Adaptive Fuzzy Observer based Hierarchical Sliding Mode Control for Uncertain 2D Overhead Cranes

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This paper proposes a new approach to robustly control a 2D under-actuated overhead crane system, where a payload is effectively transported to a destination in real time with small sway angles, given its inherent uncertainties such as actuator nonlinearities and external disturbances. The control law is proposed to be developed by the use of the robust hierarchical sliding mode control (HSMC) structure in which a second-level sliding surface is formulated by two first-level sliding surfaces drawn on both actuated and under-actuated outputs of the crane. The unknown and uncertain parameters of the proposed control scheme are then adaptively estimated by the fuzzy observer, where the adaptation mechanism is derived from the Lyapunov theory. More importantly, stability of the proposed strategy is theoretically proved. Effectiveness of the proposed adaptive fuzzy observer based HSMC (AFHSMC) approach was extensively validated by implementing the algorithm in both synthetic simulations and real-life experiments, where the results obtained by our method are highly promising.

Keywords: 2D overhead crane, hierarchical sliding mode control, fuzzy observer, under-actuated systems, Lyapunov function

1. Introduction

An overhead crane, also called a bridge crane, is more and more popular in many manufactures or industries due to its universal ability to transport a heavy object or hazardous material from one place to another place. It is also widely utilized in harbour ports to load and unload cargo from ships [1,2]. The model of the overhead crane is classified into a class of under-actuated mechanical systems, where a number of control inputs is less than a number of actuators. For instance, in a 2D overhead crane, both the position of the trolley and the swing angle of an object is dependent on the control force. Due to oscillations of the payload during operations, the bridge crane may severely cause unsafety on operators, loads or surrounding items [3]. Furthermore, the oscillation of the payload is nonlinear and highly coupled with motion of the trolley, which make the overhead crane cumbersome to control. As a result, expectations in controlling the 2D overhead crane are to effectively drive the trolley on a desired path while minimizing the swing angle of the payload.

Many approaches to efficiently control an overhead crane have been proposed in the past decades, from the open-loop to closed-loop control techniques. While the open-loop control method [4] is straightforward to implement on a crane but very sensitive to external disturbances, the closed-loop approaches, by employing sensor measurements and system state estimations, are more efficient to control crane actuators due to their less sensitivity to the external disturbances as well as system uncertainties [5]. For instance, Yu *et al.* [6] proposed a nonlinear two closed loop controller to fast transport an object to a desired destination while minimizing its oscillations. Another nonlinear control law based on partial feedback linearization was developed for a 2D crane system [7]. Furthermore, in the works [8,9], Le *et al.* extended the nonlinear partial feedback linearization controllers for a 3D overhead crane system by considering nonlinear feedback of actuated and un-actuated states in a superposition fashion. Due to strong adaptability to complexity and nonlinearity of a crane, a fuzzy model was used to present an overhead crane, which leads to a fuzzy logic controller in the works [10,11].

To maintain robustness of a crane system under its actuator nonlinearities and parameter uncertainties, sliding mode control (SMC) has attracted much attention from researchers and practitioners [12-14]. However, how to formulate the sling surfaces so as to guarantee stability of actuators and un-actuators in a crane is really challenging. For instance, the work [15] first developed an intermediate variable based upon state errors and then formed as second-level sliding surface. Among approaches of defining a sliding surface for a SMC scheme, the hierarchical SMC (HSMC) law has been widely employed in a class of under-actuated systems [16,17,27,28,33]. Wang *et al.* [16] proposed the HSMC strategy for a class of second-order under-actuated systems, where a first-level sliding surface is defined for each subsystem. A second-level sliding surface is then formulated as a linear combination of the first-level sliding surfaces. Similarly, the authors of the work [17] developed another hierarchical structure of sliding surfaces in designing a control law for a single-input-multiple-output under-actuated system. The first-layer sliding surface based on the first subsystem is computed, which is then incorporated into the sliding surface of a second subsystem to formulate a second-layer sliding surface. The chain is subsequently produced until the last subsystem.

It is noted that in the SMC laws all the system parameters are assumed to be known and certain. Though, in fact, those parameters can be deterministically estimated, they are highly uncertain due to the system nonlinearities and external disturbances. That is, it is impractical to certainly determine the system parameters Therefore, the more accurately the system parameters are computed, the more efficiently the control scheme works. Some adaptive control strategies have been developed to consider the system parameter uncertainties. For instance, in the works [18,19], a nonlinear coupling control law for a crane system is discussed, where its parameters are adaptively updated. Likewise, Le *et al.* [20] proposed a model reference adaptation mechanism, derived from Lyapunov theory, to approximate system parameters in a crane system. Moreover, a fuzzy disturbance observer was introduced by Kim [21] for designing a controller in nonlinear systems. By employing the fuzzy observer [21], Park *et al.* [22,23] discussed a method to represent crane system uncertainties as well as actuator nonlinearities. In the previous work [30], we considered a SMC law for an offshore container crane, where system parameters are adaptively learned by a radial basis function network. Moreover, in the work [32], Wang *et al.* employed a finite-time disturbance observer to identify unknown disturbances in complex marine environments. The proposed observer is utilized in conjunction with the nonsingular fast terminal sliding mode manifold to form a new finite-time control law for effectively tracking trajectories of a surface vehicle. Some other SMC strategies were designed for non-linear systems in which unknown parameters are adaptively estimated by a neural network [24,29], a fuzzy-neural network [25] and a fuzzy wavelet neural network [26].

