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# Adaptive neural network based backstepping sliding mode control approach for dual arm robots

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**Abstract** The paper introduces an adaptive strategy to effectively control a nonlinear dual arm robot under external disturbances and uncertainties. By the use of the backstepping sliding mode control (BSSMC) method, the proposed algorithm first allows the manipulators to be able to robustly track the desired trajectories. Furthermore, due to the nonlinear, uncertain and unmodelled dynamics of the dual arm robot, it is proposed to employ the radial basis function network (RBFN) to adaptively estimate the robot's dynamic model. Though the estimation of the dynamics is approximate, the adaptation law is derived from the Lyapunov theory, which provides the controller with ability to guarantee stability of the whole system in spite of its nonlinearities, parameter uncertainties and external load variations. The effectiveness of the proposed RBFN-BSSMC approach is demonstrated by implementation in a simulation environment with realistic parameters, where the obtained results are highly promising.

**Keywords** Backstepping sliding mode control · Adaptive neural network · Dual arm robots · Lyapunov method

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## 1 Introduction

Robots have been increasingly moving into humans based environments to replace or assist human workers. More specifically, anthropomorphic or dual arm robots (DAR) have more and more played a vital role in many industrial, health care or household environments (Do et al. (2012); Zheng et al. (1989); Smith et al. (2012); Dauchez et al. (1991); Tanie (2003)). For instance, dual arm manipulators (DAM) have been effectively employed in a diversity of tasks including assembling a car, grasping and transporting an object or nursing the elderly (Liu et al. (2015)). In those scenarios, the DAR have been expected to behave like a human, which is they should be able to manipulate an object similarly to what a person does (Smith et al. (2012)). As compared to a single arm robot, some works (Lee (1989); Meier and Graf (1991)) have shown that the DAR have significant advantages such as more flexible movements, higher precision and greater competence for handling large objects. Nevertheless, since the kinematic and dynamic models of the DAM are much more complicated than those of a single arm robot, it has more challenges to effectively and efficiently control the DAR.

In order to track the robot manipulators along desired trajectories, a robust controller is highly expected to synchronously coordinate the robot arms. A number of the control strategies have been proposed to guarantee the accuracy and stability of the manipulator operations. For instance, the traditional methods such as nonlinear feedback control (Yun and Kumar (1991)) or hybrid force/position control relied on the kinematics and statics (Yamano et al. (2004); Hayati (1986)) have been proposed to simultaneously control both the arms. In the works (Schneider and Cannon (1992); Caccavale et al. (2008); Lee et al. (2014)), the authors have proposed to utilize the impedance control by considering the dynamic interaction between the robot and its surrounding environment while guaranteeing the desired movements. Nonetheless, these traditional control techniques are not really practical when they require to accurately model all the nonlinear dynamics of the DAR system, where its unknown parameters are highly uncertain and not easily estimated. It is noted that uncertainties of the DAM system can practically lead to degradation of its control performance. Furthermore, a number of unexpected disturbances and obstacles in the working environments can cause the DAR system to be unstable.

To address the aforesaid issues of accurately modelling all the nonlinear dynamics and estimating the unknown and uncertain parameters, some modern control approaches based on fuzzy logic or artificial neural network have been proposed in the past decades. For instance, by the use of the adaptive learning and function approximations, Lee and Choi (Lee and Choi (2004)) introduced a radial basis function network (RBFN) for approximating the nonlinear dynamics of a SCARA-type robot manipulator. Similarly, Wang et al. (Wang et al. (2009)) has employed the approximation of a neural network to deal with the nonlinearities and uncertainties of a single robot manipulator, where errors caused by the neural network approximation can be estimated by a proposed control robust term. In addition, the authors in (Liu et al. (2015)) have designed an adaptive control system for a humanoid robot by using the RBFN to develop a scheme to adaptively estimate unknown and uncertain dynamics of the robot. Based on a multi-input multi-output fuzzy logic unit, Jiang et al. (Jiang et al. (2015)) have proposed an algorithm to adaptively estimate the dynamics of the DAM, given its nonsysmmetric deadzone nonlinearity.

More importantly, robustness of the control performance is also highly prioritized in consideration of designing a controller for a highly uncertain and nonlinear DAR system. In literature of the modern control theory, sliding mode control (SMC) demonstrates a diverse ability to robustly control any system. Since the pioneer paper (Utkin (1977)), the variable structure SMC has enjoyed widespread use and attention in many applications (Hashimoto et al. (1987); Herman (2005); Yannier et al. (2005); Yagiz et al. (2010)). For the DAR system, Yagiz et al. (Yagiz et al. (2010)) has developed a non-chatting sliding mode controller for handling an object, which has been proved to be more efficient than PID controller. Moreover, the authors in (Tang et al. (2006); Wang et al. (2009)) have proposed a terminal SMC approach for a single arm robot, which enhances the contradiction between control efforts in the transient state and tracking errors in the steady state. More importantly, by the use recursive feature of the standard backstepping method, Zhou et al. (Zhou et al. (2007)) have proposed a control scheme for robustly tracking outputs of an uncertain MIMO nonlinear system, where its tracking errors are proved to be bounded. Similarly, Chen et al. (Chen et al. (2013)) have proposed a backstepping sliding mode controller to enhance the global ultimate asymptotic stability and invariability to uncertainties in a nonholonomic wheeled mobile manipulator.

