Finite-Control-Set Model Predictive Direct Torque Control of PMSMs with Virtual Space Vectors

Tianshi Wang , Youguang Guo and Jianguo Zhu
School of Electrical, Mechanical and Mechatronic Systems
University of Technology Sydney
Sydney, Australia

***Abstract*— The finite-control-set model predictive direct torque control (FCS-MPDTC) is a novel control scheme for permanent magnet synchronous motors (PMSMs). A key feature of FCS-MPDTC is that the possible voltage space vectors or switching combinations of the power converters are directly taken into account as the control input of the drive system. A cost function is used in FCS-MPDTC to evaluate each possible voltage space vector and the one with minimum cost is selected. Due to the considerable torque and flux ripples of FCS-MPDTC, this paper presents a novel FCS-MPDTC with virtual voltage space vectors. To mitigate the computational burden caused by increased number of voltage space vectors, a pre-selective scheme is designed for the proposed FCS-MPDTC to filter out the impractical voltage vectors instead of evaluating all twenty voltage space vectors.**

Keywords*— *model predictive direct torque control; permanent magnet synchronous motors; ripple reduction; system efficiency

I. Introduction

Direct torque control (DTC) features simple structure and fast dynamic response. In DTC, the voltage space vector (VSV) is selected by using two hysteresis comparators and a pre-defined switching table. The major drawback of DTC is large torque and flux ripples since the VSV is selected heuristically [1] [2]. Recently, the finite-control-set model predictive direct torque control (FCS-MPDTC) was introduced to overcome the problem [3]-[11]. Based on the system model, the FCS-MPDTC can predict the future behaviour of the controlled variables, such as torque, stator flux and switching transitions, etc. Each possible VSV is evaluated by the cost function and the one minimizing the cost function is selected as the output VSV [3]. The implementation of the cost function offers a flexible and simple way to incorporate various constraints and other control objectives, such as switching frequency reduction [12], common-mode voltage decrease [13] and the peak current restriction [3]. Compared to conventional DTC, the voltage vector selected by FCS-MPDTC is more effective owing to the real-time optimization nature of the cost function. However, the FCS-MPDTC still suffers from relatively high torque and flux ripples due to the limited number of VSVs.

This paper proposes a novel FCS-MPDTC with an extended set of twenty modulated VSVs, which are formed by eight basic VSVs and twelve virtual VSVs by modulating eight basic VSVs with fixed duty ratio. By evaluating all twenty VSVs, the concept of duty ratio control is naturally integrated into the proposed algorithm. In contrast with the cascaded duty ratio processing methods, the proposed algorithm can articulate the duty ratio control in the motor model and make better utilization of concept of model predictive control. To mitigate the computational burden caused by the increased number of VSVs, a pre-selective scheme is designed for the proposed FCS-MPDTC to filter out the impractical VSVs instead of evaluating all twenty VSVs. Numerical simulation results are presented to demonstrate the effectiveness of the proposed control scheme.

II. Model of PMSM

In the rotor synchronous d-q coordinate system, the model of PMSM can be expressed as

$$u_d = R_s i_d + L_d \frac{di_d}{dt} - \omega L_q i_q \tag{1}$$

$$u_q = R_s i_q + L_q \frac{di_q}{dt} + \omega L_d i_d + \omega \psi_f \tag{2}$$

$$\psi_d = L_d i_d + \psi_f \tag{3}$$

$$\psi_q = L_q i_q \tag{4}$$

In the form of state equations, (1) and (2) can be written as

$$\frac{di_d}{dt} = \frac{-R_s i_d + \omega L_q i_q + u_d}{L_d} \tag{5}$$

$$\frac{di_q}{dt} = \frac{-\omega L_d i_d - R_s i_q + u_q - \omega \psi_f}{L_q} \tag{6}$$

The electromagnetic torque can be expressed as

$$T_e = \frac{3}{2} p\left(\psi_d i_q - \psi_q i_d\right) = \frac{3}{2} p\left[\psi_f i_q + \left(L_d - L_q\right) i_d i_q\right] \tag{7}$$

III. The Conventional FCS-MPDTC

The principle of VSV selection in FCS-MPDTC is based on evaluating a pre-defined cost function. The selected VSV from the switching table in the conventional DTC is not necessarily the best one in terms of torque and flux ripples reduction. Due to the limited number of discrete VSVs in the two-level inverter-fed PMSM drives, it is possible to evaluate the effect of each VSV and select the best one.

