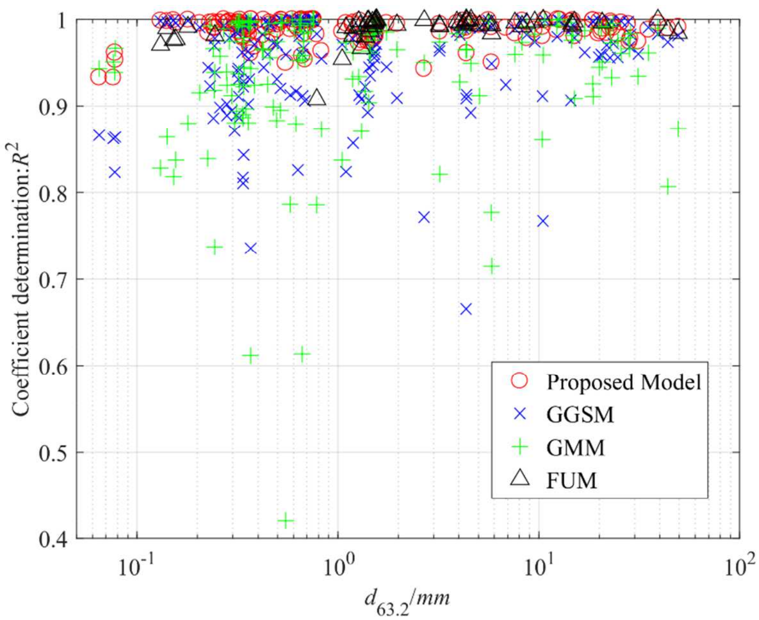
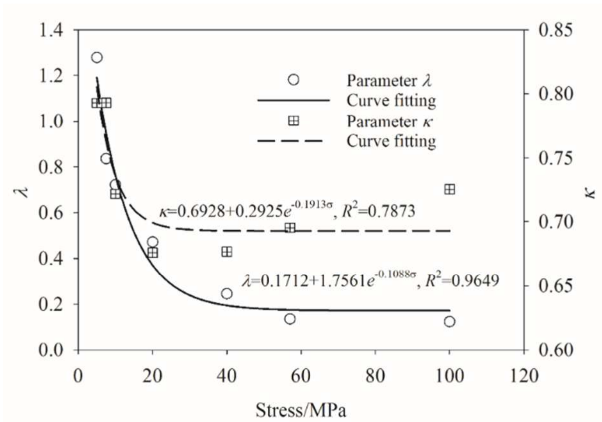
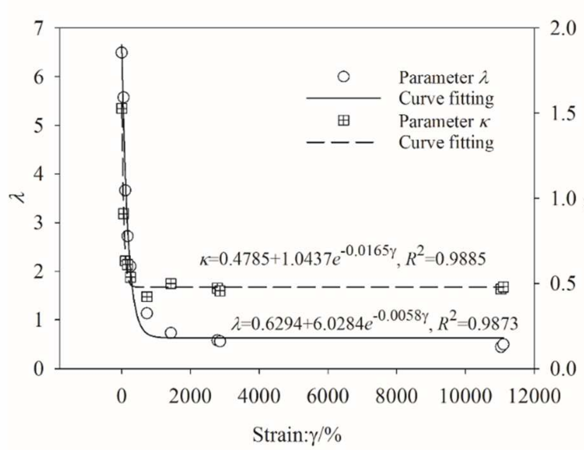
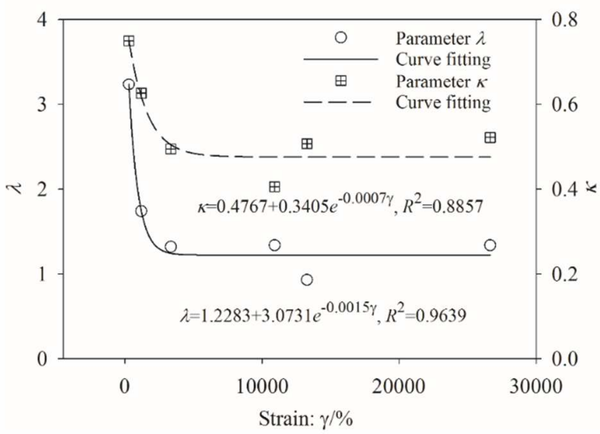