In this paper, we propose an adaptive control strategy based on the backstepping sliding mode control (BSSMC) method and the RBFN to effectively and efficiently control the DAR. The proposed approach provides the DAM system not only adaptive estimation of its nonlinear dynamics but also robustness to its uncertainties. In other words, the BSSMC enables the manipulators to be capable of efficiently tracking the desired trajectories given large variation of the system information such as the undetermined volume and mass of the payload and significantly reducing chattering influences. The RBFN allows the proposed controller to be able to adaptively learn the nonlinear and uncertain dynamics of the DAR system. More importantly, the adaptation mechanism is designed based on Lyapunov method, which mathematically guarantees the stability of the control system.

The rest of the paper is arranged as follows. We first introduce a model of the DAR system in Section 2. We then present how to construct a RBFN-BSSMC controller for the DAM based on the BSSMC and RBFN in Section 3. Section 4 discusses validation of the proposed approach in a simulation environment before conclusions are drawn in Section 5.

## 2 Dual arm robot model

Lets consider a dual two degree of free (DoF) arm robot that cooperatively manipulates an object with mass of $m$ as pictorically shown in Fig. 1. It is assumed that both the manipulators rigidly attach to the load so that there is no slip between the grasping points and the grasped load. Let $m_i, I_i, l_i$ denote the mass, mass moment of inertia and length of the corresponding link in the model, respectively. We also define $d_1$ and $d_2$ as the length of the object and distance between the two arms at the robot's base. The distance from the mass centre of a link to a joint is denoted as $k_i$ while the joint angle between a link and the base or its preceding link is denoted as $q_i$.

Operationally, in this work we consider that the robot manipulators make motions on the horizontal $xy$

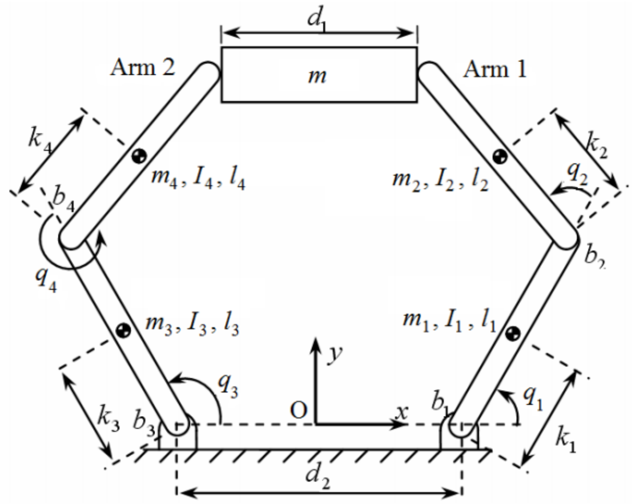


Fig. 1: Dual arm robot modelling

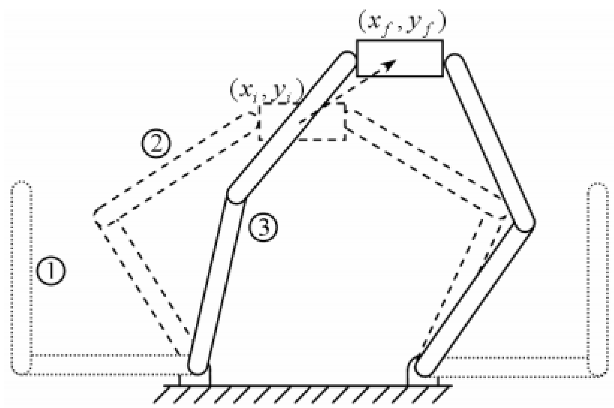


Fig. 2: Operational motions of dual arm robot

plane. In other words, the robot arms first move towards the object. After the manipulators are firmly attached to the load, the robot then picks the object up and transports it to a new position by adjusting the motions to robustly follow the given trajectory, demonstrated in Fig. 2. We let $x_m$ and $y_m$ denote the mass center of the payload on the $xy$ plane, the trajectory of the object can be specified by

$$x_m = \frac{d_2}{2} + l_1 \cos q_1 + l_2 \cos(q_1 + q_2) - \frac{d_1}{2} \quad (1)$$
$$= -\frac{d_2}{2} + l_3 \cos q_3 + l_4 \cos(q_3 + q_4) + \frac{d_1}{2},$$
$$y_m = l_1 \sin q_1 + l_2 \sin(q_1 + q_2) \quad (2)$$
$$= l_3 \sin q_3 + l_4 \sin(q_3 + q_4).$$

In order to transport the object to a new position, the robot manipulators apply forces $F_1$ and $F_2$ to the payload as illustrated in Fig. 3. On the other hands, to rigidly hold the load up, friction forces $F_{s1}$ and $F_{s2}$

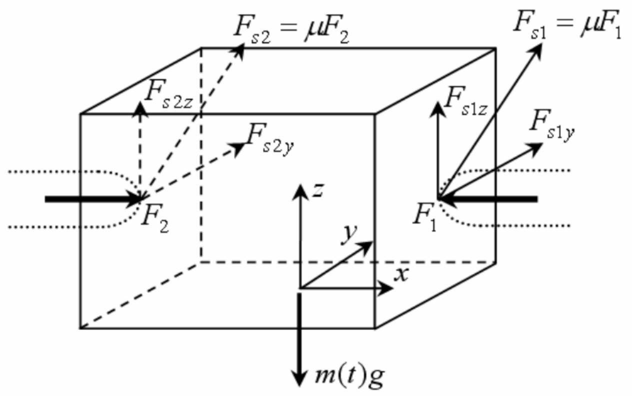


Fig. 3: Physical model of the robot arms

are needed. Let $F_{siy}$ and $F_{siz}$ denote the components of the friction forces in $y$ and $z$ directions, respectively. To prevent the load from rotating around $y$ and $z$ axes, it is supposed that $F_{s1y} = F_{s2y}$ and $F_{s1z} = F_{s2z}$. Then the dynamic equations of the object are as follows,

$$m\ddot{x}_m = F_2 - F_1, \quad (3)$$
$$m\ddot{y}_m = 2F_{s1y} = 2F_{s2y}, \quad (4)$$
$$mg = 2F_{s1z} = 2F_{s2z}, \quad (5)$$

where $g = 9.8m/s^2$. And the relationship between the applied forces and the friction forces is presented by

$${F_{s1y}}^2 + (\frac{mg}{2})^2 < (\mu F_1)^2, \quad (6)$$
$${F_{s2y}}^2 + (\frac{mg}{2})^2 < (\mu F_2)^2, \quad (7)$$

where $\mu$ is the friction coefficient in dry condition.