The key technology of FCS-MPDTC lies in the definition of the cost function, which is related to the control objectives. The greatest concern of PMSM drive applications are torque and stator flux and the cost function is defined in such a way that both torque and stator flux at the end of control period is as close as possible to the reference value. In this paper, the cost function is defined as

$$\min.\ G = \left|T_e^* - T_e^{k+1}\right| + k_1 \left\| \psi_s^* \right| - \left| \psi_s^{k+1} \right\| \quad s.t.\ \ u_s^k \in \{V_0, V_1, \ldots V_6, V_7\} \tag{8}$$

where T_e^* , ψ_s^* , T_e^{k+1} and ψ_s^{k+1} are the reference and predicted values of torque and flux, respectively, and k_1 is the weighting factor, introduced to unify the terms of torque and flux which are different in physical nature. In this paper, k_1 is selected to be $T_n\ /\ \psi_n$, where T_n and ψ_n are the rated values of torque and stator flux, respectively.

The stator current in the *d-q* frame at the (*k+1*)*th* time instant can be obtained by discretizing (5) and (6) as

$$i_d^{k+1} = i_d^k + \frac{1}{L_d}\left(-R_s i_d^k + \omega^k L_q i_q^k + u_d^k\right) T_s \tag{9}$$

$$i_q^{k+1} = i_q^k + \frac{1}{L_q}\left(-\omega^k L_d i_d^k - R_s i_q^k + u_q^k - \omega^k \psi_f\right) T_s \tag{10}$$

With the knowledge of future stator current, both torque and flux at the (*k+1*)*th* time instant can be obtained from (3), (4) and (7). Fig.1 illustrates the block diagram of conventional FCS-MPDTC. The inputs of the system are the reference and estimated values of torque and flux. By evaluating the effects of each VSV when applied to the system, the VSV which minimize the difference between the reference and predicted values is selected.

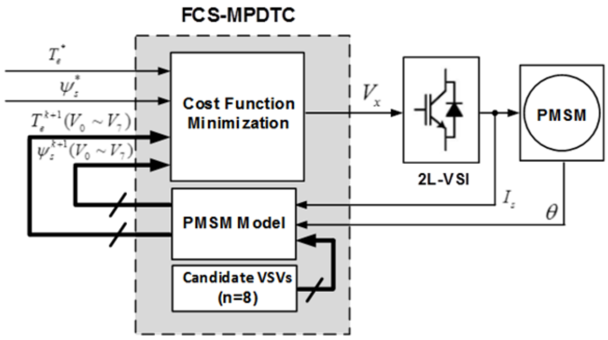


Fig. 1. Block diagram of conventional FCS-MPDTC drive system.

TABLE I. MODULATION OF EXTENDED VSVS

Vector	Modulated by	Gate signal (d=0.5)
V_7	$V_1 + V_{19}$	d,0,0
V_8	$V_2 + V_{20}$	1,1,d
V_9	$V_3 + V_{19}$	0,d,0
V_{10}	$V_4 + V_{20}$	d,1,1
V_{11}	$V_5 + V_{19}$	0,0,d
V_{12}	$V_6 + V_{20}$	1,d,1
V_{13}	$V_1 + V_2$	1,d,0
V_{14}	$V_2 + V_3$	d,1,0
V_{15}	$V_3 + V_4$	0,1,d
V_{16}	$V_4 + V_5$	0,d,1
V_{17}	$V_5 + V_6$	d,0,1
V_{18}	$V_1 + V_6$	1,0,d
V_{19}		0,0,0
V_{20}		1,1,1

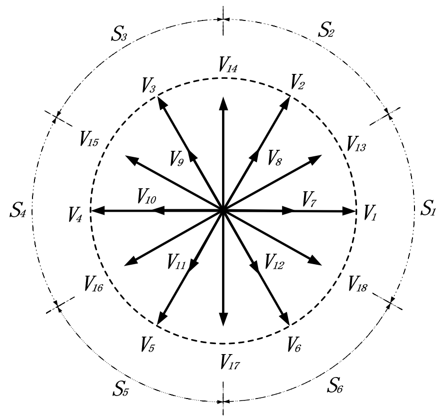


Fig. 2. Basic VSVs and extended virtual VSVs.

IV. PRINCIPLE OF THE PROPOSED FCS-MPDTC

A. *Definition of Virtual VSVs*

The virtual VSVs are created by modulating two adjacent basic active VSVs or one basic active VSV and one zero VSV with a fixed duty ratio of 0.5. The virtual VSVs are presented in Table I and Fig. 2.

