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Error Exponent Analysis in Quantum Information Theory

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Certification of Original Authorship

I certify that this thesis “Error Exponent Analysis in Quantum Information Theory” has been written by me. This thesis is the result of a research candidature conducted with another University as part of a collaborative Doctoral degree. Any help that I have received in my research and in the preparation of the thesis itself has been fully acknowledged. In addition, I certify that all information sources and literature used are quoted in the thesis

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Abstract

Error exponent analysis aims at evaluating the exponential behaviour of the performance of the underlying system given a certain fixed coding rate. It is arguably a significant research topic in information theory because the analysis characterizes the trade-offs between the error probability of an information task, the size of the coding scheme, and the coding rate that determines the efficiency of the task. In this thesis, we give an exposition of error exponent analysis to two important quantum information processing protocols—classical data compression with quantum side information, and classical communications over quantum channels.

We first prove substantial properties of various exponent functions, which allow us to better characterize the error behaviours of the tasks. Second, we establish accurate achievability and optimality finite blocklength bounds for the optimal error probability, providing useful and measurable benchmarks for future quantum information technology design. Finally, we extend the error exponent analysis to a more general setting where the coding rate is not fixed anymore, a research topic known as moderate deviation analysis. In other words, we show that the data recovery can be reliable when the compression rate approaches the conditional entropy slowly, and the reliable communication over a classical-quantum channel is possible as the transmission rate approaches channel capacity slowly.

This line of research lies in the intersection of statistical analysis, matrix analysis, and information theory. Thus, the techniques employed in this studies could potentially be applicable to various areas such as classical and quantum information community, detection and estimation theory, statistics, and secrecy.

Keywords: error exponent analysis, moderate deviation analysis, quantum information theory, classical-quantum channel, Slepian-Wolf coding, quantum side information, reliability function, large deviation theory, matrix analysis.

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Contents

List of Figures

List of Tables