If $\ddot{x}_{m(t)} \geq 0$, both the applied forces $F_1$ and $F_2$ can be computed by

$$F_1 = \frac{1}{\mu}\sqrt{\left(\frac{m\ddot{y}_m}{2}\right)^2 + \left(\frac{mg}{2}\right)^2}, \quad (8)$$
$$F_2 = \frac{1}{\mu}\sqrt{\left(\frac{m\ddot{y}_m}{2}\right)^2 + \left(\frac{mg}{2}\right)^2} + m\ddot{x}_m. \quad (9)$$

Nonetheless, if $\ddot{x}_{m(t)} < 0$, those forces can be obtained by

$$F_1 = \frac{1}{\mu}\sqrt{\left(\frac{m\ddot{y}_m}{2}\right)^2 + \left(\frac{mg}{2}\right)^2} - m\ddot{x}_m, \quad (10)$$
$$F_2 = \frac{1}{\mu}\sqrt{\left(\frac{m\ddot{y}_m}{2}\right)^2 + \left(\frac{mg}{2}\right)^2}. \quad (11)$$

By the use of Lagrange multipliers, the dynamic model of the dual arm robot manipulating the payload can be summarized as follows,

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u + J^T(q)F(q,\dot{q},\ddot{q}) - T_d - \beta, \quad (12)$$

where u is a $4 \times 1$ control torque input vector, $T_d$ is a $4 \times 1$ vector presenting the noise effects on the robot arms and $\beta$ denotes the viscous friction forces on all the joints, which are specified as follows,

$$q = \left[q_1\ q_2\ q_3\ q_4\right]^T,$$
$$u = \left[u_1\ u_2\ u_3\ u_4\right]^T,$$
$$F = \left[F_1\ F_{s1y}\ F_2\ F_{s2y}\right]^T,$$
$$T_d = \left[T_{d1}\ T_{d2}\ T_{d3}\ T_{d4}\right]^T,$$
$$G(q) = \left[0\ 0\ 0\ 0\right]^T,$$
$$\beta = \left[b_1\dot{q}_1\ b_2\dot{q}_2\ b_3\dot{q}_3\ b_4\dot{q}_4\right]^T.$$

$M(q)$ is a $4 \times 4$ matrix of the mass moment of inertia, whose components are specified by

$$m_{11} = A_1 + A_2 + 2A_3 \cos q_2,$$
$$m_{12} = m_{21} = A_2 + A_3 \cos q_2,$$
$$m_{22} = A_2,$$
$$m_{13} = m_{14} = m_{23} = m_{24} = 0,$$
$$m_{33} = A_4 + A_5 + 2A_6 \cos q_4,$$
$$m_{34} = m_{43} = A_5 + A_6 \cos q_4,$$
$$m_{44} = A_5,$$
$$m_{31} = m_{32} = m_{41} = m_{42} = 0$$

with

$$A_1 = m_1k_1^2 + m_2l_1^2 + I_1,$$
$$A_2 = m_2k_2^2 + I_2,$$
$$A_3 = m_2l_1k_2,$$
$$A_4 = m_3k_3^2 + m_4l_3^2 + I_3,$$
$$A_5 = m_4{k_4}^2 + I_4,$$
$$A_6 = m_4l_3k_4.$$

$C(q,\dot{q})$ is a $4 \times 1$ Coriolis-centripetal vector, whose elements are computed by

$$c_{11} = -A_3 \sin q_2(\dot{q}_2^2 + \dot{q}_1\dot{q}_2) + b_1\dot{q}_1,$$
$$c_{21} = A_3\dot{q}_1^2 \sin q_2 + b_2\dot{q}_2,$$
$$c_{31} = -A_6 \sin q_4(\dot{q}_4^2 + \dot{q}_3\dot{q}_4) + b_3\dot{q}_3,$$
$$c_{41} = A_6\dot{q}_3^2 \sin q_4 + b_2\dot{q}_4.$$

Furthermore, $J$ is a $4 \times 4$ Jacobian matrix with the elements obtained by

$$J_{11} = -L_1 \sin q_1 - L_2 \sin(q_1 + q_2),$$
$$J_{12} = -L_1 \cos q_1 - L_2 \cos(q_1 + q_2),$$
$$J_{13} = J_{14} = 0,$$
$$J_{21} = -L_2 \sin(q_1 + q_2),$$
$$J_{22} = -L_2 \cos(q_1 + q_2),$$
$$J_{23} = J_{24} = 0,$$
$$J_{31} = J_{32} = 0,$$
$$J_{33} = L_3 \sin q_3 + L_4 \sin(q_3 + q_4),$$
$$J_{34} = -L_3 \cos q_3 - L_4 \cos(q_3 + q_4),$$
$$J_{41} = J_{42} = 0,$$
$$J_{43} = L_4 \sin(q_3 + q_4),$$
$$J_{44} = -L_4 \cos(q_3 + q_4).$$

## 3 Controller design

In order to design a control system to effectively control the dual arm robots, we first introduce a controller based on the backstepping sliding mode control method. Nevertheless, many parameters of the designed controller are uncertain and practically unknown, we then present a neural network based technique that allows the system to adaptively estimate those uncertain and unknown dynamics.