In this work, an adaptive HSMC strategy employing the fuzzy observer is investigated to address the control problem in a 2D overhead crane system. There are two hierarchical layers in the proposed SMC law. Two first-level sliding surfaces are formulated from two subsystems, including actuated and under-actuated outputs of the 2D overhead crane. Then the sliding surface in the second layer is simply formed by linearly combining two first-level sliding surfaces. Moreover, to deal with challenges in computing the system parameters due to system uncertainties such as parameter variations and nonlinearities or external disturbances, it is proposed to utilize the fuzzy observer to approximately but adaptively estimate the dynamics of the crane. The adaptive fuzzy observer based HSMC (AFHSMC) algorithm was extensively validated in both synthetic simulations and real-life experiments. The results obtained by the proposed approach are highly promising. More importantly, based on the Lyapunov theory, stability of the 2D overhead crane system is theoretically proven.

The remaining of the paper is arranged as follows. The dynamic model of a 2D overhead crane is described in Section 2 while the conventional HSMC scheme is introduce in Section 3. Section 4 discusses the proposed AFHSMC law, where stability of the system is theoretically analyzed. Validations of the proposed method in both the simulations and experiments are delineated in Section 5 before conclusions are drawn in

Section 6.

2. Dynamic Model of 2D Overhead Crane

Let's consider an overhead crane on a 2D xy plane as demonstrated in Fig. 1. The system comprises a trolley moving on a rail, a payload that is considered as a point mass and a hoisting cable. It is assumed that the static friction and elasticity are trivial while stiffness and mass of the hoisting cable are neglected. Moreover, both the trolley and payload are considered as material particles.

We denote x, θ and F are variables presenting for the trolley position, the swing angle of the payload with respect to the vertical line and the trolley driving force, respectively. On the other hands, M, m and l are also defined as constant parameters delineating the total mass of the trolley, the mass of the payload and the hoisting cable length. Furthermore, in this work, it is assume that the overhead crane operates in indoor environments, hence the effects of disturbances caused by wind are not examined. Then, the mathematical dynamic model of the 2D overhead crane [5] can be specified by

$$\begin{gathered}(M+m)\ddot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=F,\\ l\ddot{\theta}+g\sin\theta+\ddot{x}\cos\theta=0,\end{gathered} \tag{1}$$

where g is the gravitational acceleration. It is noted that in the state space, if we let $u=F$ and $X^T=[x_1 \quad x_2 \quad x_3 \quad x_4]=[x \quad \dot{x} \quad \theta \quad \dot{\theta}]$, then the dynamic model can be represented as follows,

$$\begin{gathered}\dot{x}_1=x_2,\\ \dot{x}_2=f_1(X)+g_1(X)u,\\ \dot{x}_3=x_4,\\ \dot{x}_4=f_2(X)+g_2(X)u,\end{gathered} \tag{2}$$

where

$$f_1(X)=\frac{ml\dot{\theta}^2\sin\theta+mg\sin\theta\cos\theta}{M+m\sin^2\theta}, \tag{3}$$

$$f_2(X) = -\frac{(M+m)g\sin\theta + ml\dot{\theta}^2\sin\theta\cos\theta}{(M+m\sin^2\theta)l},$$

$$g_1(X) = \frac{1}{M+m\sin^2\theta},$$

$$g_2(X) = -\frac{\cos\theta}{(M+m\sin^2\theta)l}.$$

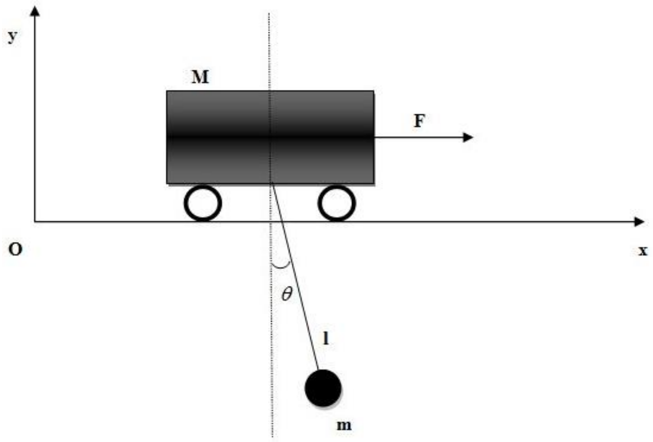

Figure 1. Dynamic model of a 2D overhead crane

It can be clearly seen that there is one input as the driving force while there are two outputs as the motion of the trolley and the swing angle of the payload. Therefore, the primary difficulty in controlling an under-actuated overhead crane is how to manage the coupled nature of the outputs from a single input.

3. Hierarchical Sliding Mode Control

In order to overcome issues in controlling an overhead crane system, many control strategies have been proposed, including feedback linearization [6,8], fuzzy control [10,11], and adaptive control [18,21,22]. Nonetheless, the control designers have remarkably paid more attention on the SMC approach due to its well-known robustness under uncertain conditions. Among the SMC schemes, a hierarchical SMC (HSMC) has been considered as an appropriate approach for controlling a class of under-actuated systems [17]. In this sections, we discuss the HSMC strategy for a 2D overhead crane, which will be then utilized to adapt to its adaptive paradigm in the next section.