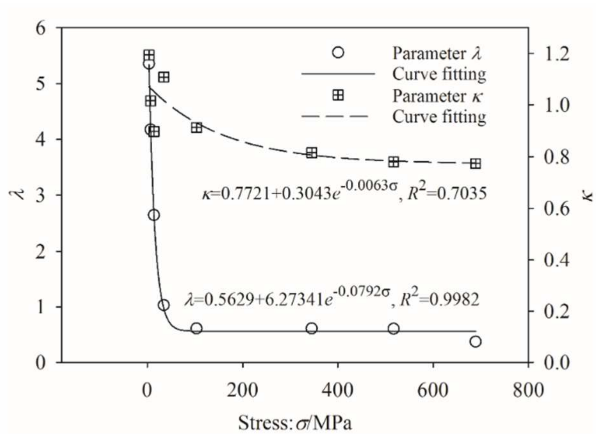
Fig. 8. Performance of the four PSD models at different particle diameters $d_{63.2}$.



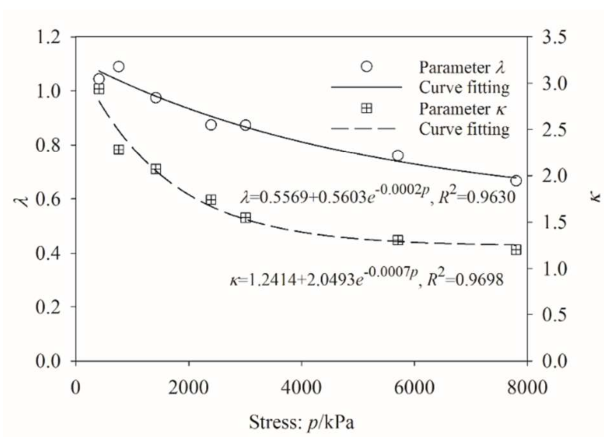


(a) (b)





(c) (d)



(e)

Fig. 9. Evolution of model parameters with particle breakage: (a) data from Bard (1993); (b)-(c) data from Coop et al. (2004); (d) data from Hagerty et al. (1993); (e) data from Russell & Khalili (2004).

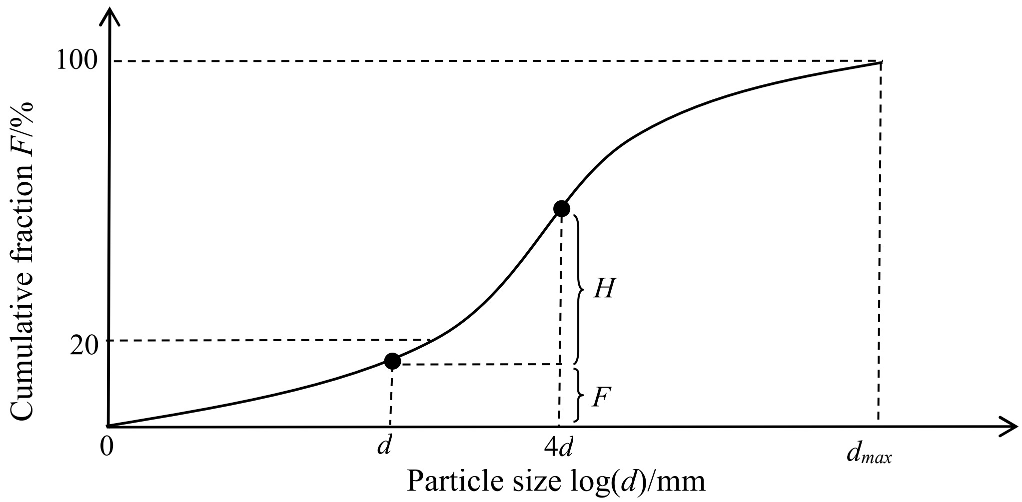


Fig. 10. Illustration of Kenny and Lau’s criterion.