Without loss of generality, the dynamic model of the dual arm robot (12) can be rewritten as follows,

$$\dot{x}_1 = x_2, \quad (13)$$
$$\dot{x}_2 = M^{-1}(q).u + M^{-1}(q).K(q,\dot{q},\ddot{q}), \quad (14)$$

where $x_1 = (q_1, q_2, q_3, q_4)^T$ and

$$K(q,\dot{q},\ddot{q}) = J^T(q)F(q,\dot{q},\ddot{q}) - C(q,\dot{q})\dot{q} - G(q) - \beta - T_d. \quad (15)$$

It is noted that while $M(q)$ is assumed to be deterministic, $K(q,\dot{q},\ddot{q})$ is the complex nonlinear dynamics of the system that cannot be fully analytically modelled in reality. The nonlinear dynamics in the dual arm robot comprise a sudden change in load mass, viscous and static friction coefficients, dynamic damping and external disturbances, which are embedded in the dynamic parameters such as $J(q)$, $F(q,\dot{q},\ddot{q})$ and $C(q,\dot{q})$. Therefore, in this work, $K(q,\dot{q},\ddot{q})$ will be approximately estimated via an adaptive mechanism using a network of the radial basis functions.

### 3.1 Backstepping sliding mode control for dual arm robot

The ultimate objective in controlling the dual arm robot is to track the motion of its end-effectors on the given trajectories. In other words, in designing a backstepping sliding mode controller (BSSMC), if $z_1 = x_1 - x_{1ref}$ is defined as a tracking error, where $x_{1ref} = q_{ref}$ is the reference vector, then

**Step 1**: Let

$$\alpha = -c_1 z_1 + \dot{x}_{1ref} \tag{16}$$

define the virtual control with a positive $c_1$ so that

$$\lim_{t\to\infty} z_1(t) = 0.$$

If $z_2 = x_2 - \alpha$, and differentiating $z_1$ with respect to time, it yields

$$\dot{z}_1 = \dot{x}_1 - \dot{x}_{1ref} = z_2 + \alpha - \dot{x}_{1ref} = z_2 - c_1 z_1. \tag{17}$$

Hence, the first Lyapunov function candidate can be defined by

$$V_1 = \frac{1}{2} {z_1}^T z_1. \tag{18}$$

Then, the derivative of $V_1$ is computed as follows,

$$\dot{V}_1 = {z_1}^T \dot{z}_1 = -{z_1}^T c_1 z_1 + {z_1}^T z_2. \tag{19}$$

**Step 2**: To design a sliding mode controller, a sliding surface can be presented by

$$s = \lambda z_1 + M\, z_2, \tag{20}$$

where $\lambda = diag(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ is a matrix of positive gains that characterize for the convergence rate of $s$ and $z$. If one differentiates $s$ with respect to time, it is given

$$\begin{aligned} \dot{s} &= \lambda \dot{z}_1 + M \dot{z}_2 = \lambda \dot{z}_1 + M(\dot{x}_2 - \dot{\alpha}) \\ &= \lambda \dot{z}_1 + M(M^{-1}K + M^{-1}u - \dot{\alpha}) \\ &= \lambda \dot{z}_1 + K + u - M\dot{\alpha}. \end{aligned} \tag{21}$$

Moreover, the second Lyapunov function candidate can be obtained by

$$V_2 = V_1 + \frac{1}{2} s^T s, \tag{22}$$

and its derivative can be specified by

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + s^T \dot{s} \\ &= -{z_1}^T c_1 z_1 + {z_1}^T z_2 + s^T(\lambda \dot{z}_1 + K + u - M\dot{\alpha}). \end{aligned} \tag{23}$$

$\dot{V}_2$ can be rearranged by adding the function of $sign(s)$ as follows,

$$\begin{aligned} \dot{V}_2 = &- {z_1}^T c_1 z_1 - s^T c_2 sign(s) \\ &+ s^T \left( \frac{s {z_1}^T z_2}{s^T s} + c_2 sign(s) + \lambda \dot{z}_1 + K + u - M\dot{\alpha} \right), \end{aligned} \tag{24}$$

where $c_2$ is a positive number. If the control input is chosen by

$$u = -\left( \frac{s {z_1}^T z_2}{s^T s} + c_2 sign(s) + \lambda \dot{z}_1 + K - M\dot{\alpha} \right), \tag{25}$$

then

$$\dot{V}_2 = -{z_1}^T c_1 z_1 - s^T c_2 sign(s) < 0. \tag{26}$$

As a result, the outputs of the system proximally approach to the desired references.

It is noticed that when the sliding surface $s \to 0$, $u \to -\infty$. Therefore, in practice, the control input for the BSSMC controller is proposed to be given by

$$u = -\left( \frac{s {z_1}^T z_2}{s^T s + \sigma} + c_2 sign(s) + \lambda \dot{z}_1 + K - M\dot{\alpha} \right), \tag{27}$$

where $\sigma$ is a very small positive number.