To ascertain the stability of the sliding surfaces, based on the Lyapunov theory, let's define the vectors of the errors as follows,

$$e(t)=\begin{bmatrix} x_1-x_d \\ x_3-\theta_d \end{bmatrix}=\begin{bmatrix} x-x_d \\ \theta-\theta_d \end{bmatrix}=\begin{bmatrix} e_1 \\ e_3 \end{bmatrix}$$

where x_d and θ_d are the desired trolley position and swing angle of the payload. It is assumed that $\dot{x}_d$ and $\ddot{x}_d$ exist and are bounded, then derivatives of the errors derived from (2) can be specified by,

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= f_1(X)+g_1(X)u-\ddot{x}_d, \\ \dot{e}_3 &= e_4, \\ \dot{e}_4 &= f_2(X)+g_2(X)u, \end{aligned} \tag{4}$$

Now, we define two first-level sliding surfaces for the corresponding actuated output x and under-actuated output θ as follows,

$$\begin{aligned} s_1 &= c_1 e_1 + e_2, \\ s_2 &= c_2 e_3 + e_4, \end{aligned} \tag{5}$$

where c_1 and c_2 are positive constants. The second-level sliding surface for the whole system is then aggregated by

$$s=\alpha s_1+\beta s_2, \tag{6}$$

where α and β are positive parameters. Note that if an appropriate HSMC scheme is given, the second-level sliding surface can ultimately converge to zero. As pointed out by Qian *et al.* [13], such SMC scheme should comprise two laws. The first is the switch control law that is utilized to drive the system states towards a particular sliding surface, while the second is the equivalent control law that is employed to keep those states on the sliding surface. In other words, the HSMC scheme can be presented by

$$u=u_{eq}+u_{sw}.$$

To design that SMC law, let's compute the derivative of the second-level sliding surface from (5) and (6) as follows,

$$\begin{aligned}\dot{s} &= \alpha\dot{s}_1 + \beta\dot{s}_2 \\ &= \alpha(c_1e_2 + f_1 + g_1u - \ddot{x}_d) + \beta(c_2e_4 + f_2 + g_2u).\end{aligned} \tag{7}$$

If we consider the Lyapunov function candidate as

$$V = \frac{1}{2}s^2$$

and differentiate it with respect to time, we obtain

$$\begin{aligned}\dot{V} &= s\dot{s} \\ &= s\left(\alpha(c_1e_2 + f_1 + g_1u - \ddot{x}_d) + \beta(c_2e_4 + f_2 + g_2u)\right) \\ &= s\left(\alpha c_1e_2 + \beta c_2e_4 + \alpha f_1 + \beta f_2 - \alpha\ddot{x}_d + (\alpha g_1 + \beta g_2)u\right).\end{aligned} \tag{8}$$

In order to guarantee the stability of the second-level sliding surface, we define

$$\begin{cases}(\alpha g_1 + \beta g_2)u_{eq} + \alpha c_1e_2 + \beta c_2e_4 + \alpha f_1 + \beta f_2 - \alpha\ddot{x}_d = 0, \\ (\alpha g_1 + \beta g_2)u_{sw} + k_1s + k_2\,\mathrm{sgn}(s) = 0,\end{cases} \tag{9}$$

where k_1 and k_2 are positive. Therefore, the HSMC scheme is given by

$$u = -\frac{\alpha c_1e_2 + \alpha f_1 + \beta c_2e_4 + \beta f_2 - \alpha\ddot{x}_{1d} + k_1s + k_2\,\mathrm{sgn}(s)}{\alpha\hat{g}_1 + \beta\hat{g}_2}, \tag{10}$$

where f_1, g_1, f_2 and g_2 rely on the system parameters, comprising M, m, l, θ, as presented in (3). If substituting (10) into (9), we obtain

$$\dot{V} = s\dot{s} = s\left(-k_1s - k_2\,\mathrm{sgn(s)}\right) = -k_1s^2 - k_2\left|s\right| \le 0,$$

which guarantees that the sliding mode is reachable in finite time.

4. Fuzzy observer based adaptive hierarchical sliding mode control

It can be clearly seen that the deterministic HSMC scheme introduced in Section 3 can be effectively employed provided that the system parameters are certain. Nevertheless, in practice, an overhead crane operates under system uncertainties such as parameter variations and nonlinearities or external disturbances; that is, the dynamic models presented in the previous sections do not fully delineate the characteristics of the crane system. In other words, it is impractical to accurately determine the system parameters. To deal with these challenges, it is proposed to utilize the fuzzy observer to approximately estimate those dynamic models. In this section, we first introduce the fuzzy observer, which is then incorporated into the HSMC law to design an adaptive controller for a 2D overhead crane.

4.1. Fuzzy observer

Let's consider approximation of a fuzzy system as introduced by Park *et al.* [22]. A fuzzy system comprises three fundamental elements including fuzzifier, fuzzy inference engine and defuzzifier. The fuzzy inference engine employs the fuzzy IF-THEN rules to compute fuzzy sets in the input space to fuzzy sets in the output space. Moreover, while the fuzzifier is considered to map a real-valued point to fuzzy set, the defuzzifier maps a fuzzy set as an output of the fuzzy inference engine to a crisp point. For instance, in this work, we propose to employ the fuzzy IF-THEN rules, whose i^{th} law is specified by

$$\text{If } x_1 \text{ is } A_1^i,\ x_2 \text{ is } A_2^i,\ \dots,\ x_n \text{ is } A_n^i \text{, then } y \text{ is } y^i, \tag{11}$$

where $x=\left(x_1,x_2,\dots,x_n\right)\in R^n$ is the input of the fuzzy system. A_j^i the i^{th} fuzzy set of the input variable x_j. $y\in R$ is the output of the fuzzy system, where its element y^i is a singleton number, $i=1,2,\dots,r$ and $j=1,2,\dots,n$.