B. *The Pre-selective Scheme*

To mitigate the computational burden caused by increased number of VSVs, a pre-selective scheme is designed for proposed FCS-MPDTC to filter out the impractical VSVs instead of evaluating all twenty VSVs as shown in Table II. In the table, $\Delta\varphi$ represent the error between the actual stator flux and the reference stator flux, ΔT_e represent the error between the actual torque and the command torque, e.g. if the command stator flux is greater than the actual stator flux, the command torque is greater than the actual torque and the rotor is located in space sector S_1, then only V_1, V_2, V_7, V_8, V_{13} and V_{14} are considered as candidate VSVs and then provided to the cost function for evaluation. This pre-selective scheme is inspired by the switching table used in conventional DTC. With the help of proposed scheme, the number of candidate VSVs is reduced from 20 to 6 and the proposed FCS-MPDTC requires less computing time than conventional FCS-MPDTC. To be more specific, the computing time of conventional FCS-

TABLE II. THE PRE-SELECTIVE SCHEME

$\Delta\varphi$	ΔT_e	**Candidate VSVs in Corresponding Space Sector (S_x)**					
		S_1	**S_2**	**S_3**	**S_4**	**S_5**	**S_6**
1	1	$V_1V_2V_7V_8V_{13}V_{14}$	$V_2V_3V_8V_9V_{14}V_{15}$	$V_3V_4V_9V_{10}V_{15}V_{16}$	$V_4V_5V_{10}V_{11}V_{16}V_{17}$	$V_5V_6V_{11}V_{12}V_{17}V_{18}$	$V_1V_6V_7V_{12}V_{13}V_{18}$
	-1	$V_1V_6V_7V_{12}V_{17}V_{18}$	$V_1V_2V_7V_8V_{13}V_{18}$	$V_2V_3V_8V_9V_{13}V_{14}$	$V_3V_4V_9V_{10}V_{14}V_{15}$	$V_4V_5V_{10}V_{11}V_{15}V_{16}$	$V_5V_6V_{11}V_{12}V_{16}V_{17}$
-1	1	$V_3V_4V_9V_{10}V_{14}V_{15}$	$V_4V_5V_{10}V_{11}V_{15}V_{16}$	$V_5V_6V_{11}V_{12}V_{16}V_{17}$	$V_1V_6V_7V_{12}V_{17}V_{18}$	$V_1V_2V_7V_8V_{13}V_{18}$	$V_2V_3V_8V_9V_{13}V_{14}$
	-1	$V_4V_5V_{10}V_{11}V_{16}V_{17}$	$V_5V_6V_{11}V_{12}V_{17}V_{18}$	$V_1V_6V_7V_{12}V_{13}V_{18}$	$V_1V_2V_7V_8V_{13}V_{14}$	$V_2V_3V_8V_9V_{14}V_{15}$	$V_3V_4V_9V_{10}V_{15}V_{16}$

MPDTC, FCS-MPDTC with 20 VSVs and FCS-MPDTC with both 20 VSVs and pre-selective scheme are 53.4μs, 95.3μs and 43.7μs.

V. SIMULATION STUDY

In this section, the proposed FCS-MPDTC is simulated in the environment of MATLAB/Simulink. In the figures, the conventional DTC, conventional FCS-MPDTC and proposed FCS-MPDTC are abbreviated to DTC-8, MPDTC-8 and MPDTC-20, respectively. The block diagram of proposed FCS-MPDTC is illustrated in Fig. 3 and the parameters of motor and control system are listed in Table III. The sampling frequency is 5000 Hz in all methods.

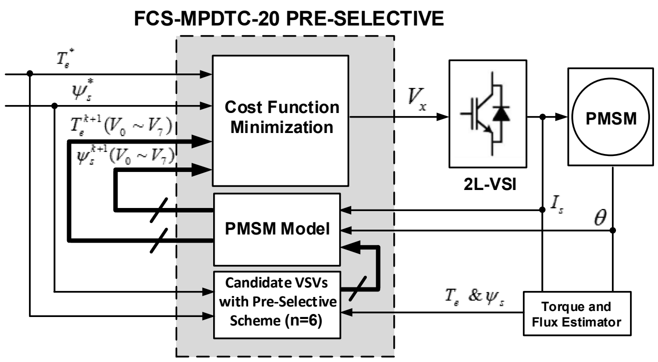
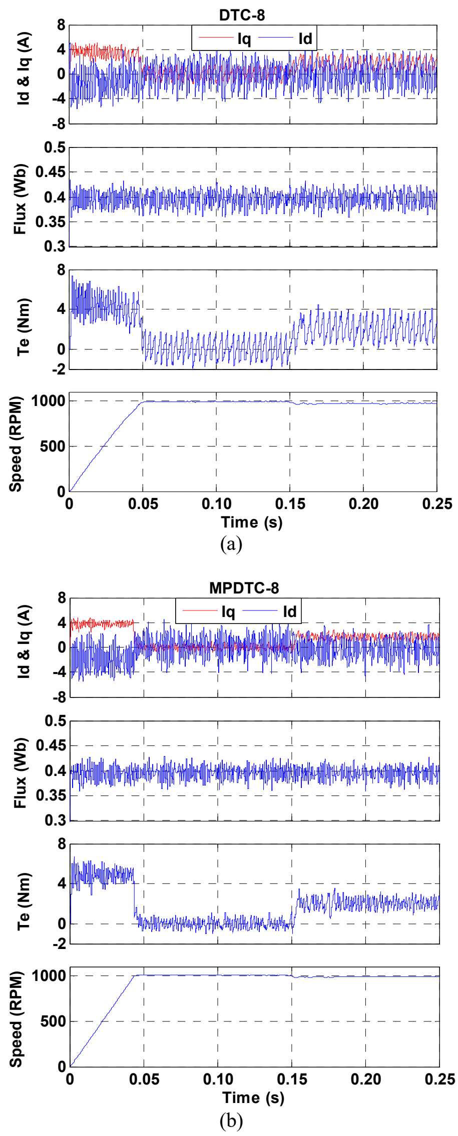