### 3.2 Adaptive neural backstepping sliding mode controller and system stability

As discussed in the previous section, the complex nonlinear dynamic $K$ in (27) is not fully analytically modelled in practice. In other words, computing the control signal $u$ in (27) is analytically impractical. Therefore, in this work, we propose to employ the radial basis function network to approximately estimate the undetermined dynamic parameters including $J(q)$, $F(q, \dot{q}, \ddot{q})$, $C(q, \dot{q})$.

Let $f(Z) : R^a \to R^b$ denote the radial basis function network,

$$f(Z) = \mathrm{W}^T H(Z), \tag{28}$$

where $\mathrm{W} = [\mathrm{W}_1, \mathrm{W}_2, ..., \mathrm{W}_l]^T \in R^{b\times l}$ is the ideal weight matrix, and $l$ is the number of neurons in a hidden layer. $H(Z) = [h_1(Z), h_2(Z), ..., h_l(Z)]^T$, where $h_i(Z)$ is an activation function. The widely used activation function, which is also employed in this work, is Gaussian,

$$h_i(Z) = \exp\left[ \frac{-(Z-\mu_i)^T (Z-\mu_i)}{2\eta_i^2} \right], \tag{29}$$

and

$$0 < h_i(Z) \le 1, \tag{30}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, ..., \mu_{ia}]^T$ is the center vector of the receptive field, and $\eta_i$ is the width of the Gaussian function. $Z \in R^a$ is a matrix of the neural inputs. In this design, we define

$$Z = [{x_1}^T, \dot{x}_1^T, \ddot{x}_1^T]^T \in \Omega_Z \subset R^{12}, \tag{31}$$

where $x_1$ and $x_2$ are given in (13) and (14). If $\hat{W}$ denotes estimation of the weight matrix, the output of the radial basis function $f(Z)$ is approximated by

$$\hat{f}(Z) = \hat{W}^T H(Z). \tag{32}$$

As a consequence, the approximation of the control signal in (27) can be computed by

$$u = -\left(\frac{s{z_1}^T z_2}{s^T s + \sigma} + c_2 sign(s) + \lambda \dot{z}_1 + \hat{f}(Z) - M\dot{\alpha}\right). \tag{33}$$

It is noted that from now onward we define the control approach with a control input presented in (33) as the radial basis function network based backstepping sliding mode control (RBFN-BSSMC).

Now let's consider an adaptive approach based on the Lyapunov stability to effectively estimate $\hat{W}$. Controllably, the Lyapunov candidate is formulated by

$$V_2 = V_1 + \frac{1}{2}s^T s + \frac{1}{2}\sum_{i=1}^{4} \tilde{W}_i^T \Gamma^{-1} \tilde{W}_i, \tag{34}$$

where $\Gamma = diag(\Gamma_1, \Gamma_2, ..., \Gamma_4)$ is a positive definite diagonal matrix of the adaptation gains. $\tilde{W} = \hat{W} - W$ is error between the estimated weights $\hat{W}$ and the ideal weights $W$. Then, derivative of $V_2$ can be computed by

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + s^T \dot{s} + \sum_{i=1}^{4} \tilde{W}_i^T \Gamma^{-1} \dot{\tilde{W}}_i \\ &= -c_1 {z_1}^T z_1 + {z_1}^T z_2 \\ &+ s^T(\lambda \dot{z}_1 + K + u - M\dot{\alpha}) + \sum_{i=1}^{4} \tilde{W}_i^T \Gamma^{-1} \dot{\tilde{W}}_i \end{aligned} \tag{35}$$

Substituting the control law in (33) into (35) leads to

$$\begin{aligned} \dot{V}_2 = &- {z_1}^T c_1 z_1 - s^T c_2 sign(s) \\ &+ \sum_{i=1}^{4} \tilde{W}_i^T \left[\Gamma^{-1} \dot{\tilde{W}}_i - s^T H(z) - \delta \hat{W}\right], \end{aligned} \tag{36}$$

where $\delta$ is a positive number. If the adaptation mechanism is chosen by

$$\dot{\tilde{W}} = \dot{\hat{W}} = \Gamma[H(Z)s^T + \delta \hat{W}], \tag{37}$$

then the derivative of $V_2$ can be rewritten as

$$\dot{V}_2 = -{z_1}^T c_1 z_1 - s^T c_2 sign(s) < 0. \tag{38}$$

In other words, the system stability holds if the estimated weights $\hat{W}$ are adaptively computed by (37).

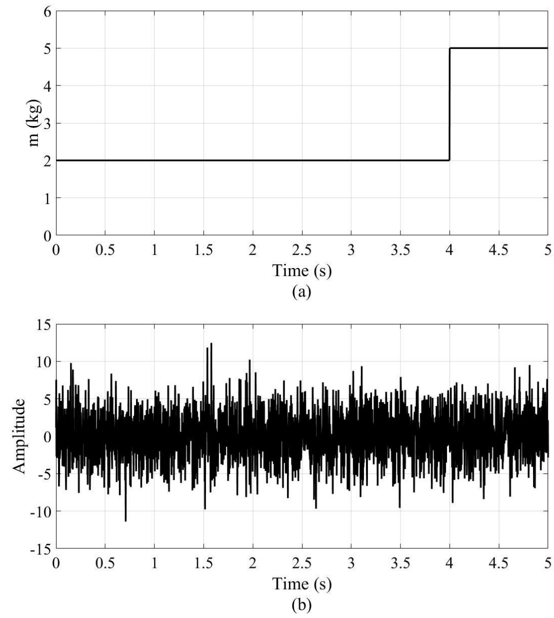


Fig. 4: Input parameters: (a) load mass variation and (b) external disturbance.