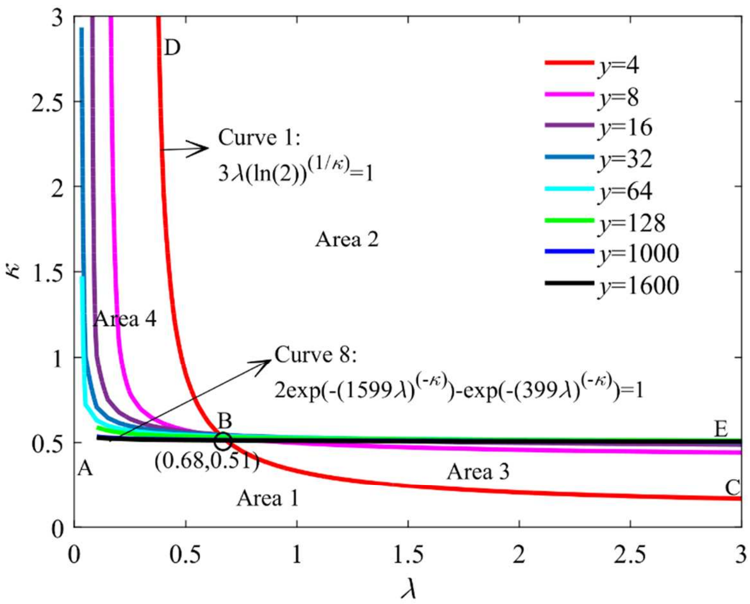


Fig. 11. Curves of $f(y)=0$ in λ-κ space

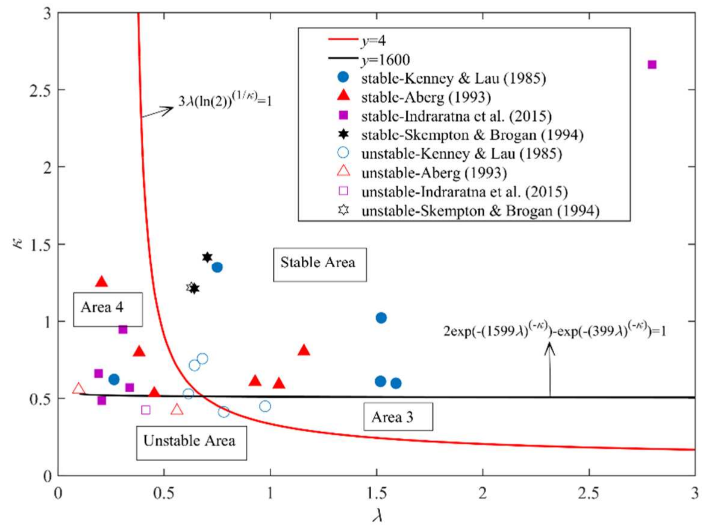


Fig. 12. Validation of internal stability of well-graded granular soil

Table 1. Performance of four PSD models for different materials

Reference	Property/Loading	Proposed Model λ	Proposed Model κ	Proposed Model R^2	GGMM R^2	GMM R^2	FUM R^2
Coop et al. (2004)	CS/UG/RS/VS1/no shearing	6.498	1.528	0.9997	0.9981	0.9959	
	CS/UG/RS/VS1/ γ=52%	5.572	0.911	0.9833	0.9545	0.9963	
	CS/UG/RS/VS1/ γ=104%	3.664	0.632	0.9935	0.9616	0.9912	
	CS/UG/RS/VS1/ γ=171%	2.716	0.611	0.9959	0.9848	0.9947	
	CS/UG/RS/VS1/ γ=251%	2.101	0.537	0.9976	0.9785	0.9746	
	CS/UG/RS/VS1/ γ=730%	1.129	0.422	0.9845	0.9444	0.8389	
	CS/UG/RS/VS1/ γ=1430%	0.729	0.500	0.9992	0.9981	0.8800	0.9908
	CS/UG/RS/VS1/ γ=2780%	0.579	0.472	0.9955	0.9955	0.8373	0.9768
	CS/UG/RS/VS1/ γ=2860%	0.556	0.458	0.9989	0.9988	0.8179	0.9757
	CS/UG/RS/VS1/ γ=11030%	0.444	0.467	0.9984	0.9979	0.8280	0.9702
	CS/UG/RS/VS1/ γ=11100%	0.496	0.480	0.9976	0.9965	0.8641	0.9896
	CS/UG/RS/VS2/ γ=285%	3.235	0.750	0.9994	0.9855	0.9982	
	CS/UG/RS/VS2/ γ=1180%	1.742	0.626	0.9996	0.9924	0.9888	
	CS/UG/RS/VS2/ γ=3350%	1.325	0.496	0.9894	0.9691	0.9181	
	CS/UG/RS/VS2/ γ=10920%	1.343	0.406	0.9905	0.9388	0.7373	0.9815
	CS/UG/RS/VS2/ γ=13280%	0.931	0.507	0.9952	0.9906	0.9154	
	CS/UG/RS/VS2/ γ=26650%	1.343	0.522	0.9943	0.9779	0.9388	
	CS/UG/RS/VS3/ γ=9040%	5.126	0.877	0.9892	0.9626	0.9979	
	CS/UG/RS/VS3/ γ=23900%	3.908	0.593	0.9900	0.9224	0.9736	
	CS/UG/RS/VS3/ γ=31700%	3.960	0.623	0.9894	0.9283	0.9782	
	CS/UG/RS/VS3/ γ=37500%	3.062	0.475	0.9964	0.9111	0.9371	
	CS/UG/RS/VS3/ γ=147000%	3.137	0.495	0.9852	0.8863	0.9248	