If the fuzzy system utilizes product inference engine, singleton fuzzifier and center average defuzzifier, its output can be presented in the following form.

$$y(x)=\frac{\sum_{i=1}^{r} y^i\left(\prod_{j=1}^{n}\mu_{A_j^i}(x_j)\right)}{\sum_{i=1}^{r}\left(\prod_{j=1}^{n}\mu_{A_j^i}(x_j)\right)}=\hat{\phi}^T\xi(x), \tag{12}$$

where, $\mu_{A_j^i}(x_j)$ is the membership function of the fuzzy set A_j^i. $\xi^T=\left(\xi^1,\xi^2,\dots,\xi^r\right)$, whose component is computed by

$$\xi^i=\frac{\prod_{j=1}^{n}\mu_{A_j^i}(x_j)}{\sum_{i=1}^{r}\left(\prod_{j=1}^{n}\mu_{A_j^i}(x_j)\right)},$$

which is the fuzzy basic function. And $\hat{\phi}^T=(y^1,y^2,\dots,y^r)$ is a vector of the adjustable parameters.

In a nonlinear system, if there exist an adjusted $\hat{\phi}^T$ so that a nonlinearity $f(x)$ approximates to $y(x)$, then $|f - y|$ is minimal. In equivalent words, in that case, the nonlinearity $f(x)$ can be approximately estimated by an output of a fuzzy system.

In the overhead system, the four dynamic models (3) as presented in the previous section are nonlinear, unknown and uncertain, which is not trivial to practically determine. Thus, we propose to estimate them by the use of the fuzzy systems. In other words, a fuzzy observer (FO) to approximate $f_1(X)$, $g_1(X)$, $f_2(X)$ and $g_2(X)$ is defined as follows,

$$\begin{cases} f_1(X) = \phi_1^T \xi(\mathrm{X}) + \varepsilon_1(X) \\ g_1(X) = \phi_2^T \xi(\mathrm{X}) + \varepsilon_2(X) \\ f_2(X) = \phi_3^T \xi(\mathrm{X}) + \varepsilon_3(X) \\ g_2(X) = \phi_4^T \xi(\mathrm{X}) + \varepsilon_4(X) \end{cases} \tag{13}$$

where ϕ_i is the ideal adjustable parameter, and $|\varepsilon_i(\mathrm{X})| \le \varepsilon_i$ is the approximation error with constant $\varepsilon_i > 0$, $i = 1,2,3,4$.

4.2. FO based adaptive HSMC scheme

Let $\hat{\phi}_1$, $\hat{\phi}_2$, $\hat{\phi}_3$ and $\hat{\phi}_4$ denote estimations of ϕ_1, ϕ_2, ϕ_3 and ϕ_4, respectively. Then the outputs of the fuzzy observer are approximations of the dynamics f_1, g_1, f_2 and g_2 and computed as follows,

$$\begin{cases} \hat{f}_1(X) = \hat{\phi}_1^T \xi(\mathrm{X}) \\ \hat{g}_1(X) = \hat{\phi}_2^T \xi(\mathrm{X}) \\ \hat{f}_2(X) = \hat{\phi}_3^T \xi(\mathrm{X}) \\ \hat{g}_2(X) = \hat{\phi}_4^T \xi(\mathrm{X}). \end{cases} \tag{14}$$

We now introduce how to employ the fuzzy system approximations $\hat{f}_1$, $\hat{g}_1$, $\hat{f}_2$ and $\hat{g}_2$ in designing the adaptive HSMC law.

Let's consider the Lyapunov function candidate

$$V = \frac{1}{2}\left(s^2 + \sum_{i=1}^{4} \tilde{\phi}_i^T \Gamma_i^{-1} \tilde{\phi}_i \right), \tag{15}$$

where $\tilde{\phi}_i = \phi_i - \hat{\phi}_i$ is the error between ideal and estimated adjustable parameters. Γ_i is positive constant.

If we differentiate the Lyapunov function candidate in (15), it yields,

$$\dot{V} = s\dot{s} + \sum_{i}^{4} \tilde{\phi}_i^T \Gamma_i^{-1} \dot{\tilde{\phi}}_i, \tag{16}$$

In a more specific form, (16) can be presented as follows,

$$\begin{aligned}
\dot{V} &= s\left(\alpha\left(c_1 e_2 + f_1 + g_1 u - \ddot{x}_{1d} \right) + \beta\left(c_2 e_4 + f_2 + g_2 u \right) \right) \\
&\quad + \sum_{i}^{4} \tilde{\phi}_i^T \Gamma_i^{-1} \dot{\tilde{\phi}}_i \\
&= s\left(\alpha c_1 e_2 + \alpha \hat{f}_1 + \beta c_2 e_4 + \beta \hat{f}_2 - \ddot{x}_{1d} + (\alpha \hat{g}_1 + \beta \hat{g}_2) u \right) \\
&\quad + s\left(\alpha\left(\tilde{\phi}_1^T \xi + \varepsilon_1 \right) + \beta\left(\tilde{\phi}_3^T \xi + \varepsilon_3 \right) + \alpha u \left(\tilde{\phi}_2^T \xi + \varepsilon_2 \right) \right) \\
&\quad + s\left(\beta u \left(\tilde{\phi}_4^T \xi + \varepsilon_4 \right) \right) + \sum_{i}^{4} \tilde{\phi}_i^T \Gamma_i^{-1} \dot{\tilde{\phi}}_i
\end{aligned} \tag{17}$$