## 4 Simulation discussion

To demonstrate effectiveness of the proposed approach, we conducted experiments in simulation environment. To simulate the dual arm robot protocol, the manipulators are first to track the reference trajectories to approach the payload. The reference trajectories in the first 2 seconds are given by

$$x_{a1}(t) = x_{f1} + (x_{i1} - x_{f1})e^{-10t^2}, \tag{39}$$

$$y_{a1}(t) = y_{f1} + (y_{i1} - y_{f1})e^{-10t^2}, \tag{40}$$

$$x_{a2}(t) = x_{f2} + (x_{i2} - x_{f2})e^{-10t^2}, \tag{41}$$

$$y_{a2}(t) = y_{f2} + (y_{i2} - y_{f2})e^{-10t^2}, \tag{42}$$

where $x_{a1}, y_{a1}, x_{a2}, y_{a2}$ are the trajectories of the manipulators. $(x_{i1}, y_{i1}, x_{i2}, y_{i2})$ and $(x_{f1}, y_{f1}, x_{f2}, y_{f2})$ are the initial and final positions of the end-effectors, respectively. After rigidly grasping the object, the robot transports the payload along the half of a circle so that it can avoid collision with an obstacle. The center of the object is expected to travel on a curve as follows,

$$x_{mr}(t) = x_0 + r_m \cos(\psi t), \tag{43}$$

$$y_{mr}(t) = y_0 + r_m sin(\psi t), \tag{44}$$

where $(x_0, y_0)$ is the position of the obstacle, which is also the center of the circle on which the object moves. $r_m$ is the radius of the circle, while $\psi$ is a polar angle that varies from $-\pi$ to 0. Note that the joint angles between the link and the base or its preceding link at the beginning $t = 0$ were known, $q_1(0) = \frac{\pi}{6}$, $q_2(0) = \frac{\pi}{2}$, $q_3(0) = \pi$ and $q_4(0) = \frac{-2\pi}{3}$.

In the simulation experiments, the dynamic models of the dual arm robot were given. Moreover, the parameters of the controllers including BSSMC and RBFN-BSSMC were known. Those information were adapted from (Hacioglu et al. (2011)) and are summarized in Table 1. It is noted that the weight matrix $W$ of the radial basis function network were initialized by zeros, which supposes that there is no prior knowledge of the robot dynamics.

Furthermore, to illustrate robustness and adaptation of the proposed controller, it was assumed that the load mass is suddenly changed at fourth second as shown in Fig. 4a. Moreover, an unexpected disturbance as illustrated in Fig. 4b, which exerts the input forces, was taken into consideration.

Table 1: Parameters of the dual arm robot system

| Dynamic model parameters |
|---|
| $m_1 = m_2 = m_3 = m_4 = 1.5\,(kg)$; $I_1 = I_2 = I_3 = I_4 = 0.18\,(kgm^2)$; $l_1 = l_2 = l_3 = l_4 = 1.2\,(m)$; $k_1 = k_2 = k_3 = k_4 = 0.48\,(m)$; $b_1 = b_2 = b_3 = b_4 = 110\,(Nm/s)$; $d_1 = 0.25\,(m)$; $d_2 = 1.2\,(m)$; $\mu = 0.35$; |
| Reference trajectory parameters |
| $(x_{i1}, y_{i1}, x_{i2}, y_{i2}) = (0.76,\ 0.6,\ -0.76,\ 0.6)$; $(x_{f1}, y_{f1}, x_{f2}, y_{f2}) = (-0.275,\ 1.4,\ -0.525,\ 1.4)$; $(x_0, y_0) = (0,\ 1.4)$; $r_m = 0.4$; $q_1(0) = \frac{\pi}{6}$; $q_2(0) = \frac{\pi}{2}$; $q_3(0) = \pi$; $q_4(0) = \frac{-2\pi}{3}$; $\dot{q}_1(0) = \dot{q}_2(0) = \dot{q}_3(0) = \dot{q}_4(0) = 0$ |
| BSSMC parameters |
| $\lambda = diag\,(20, 20, 20, 20)$; $\sigma = 10^{-10}$; $c_1 = diag\,(220, 220, 220, 220)$; $c_2 = diag\,(1200, 1200, 1200, 1200)$ |
| RBFN-BSSMC parameters |
| $\lambda = diag\,(20, 20, 20, 20)$; $\sigma = 10^{-10}$; $c_1 = diag\,(220, 220, 220, 220)$; $c_2 = diag\,(1200, 1200, 1200, 1200)$; $\hat{W}\,(0) = 0$; $\Gamma = diag\,(30, 30, 30, 30)$ |