Bard (1993)	PC/UG/1DC/ VS=5MPa	1.278	0.793	0.9989	0.9982	0.9980	0.9946
	PC/UG/1DC/ VS=7.5MPa	0.836	0.793	0.9976	0.9906	0.9923	0.9950
	PC /UG/1DC/ VS=10MPa	0.723	0.722	0.9982	0.9896	0.9871	0.9935
	PC /UG/1DC/ VS=20MPa	0.471	0.676	0.9945	0.9635	0.9737	0.9932
	PC /UG/1DC/ VS=40MPa	0.245	0.677	0.9945	0.9091	0.9649	0.9935
	PC /UG/1DC/ VS=57MPa	0.135	0.695	0.9958	0.8569	0.9618	0.9894
	PC /UG/1DC/ VS=100MPa	0.123	0.726	0.9953	0.8238	0.9729	0.9906
Russell & Khalili (2004)	QS/NG (initial)	1.478	1.731	0.9912	0.9022	0.8804	
	QS/NG/DT/MES=410kPa	1.044	2.939	0.9944	0.8711	0.8805	
	QS/NG/DT/MES=760kPa	1.090	2.283	0.9930	0.8893	0.8939	
	QS/NG/DT/MES=1417kPa	0.975	2.071	0.9944	0.8946	0.9117	
	QS/NG/DT/MES=2395kPa	0.874	1.741	0.9957	0.8890	0.9242	
	QS/NG/DT/MES=3006kPa	0.873	1.547	0.9953	0.9022	0.9393	
	QS/NG/DT/MES=5705kPa	0.760	1.306	0.9959	0.8983	0.9573	
	QS/NG/DT/MES=7800kPa	0.666	1.204	0.9967	0.8859	0.9657	
Zhang et al. (2017)	ST/NG (initial)	1.666	1.185	0.9745	0.9598	0.9342	
	ST/NG/1DC/VS=2MPa	1.304	1.139	0.9742	0.9680	0.9657	
	ST/NG/1DC/VS=5MPa	0.977	1.121	0.9823	0.9561	0.9759	
	ST/NG/1DC/VS=10MPa	0.883	0.845	0.9902	0.9719	0.9949	
	ST/NG/1DC/VS=15MPa	0.771	0.763	0.9839	0.9701	0.9913	
	ST/NG/1DC/VS=20MPa	0.689	0.733	0.9857	0.9594	0.9897	
	SM/NG (initial)	2.351	0.991	0.9879	0.9905	0.9612	
	SM/NG/1DC/VS=2MPa	1.336	0.958	0.9778	0.9822	0.9832	
	SM/NG/1DC/VS=5MPa	0.826	0.914	0.9903	0.9556	0.9913	
	SM/NG/1DC/VS=10MPa	0.657	0.755	0.9835	0.9565	0.9905	
	SM/NG/1DC/VS=15MPa	0.512	0.682	0.9887	0.9617	0.9866	
	SM/NG/1DC/VS=20MPa	0.406	0.787	0.9969	0.9066	0.9959	
Xiao et al.	CS/UG/IL/SH1, IW=4.71KJ	1.808	0.669	0.9982	0.9895	0.9867	0.9922