Moreover, given the estimations $\hat{f}_1$, $\hat{g}_1$, $\hat{f}_2$ and $\hat{g}_2$ obtained by the FO, the control signal of the HSMC scheme in (10) can be approximately computed by

$$u = -\frac{\alpha c_1 e_2 + \alpha \hat{f}_1 + \beta c_2 e_4 + \beta \hat{f}_2 - \alpha \ddot{x}_{1d} + k_1 s + k_2 \,\mathrm{sgn}(s)}{\alpha \hat{g}_1 + \beta \hat{g}_2} \tag{18}$$

From (16) to (18), the derivative of the Lyapunov function candidate can be rewritten as follows,

$$\begin{aligned}
\dot{V} &= -k_1 s^2 - k_2 \,\mathrm{sgn}(s).\mathrm{s} + \sum_{i}^{4} \tilde{\phi}_i^T \Gamma_i^{-1} \dot{\tilde{\phi}}_i s \\
&\quad + \alpha\left(\tilde{\phi}_1^T \xi + \varepsilon_1 \right) + \beta\left(\tilde{\phi}_3^T \xi + \varepsilon_3 \right) \\
&\quad + \alpha u \left(\tilde{\phi}_2^T \xi + \varepsilon_2 \right) + \beta u \left(\tilde{\phi}_4^T \xi + \varepsilon_4 \right) \\
&= -k_1 s^2 - k_2 |s| + \tilde{\phi}_1^T \left(\alpha s \xi + \Gamma_1^{-1} \dot{\tilde{\phi}}_1 \right) + \tilde{\phi}_2^T \left(\alpha u s \xi + \Gamma_2^{-1} \dot{\tilde{\phi}}_2 \right) \\
&\quad + \tilde{\phi}_3^T \left(\beta s \xi + \Gamma_3^{-1} \dot{\tilde{\phi}}_3 \right) + \tilde{\phi}_4^T \left(\beta u s \xi + \Gamma_4^{-1} \dot{\tilde{\phi}}_4 \right) \\
&\quad + s\left(\alpha \varepsilon_1 + \beta \varepsilon_3 + \alpha u \varepsilon_2 + \beta u \varepsilon_4 \right)
\end{aligned} \tag{19}$$

If the approximations of the adjustable parameters, including $\hat{\phi}_1$, $\hat{\phi}_2$, $\hat{\phi}_3$ and $\hat{\phi}_4$, are adaptively selected, where their derivatives are formulated by

$$\begin{aligned}
\dot{\hat{\phi}}_1 &= \Gamma_1 \alpha s \xi, \\
\dot{\hat{\phi}}_2 &= \Gamma_2 \alpha u s \xi, \\
\dot{\hat{\phi}}_3 &= \Gamma_3 \beta s \xi, \\
\dot{\hat{\phi}}_4 &= \Gamma_4 \alpha u s \xi,
\end{aligned} \tag{20}$$

then (19) can be simplified as follows,

$$\dot{V} = -k_1 s^2 - k_2 |s| + s\left(\alpha\varepsilon_1 + \beta\varepsilon_3 + \alpha u\varepsilon_2 + \beta u\varepsilon_4\right) \tag{21}$$

We assume that $|\alpha\varepsilon_1 + \beta\varepsilon_3 + \alpha u\varepsilon_2 + \beta u\varepsilon_4| \le \varepsilon_N$, where ε_N is a small positive number, if $k_2 - \varepsilon_N > 0$ then $\dot{V} \le 0$.

It can be clearly seen that given the stability conditions based upon the Lyapunov function, the adaptation mechanism in (20) allows the fuzzy system to adaptively estimate dynamics of the overhead crane as presented in (14), which ultimately leads to the adaptive control law in (18).

4.3. Stability analysis

The effectiveness of the proposed FO based adaptive HSMC (AFHSMC) scheme for a 2D overhead crane can be mathematically demonstrated by stability of the closed loop control system in the following theorem.

Theorem 1: Given the adaptation mechanism (20), the FO-AHSMC law (18) can asymptotically stabilize the second-level sliding surface s defined in (5) if $k_2 - \varepsilon_N > 0$.

Proof: Let's take integrals of both sides of (21), it yields,

$$\begin{aligned}
\int_0^t dV &= \int_0^t \left(-k_1 s^2 - k_2 |s| + s\left(\alpha\varepsilon_1 + \beta\varepsilon_3 + \alpha u\varepsilon_2 + \beta u\varepsilon_4\right)\right) dt, \\
V(\mathrm{t}) - \mathrm{V}(0) &= \int_0^t \left(-k_1 s^2 - k_2 |s| + s\left(\alpha\varepsilon_1 + \beta\varepsilon_3 + \alpha u\varepsilon_2 + \beta u\varepsilon_4\right)\right) dt,
\end{aligned} \tag{22}$$

or

$$\begin{aligned}
V(0) &= V(t) + \int_0^t \left(k_1 s^2 + k_2 |s| - s\left(\alpha\varepsilon_1 + \beta\varepsilon_3 + \alpha u\varepsilon_2 + \beta u\varepsilon_4\right)\right) dt \\
&\ge \int_0^t \left(k_1 s^2 + \left(k_2 - \varepsilon_N\right)|s|\right) dt
\end{aligned} \tag{23}$$

In other words,

$$V(t) = \frac{1}{2}s^2 \\ \le V(0) - \int_0^t \left(k_1 s^2 + (k_2 - \varepsilon_N)|s|\right)dt \le V(0) < \infty \tag{24}$$

Therefore, we have

$$s \in L_\infty, \text{ i.e. } \sup_{t\ge 0}|s| = \|s\|_\infty < \infty. \tag{25}$$