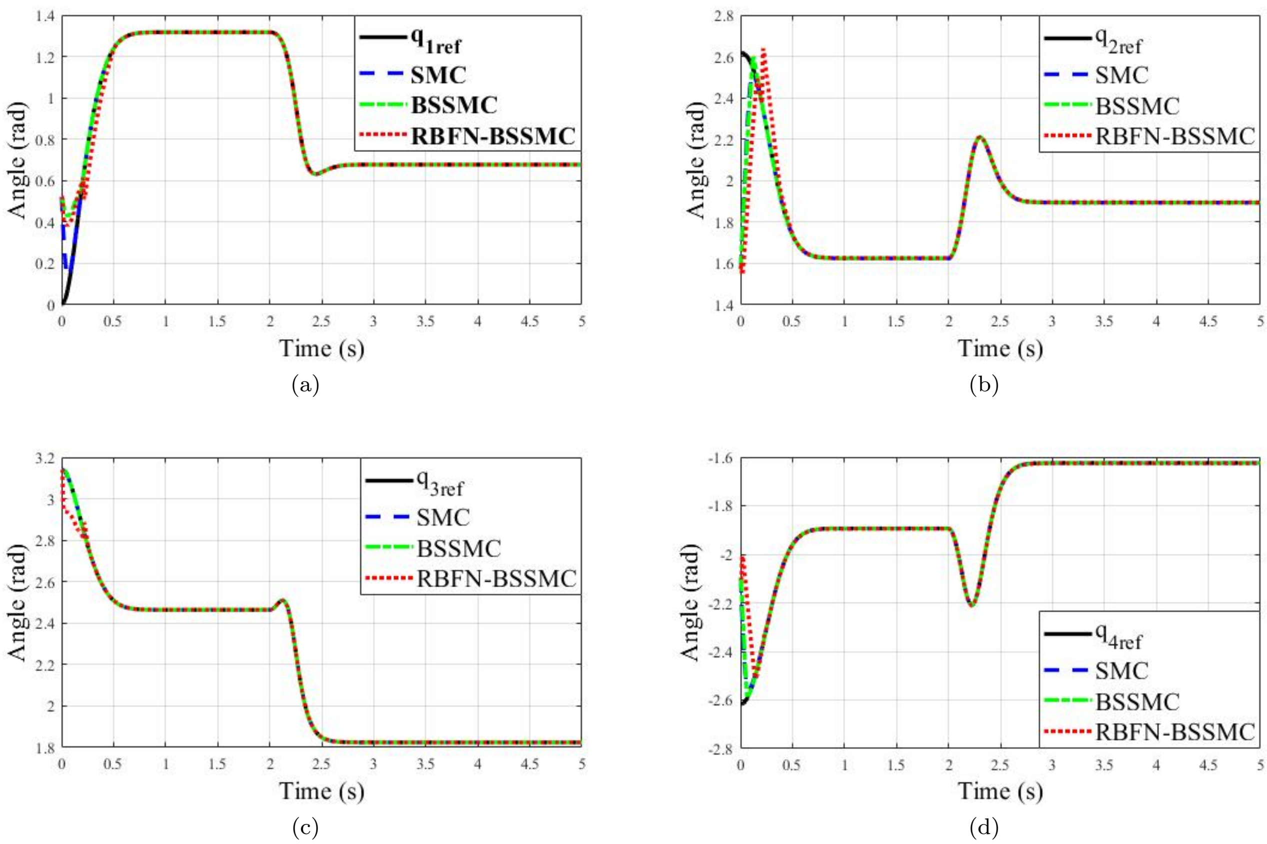


Fig. 5: Joint angles of the link and the base or its preceding link: (a) first link, (b) second link, (c) third link and (d) fourth link.