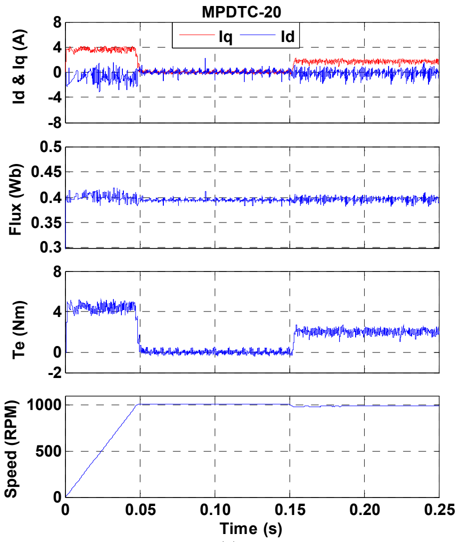
Fig. 3. Block diagram of proposed FCS-MPDTC drive system

TABLE III. MACHINE AND CONTROL PARAMETERS

Stator resistance	R_s	0.47 Ω
d-axis inductance	L_d	7.93 mH
q-axis inductance	L_q	27.77 mH
Permanent magnet flux	ψ_f	0.394 Wb
Number of pole pairs	p	2
DC bus voltage	V_{dc}	200 V
Rated torque	T_{rated}	2 Nm
Rated speed	ω_{rated}	1000 r/min
Weighting factor of flux	k_1	5
Sampling frequency	f_{sp}	5000 Hz

In Fig. 4, a comparative study of conventional DTC, conventional FCS-MPDTC and proposed FCS-MPDTC is presented. The motor runs at 1000 r/min and an external load of 2 Nm is applied at t=0.15s. It can be found that the proposed FCS-MPDTC presents the best steady-state performance in terms of lower torque/flux ripples and less current harmonics.





(c)

Fig. 4. Simulated responses of *dq*-axis current, stator flux and torque at 1000 r/min with load variation, (a) conventional DTC, (b) conventional FCS-MPDTC and (c) proposed FCS-MPDTC

The quantitative comparison of three methods under rated speed and load is presented in Table IV. The torque/flux ripples, total harmonic distortion (THD) of phase current and average switching frequency are calculated based on the simulation results from 0.15s to 0.25s. The torque/flux ripples are defined as the standard deviation of torque/flux values. The average switching frequency is calculated by counting the total switching jumps N of six legs of the inverter over a fixed period of 0.05 s.

TABLE IV. QUANTITATIVE COMPARISON OF SIMULATION RESULTS

Method	Torque Ripple T_{rip} (Nm)	Flux Ripple ψ_{rip} (Wb)	Phase Current THD	Average Switching Frequency f_{av} (Hz)
DTC-8	0.9933	0.0145	62.73%	879.96
MPDTC-8	0.4597	0.0117	44.58%	899.16
MPDTC-20	0.2790	0.0047	14.72%	2653.71

VI. CONCLUSION

This paper proposes a FCS-MPDTC with virtual VSVs. Compared to the conventional DTC and conventional FCS-MPDTC, the choices of VSVs for the control algorithm are extended from eight to twenty by inherently modulating the basic VSVs. Compared to other cascaded duty ratio processing methods, the proposed algorithm can articulate the duty ratio control in the motor model and make better utilization of concept of model predictive control. To mitigate the computational burden caused by the increased number of VSVs, a pre-selective scheme is designed to filter out the impractical VSVs instead of evaluating all twenty VSVs. Compared to the conventional FCS-MPDTC, the proposed FCS-MPDTC requires less computing time and features lower torque and flux ripples, lower phase current THD and higher system efficiency.

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