Moreover, from (21) it shows that

$$|\dot{V}| = |\dot{s}||s| \\ \le k_1 s^2 + k_2|s| + s(\alpha\varepsilon_1 + \beta\varepsilon_3 + \alpha u\varepsilon_2 + \beta u\varepsilon_4) \\ \le k_1 s^2 + k_2|s| + |s|\varepsilon_N, \tag{26}$$

which results in

$$|\dot{s}| \le k_1|s| + k_2 + \varepsilon_N < \infty.$$

Hence,

$$\dot{s} \in L_\infty, \text{ i.e. } \sup_{t\ge 0}|\dot{s}| = \|\dot{s}\|_\infty < \infty. \tag{27}$$

(23) can now be rewritten by

$$V(0) \ge \int_0^\infty \left(k_1 s^2 + (k_2 - \varepsilon_N)|s|\right)dt \\ = \int_0^\infty k_1 s^2 dt + \int_0^\infty (k_2 - \varepsilon_N)|s|dt. \tag{28}$$

It is apparent that $\int_0^\infty k_1 s^2 dt \ge 0$ and $\int_0^\infty (k_2 - \varepsilon_N)|s|\,dt \ge 0$. From (28) it shows $\int_0^\infty |s|dt < \infty$, i.e. $\|s\|_1 < \infty$, and $\int_0^\infty s^2 dt < \infty$, i.e. $s \in L_2$.

Since $s \in L_\infty$, $\dot{s} \in L_\infty$, and $s \in L_2$, according to Barbalat's lemma [31] we have $\lim_{t\to\infty} s = 0$. Therefore the second-level sliding surface s is asymptotically stable. □

5. Simulation and experimental results

In order to demonstrate efficiency of the proposed AFHSMC approach as compared with that of the conventional HSMC, we conducted the experiments both on a synthetic simulation environment and in a laboratory. It is noted that due to impracticality of determining the uncertain parameters of the dynamics of the overhead crane, we did not implement the HSMC law in the realistic system. Nevertheless, in the synthetic simulation, the deterministic HSMC scheme was expected to best perform since the system parameters were accurately obtained.

Figure 2. A real-time overhead crane system

In the first experiment, we conducted a simulation of a realistic overhead crane system as pictorially demonstrated in Fig. 2. To this end, the physical parameters of the laboratory overhead crane are summarized as follows,

$$M = 25\,kg,\ m = 8kg,\ l = 1.2m,\ g = 9.81m/s^2$$

Furthermore, in the experiments, some positive constant parameters of the control laws were given by

$$c_1 = 3,\ c_1 = 0.01,\ \alpha = 2,\ \beta = 1.4,\ \mathrm{k}_1 = 0.1,\ k_2 = 2.$$

Table 1. Parameters of the membership functions

i	$\mu_{A_1^i}(\theta)$		$\mu_{A_2^i}(\dot{\theta})$	
	σ	υ[rad]	σ	υ[rad/s]
1	-5	$-\pi/4$	-5	$-\pi/4$
2	0.3	$-\pi/6$	0.3	$-\pi/6$
3	0.3	$-\pi/12$	0.3	$-\pi/12$
4	0.3	0	0.3	0
5	0.3	$\pi/12$	0.3	$\pi/12$
6	0.3	$\pi/6$	0.3	$\pi/6$
7	5	$\pi/4$	5	$\pi/4$

Regarding the fuzzy observer, in this work, we consider only two input variables including the swing angle of the payload θ and its velocity $\dot{\theta}$. Moreover, in the construction of the fuzzy system, we propose to utilize two membership functions, comprising Gaussian function

$$\mu_{A_j^i}(x_j) = \frac{1}{1+\exp(-\sigma(x_j - \upsilon))},$$

for $i = \{1, 7\}$ and sigmoid function

$$\mu_{A_j^i}(x_j) = \exp(-0.5(x_j - \upsilon)^2 / \sigma^2)$$

for $i = \{2, ..., 6\}$. And the parameters (υ, σ) of the membership functions are specified in Table 1. It is noticed that we have only two input variables $x_j = (\theta, \dot{\theta})$, $j = \{1, 2\}$.

For the adaptation mechanism, Γ_i ($i = 1, ..., 4$) was set to 0.01, and the initial values of the estimated adjustable parameters of the fuzzy observer were all set to 0.1, $\hat{\phi}_i(0) = 0.1$.

5.1. Simulations

We now investigate the simulation results including the position of the trolley, the swing angle of the payload and the control force as pictorially illustrated in Figures 3 – 5. It was expected that the trolley horizontally travelled 0.6 m from the initial location. It can be clearly seen in Fig. 3 that given the known and certain system parameters, the HSMC law drove the trolley to reach the destination after about 2.5 s without overshoot. Comparatively, the proposed AFHSMC scheme, which, though, had to adaptively estimate the system parameters by the use of the fuzzy observer, drove the trolley to reach the destination after around 3 s. Although the proposed approach caused the overshoot in the output, it is apparently trivial. The results shown in Fig. 3 illustrate that both the deterministic HSMC and proposed AFHSMC methods well tracked the desired position of the trolley.

In terms of the swing angle of the payload as demonstrated in Fig. 4, both the HSMC and AFHSMC algorithms, at first 4 s, swung the object, though the maximum swing angle is about 0.08 rad. More importantly, as expected, due to requirement of time consumed in estimating the system parameters, the swing angle output as a result of the AFHSMC technique is a little bit lag as compared with that of the HSMC method. Nevertheless, ultimately, both approaches eliminated the swing angle of the payload after approximate 4 s, when the trolley reached the destination.