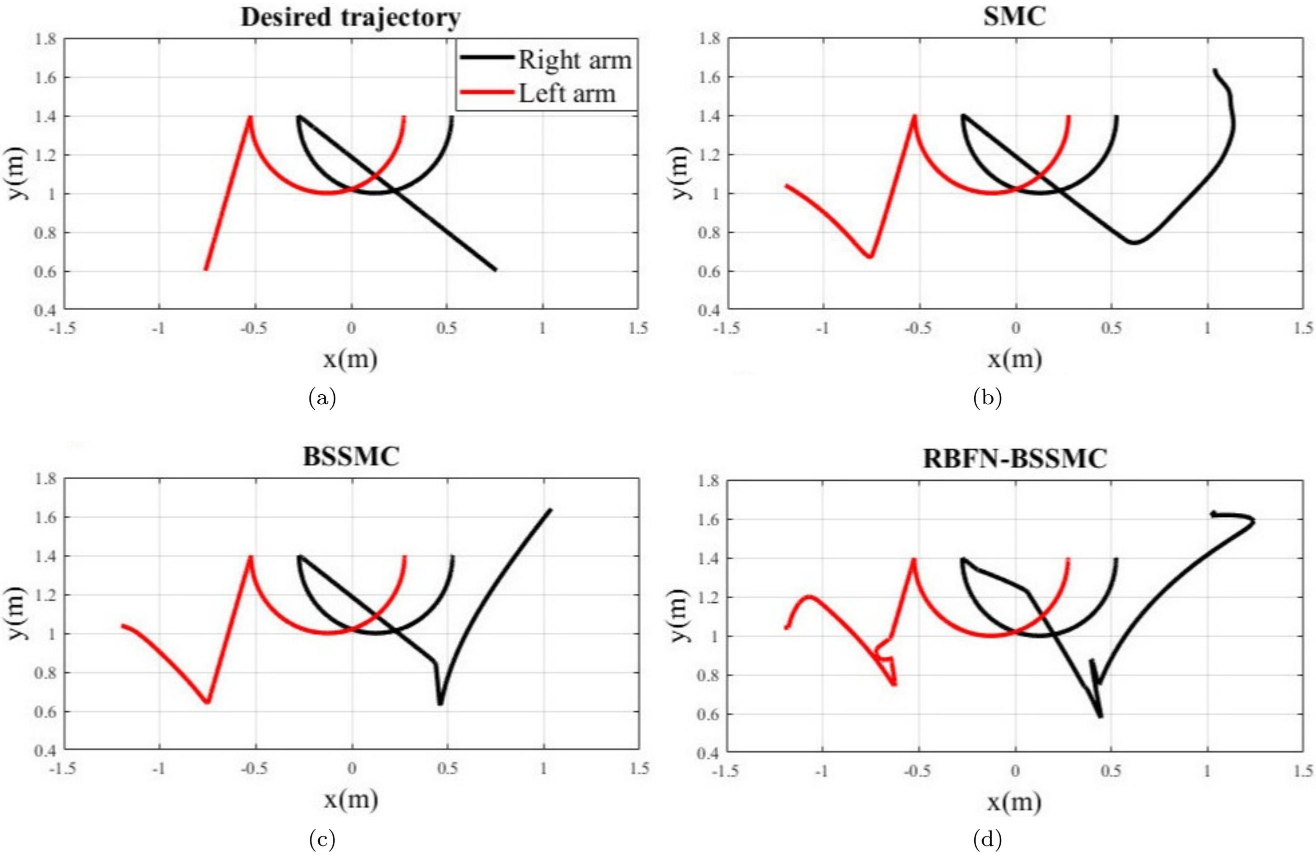


Fig. 6: Motion trajectories of the end-effectors: (a) Expected trajectories, (b) trajectories obtained by the SMC, (c) trajectories obtained by the BSSMC and (d) trajectories obtained by the RBFN-BSSMC.

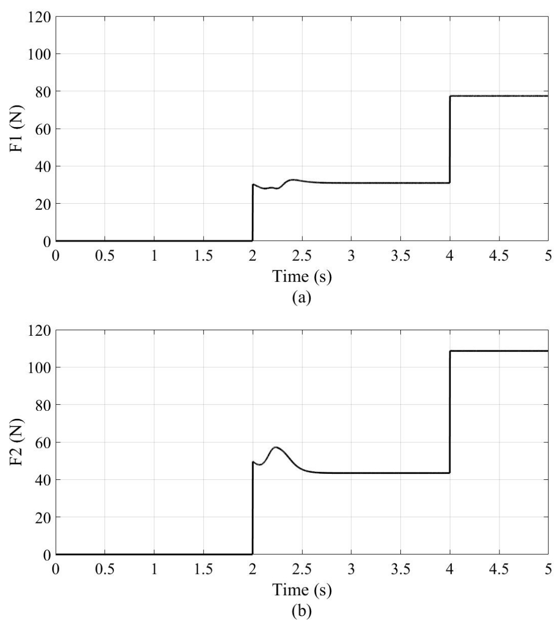


Fig. 7: Interaction forces: (a) $F_1$ (b) $F_2$.

To examine the effectiveness of the proposed technique, we first investigate motions of the four links of the dual arm robot. Practically, the best way to delineate the motions of the links on $xy - plane$ is to present the joint angles between the link and the base or its preceding link when the manipulators move as shown in Fig. 5. It can be clearly seen that for the purposes of comparisons, in this experimental example we implemented three algorithms including the classical sliding mode control (SMC) (Wu (2012); Le et al. (2017, 2019)), the BSSMC as discussed in Section 3.1 and the proposed method RBFN-BSSMC introduced in Section 3.2. The results obtained by the three implemented approaches are expected to reach the references, which are early obtained from equations (39-44), all the time. It is noticed that both the SMC and BSSMC require parameters of the model dynamics to be known, which are hardly to be acquired in reality. On the other hand, those parameters are also uncertain due to disturbances in the system. Nevertheless, the RBFN-BSSMC approach is able to effectively estimate those dynamics through a neural network. From Fig. 5, it can be seen that after about 0.2 s all the joint angles of the dual arm robot obtained by the three different approaches accurately reach to the references. Nonetheless, in the first 0.2 s time, while both the methods of the SMC and BSSMC quickly settle down with the references, the proposed RBFN-BSSMC method takes a little bit longer to do so. That is understandable where the SMC and BSSMC methods were provided the model dynamic parameters while the RBFN-BSSMC technique needs time to adaptively estimate those.

Furthermore, for the motion trajectories of the two end-effectors as demonstrated in Fig. 6, it shows that the proposed approach is effectively practical. With the aim of transporting an object along a half of a circle to avoid collision with an obstacle, the movements of both the left and right arms of the robot under control of the SMC in Fig. 6b, the BSSMC in Fig. 6c and the RBFN-BSSMC in Fig. 6d are expected to track the ideal trajectories as illustrated in Fig. 6a. It can be clearly seen that given known parameters of the model dynamics, both the SMC and BSSMC methods controlled the arms to move quite smoothly before approaching the payload. The proposed approach RBFN-BSSMC had to estimate the dynamic parameters, which made the motions of the arms of the robot before grasping the object less smooth as the expectation. However, more importantly once the arms firmly held the payload, the transportation of the object obtained by the RBFN-BSSMC controller is highly comparable to not only those obtained by the SMC and BSSMC methods but also the expectation. That is, the proposed approach guarantees an ability of the dual arm robot to adaptively learn its dynamics while safely transport the payload to a destination.

To further demonstrate the robustness and adaptation of the proposed controller, the interaction forces were summarized and are plotted in Fig. 7. It can be clearly seen that the forces on the arms of the robot present quite homogeneously, where the forces started rising when the load was handled at 2 s. More importantly, at 4 s when the load mass was suddenly varied, the forces were adaptively increased straight away to guarantee the load to be held without dropping and delivered to the destination.

## 5 Conclusions

The paper has discussed a novel but efficient scheme to design an adaptive controller based on the BSSMC and RBFN for the DAR or DAM. The proposed approach enables the DAR system to be able to adaptively estimate its nonlinear, uncertain and unmodelled dynamics. Moreover, by the use of the BSSMC, the RBFN-BSSMC controller guarantees robustness of the control performance in the DAR system given the external disturbances and its uncertainties. More particularly, the adaptation law is derived from the Lyapunov theorem, which provides the stability of the control system to be held. The proposed algorithm has been validated in a simulation environment with realistic parameters, which demonstrates the promising results.

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