(2016)	CS/UG/IL/SH1, IW=9.71KJ	1.454	0.527	0.9940	0.9773	0.9309	0.9808
	CS/UG/IL/SH1, IW=19.42KJ	1.105	0.433	0.9856	0.9709	0.8371	0.9538
	CS/UG/IL/SH1, IW=38.85KJ	0.644	0.438	0.9827	0.9830	0.786	0.9079
	CS/UG/IL/SH2, IW=4.71KJ	2.851	0.818	0.9873	0.9650	0.9871	0.9965
	CS/UG/IL/SH2, IW=9.71KJ	2.490	0.683	0.9820	0.9447	0.9632	0.9921
	CS/UG/IL/SH2, IW=19.42KJ	2.157	0.569	0.9795	0.9224	0.917	0.9796
	CS/UG/IL/SH2, IW=38.85KJ	1.890	0.514	0.9760	0.9079	0.8707	0.9666
	CS/UG/IL/SH3, IW=4.71KJ	3.351	1.070	0.9914	0.9819	0.9971	0.9990
	CS/UG/IL/SH3, IW=9.71KJ	3.145	0.897	0.9811	0.9597	0.9859	0.9966
	CS/UG/IL/SH3, IW=19.42KJ	2.924	0.814	0.9763	0.9473	0.9758	0.9918
	CS/UG/IL/SH3, IW=38.85KJ	2.637	0.715	0.9746	0.9347	0.9603	0.9860
	CS/UG/IL/SH3, IW=4.71KJ	3.566	1.208	0.9932	0.9863	0.9981	0.9995
	CS/UG/IL/SH3, IW=9.71KJ	3.449	1.060	0.9839	0.9711	0.9920	0.9980
	CS/UG/IL/SH3, IW=19.42KJ	3.298	0.969	0.9801	0.9624	0.9874	0.9967
	CS/UG/IL/SH3, IW=38.85KJ	3.125	0.895	0.9780	0.9556	0.9828	0.9951
Mayoraz et al. (2006)	ST/UG/ML/MMP=0.5MPa	3.348	1.059	0.9872	0.9799	0.9922	
	ST/UG/ML/MMP=1MPa	1.872	0.463	0.9806	0.9114	0.8608	
	ST/UG/ML/MMP=3MPa	0.748	0.300	0.9902	0.9246	0.4546	
	LT/UG/ML/MMP=0.5MPa	3.870	1.587	0.9990	0.9984	0.9996	
	LT/UG/ML/MMP=1MPa	3.522	1.321	0.9998	0.9989	0.9991	
	LT/UG/ML/MMP=3MPa	2.027	0.842	0.9990	0.9987	0.9985	

Note: CS-Carbonate Sand, PC-Petroleum Coke, QS-Quartz Sand, SM-Sandy Mudstone, ST-Sandstone, LT-Limestone

UG-uniformly graded, NG-non-uniformly graded

RS-Ring shear test, 1DC-One dimensional compression, DT-Drained triaxial test, IL-Impact loading test, ML-Monotonic loading test

VS-vertical stress, VS1-vertical stress ranges 650–930 kPa, VS2-vertical stress ranges 248–386kPa, VS3-vertical stress ranges 60–97kPa, MES- mean effective stress, MMP-maximum mean pressure

SH1-specimen height=31.8mm, SH2-specimen height=63.7mm, SH3-specimen height=95.5mm, SH4-specimen height=127.3mm, IW-input work