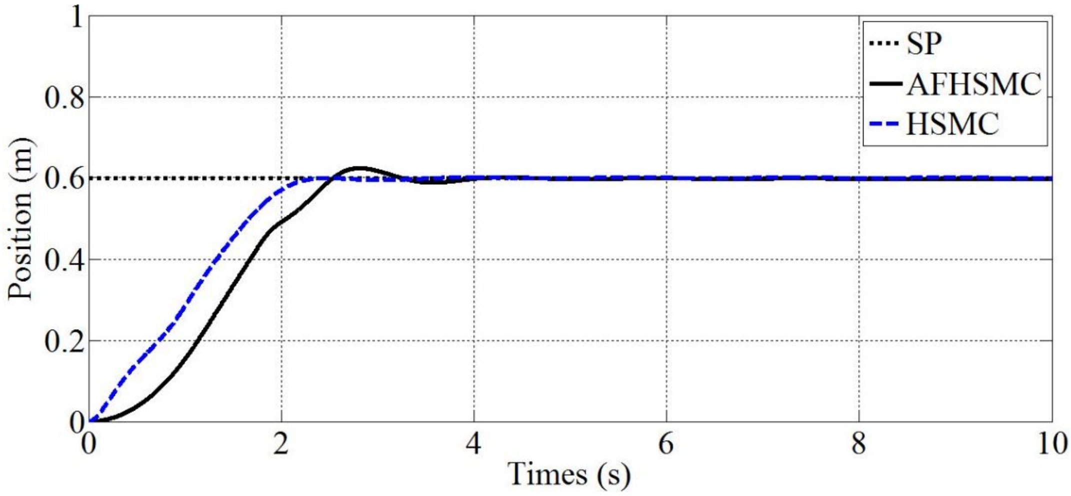

Figure 3. Position of the trolley in the simulation

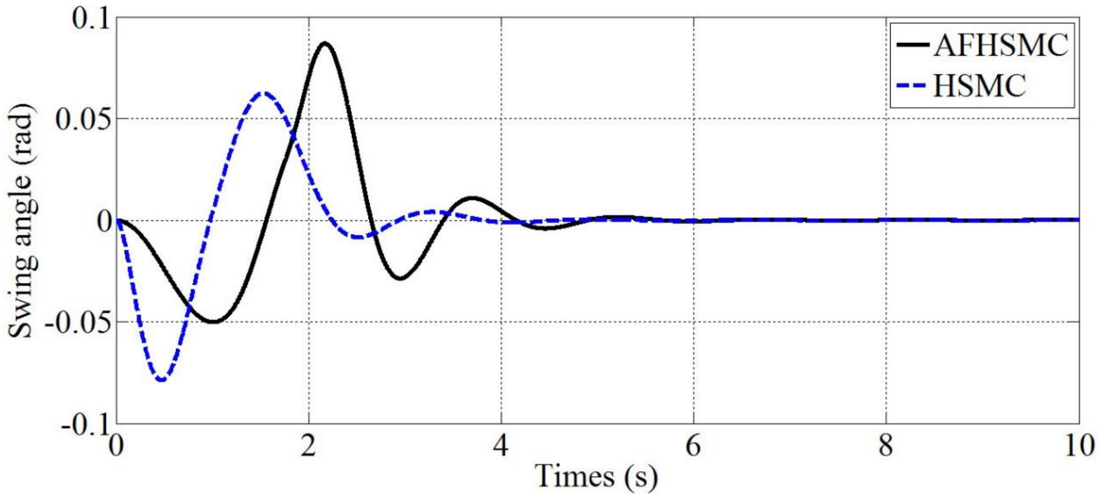

Figure 4. Swing angle of the payload in the simulation

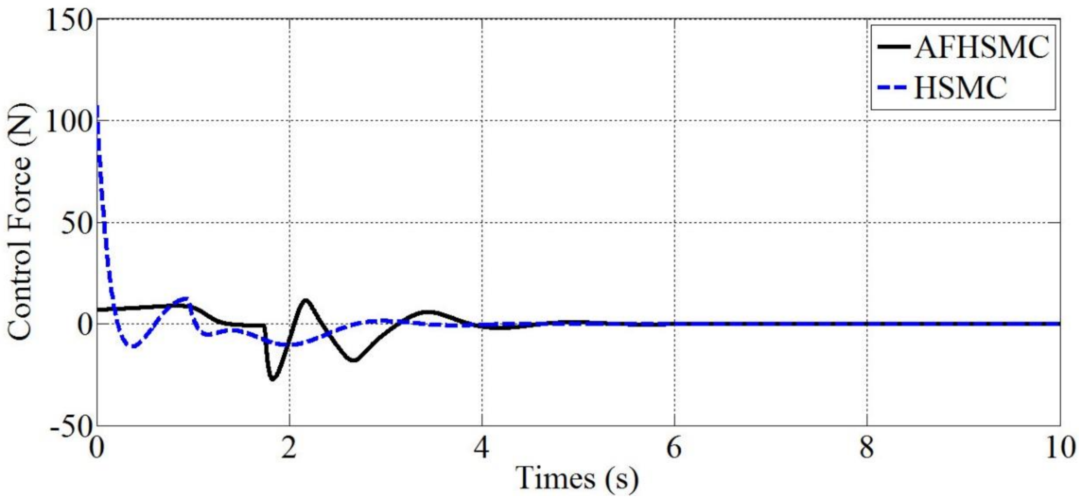

Figure 5. Control force in the simulation

In corresponding to the position of the trolley and the swing angle of the object, the control forces required in both the control laws are highly comparable as can be seen in Fig. 5. It is emphasized that given no prior information of the overhead crane system parameters, the proposed AFHSMC approach is capable of adaptively estimating those parameters, which eventually leads to the actuated and unactuated outputs highly compared with those obtained by the HSMC scheme in ideal scenarios. That is, the AFHSMC law is highly practical as demonstrated in the following section by the laboratory experiments.

5.2. Real experiments

To experimentally evaluate effectiveness of the proposed method, we implemented the algorithm in a real-time overhead crane system in a laboratory as illustrated in Fig. 2. In this example system, the trolley was driven by a three-phase asynchronous motor, where the motor was powered by the OMRON inverter 3G3JX. The position of the trolley was measured by the encoder E40S6-1024-3-T-24. Furthermore, the swing angle of the payload and its angular velocity were gathered by the sensor MPU6050. Regarding the central control unit, we implemented the proposed approach on the STM32F4 microcontroller.

Under the control of the AFHSMC law, the position of the trolley, the swing angle of the payload and the control force were recorded over time and are now plotted in Figures 6, 7 and 8, respectively. It is noted that the real-life overhead crane system operated under realistic nonlinearity of the actuators, external disturbances and system parameter uncertainty. Nonetheless, the results obtained by the proposed algorithm are appealing. It can be clearly seen that the trolley gradually reached the destination after about 5 s without overshoot as demonstrated in Fig. 6. In the first second, the controller made the object to swing up to 0.15 rad. However, the swing angle of the payload quickly declined to 0.05 rad and almost disappeared after about 5 s as can be seen in Fig. 7. In contrast to the simulation, the control force in Fig. 8 gradually reduced from beginning and reached to zero at about 5 s when the trolley stopped.

Though we set the system parameters in the simulation similarly to those in the real-time system, the realistic trolley needed about 5 s to reach the destination while the synthetic one needed about 3 s. That can be understood by the ability of the microcontroller in the laboratory experiments as compared with that to the desktop.

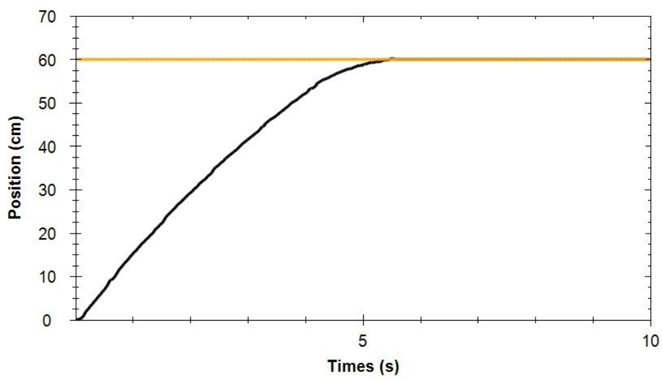

Figure 6. Position of the trolley in the realistic experiment

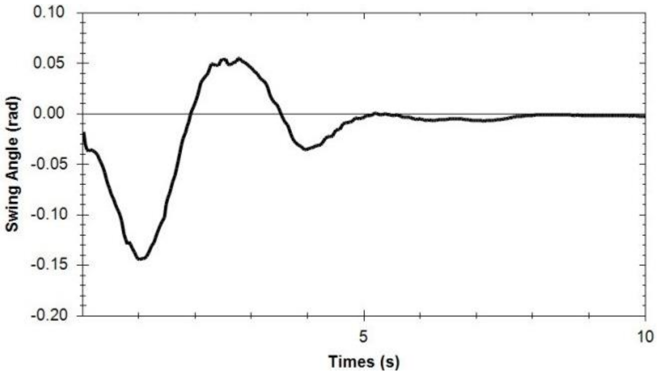

Figure 7. Swing angle of the payload in the realistic experiment

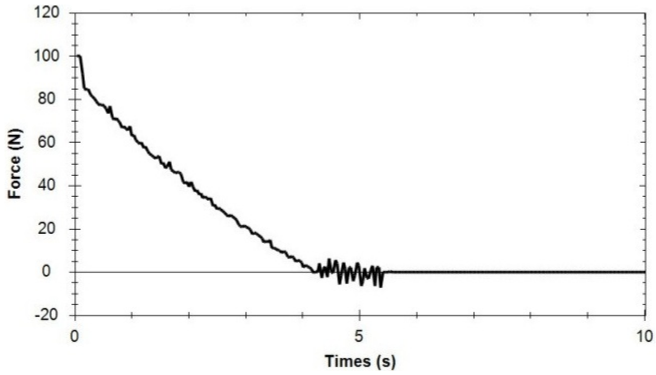

Figure 8. Control force in the realistic experiment

6. Conclusions

The paper has proposed to employ the HSMC and fuzzy observer to be deployed in a new but efficient technique to adaptively control an under-actuated overhead crane. The HSMC law enables the system to robustly transport a payload to a destination despite its unknown external disturbances and nonlinearities. More importantly, under influence of the system uncertainties, the parameters of the control law are adaptively estimated, which is derived from the Lyapunov theory. The adaptive mechanism, which theoretically proves the stability of the proposed control scheme, guarantees the crane to be able to effectively work under uncertain conditions. Implementations of the proposed algorithm in both the synthetic simulations and real-life experiments has verified effectiveness of the AFHSMC approach.

Disclosure statement

No potential conflict of interest was reported by the authors.

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