

**Quantum Computation and Combinatorial Structures**

by

Ryan L. Mann

A thesis submitted in satisfaction of the  
requirements for the degree of  
Doctor of Philosophy

in the

Faculty of Engineering and Information Technology

at the

University of Technology Sydney

February 2019

## **Abstract**

Quantum Computation and Combinatorial Structures

by

Ryan L. Mann

Doctor of Philosophy

University of Technology Sydney

This thesis explores the relationship between quantum computation and combinatorial structures, with the goal of improving our understanding of the complexity of quantum computation. We begin by studying the case when the complexity of combinatorial structures can be used to provide evidence for the hardness of classically simulating quantum computations. To this end, we show that the complexity of evaluating multiplicative-error approximations of Jones polynomials can be used to bound the classical complexity of simulating random quantum computations. We then proceed by studying the contrary case, that is, when do the combinatorial structures allow for an efficient classical simulation of quantum computations? We establish an efficient deterministic approximation algorithm for the Ising model partition function with complex parameters when the interactions and external fields are absolutely bounded close to zero. This provides an efficient classical algorithm for simulating a class of quantum computations with bounded interactions between the qubits.

In the second part of this thesis, we present some independent results on the efficient preparation of Fock states with a high number of photons from a resource of single photons. These Fock states are a fundamental resource in many quantum information protocols.

For my mother.

### **Certificate of Original Authorship**

I, Ryan L. Mann, declare that this thesis is submitted in satisfaction of the requirements for the degree of Doctor of Philosophy, in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

This research is supported by the Australian Government Research Training Program.

Production Note:

Signature removed prior to publication.

Signature of Author

## Acknowledgements

I would like to begin by thanking my supervisors Michael Bremner, Peter Rohde, and Min-Hsiu Hsieh. They have shown me exceptional support, guidance, and patience throughout my PhD. They have greatly influenced the way I conduct research, view physics, mathematics, and computer science, and approach life in general, and for this, I am eternally grateful.

I would also like to thank my colleagues and collaborators for helpful discussions. In particular, I would like to thank Gavin Brennen for his willingness to meet and discuss ideas. I would also like to thank the theory group at Sydney University for allowing me to attend their group meetings and drink their beer. I thank Jonathan Dowling for numerous whiskies.

I have been fortunate to have some very good friends in Sydney. I would first like to thank all my friends at UTSOAC and SURMC for welcome distractions from this thesis. I especially thank Krisztina Katona for her support and patience during the writing of this thesis, and Catherine Pinfold and Llewellyn Kurtz for their generous hospitality. I would also like to thank Natalie, Vicki, and Dennis Harrold for their support during the first few years of my PhD. I owe special thanks to my long-time friends from home.

This thesis would, of course, not have been possible without the support of my family: my mother Sally, to whom this thesis is dedicated, my brother Liam, my uncle Derek, my grandparents Jennifer and Walter and my great aunt Kim, and our close family friends Paul and Barbara.

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>I</b>	<b>Quantum Computation and Combinatorial Structures</b>	<b>4</b>
<b>2</b>	<b>Preliminaries</b>	<b>5</b>
2.1	Computational Complexity Theory . . . . .	5
2.1.1	Asymptotic Notation . . . . .	5
2.1.2	Computational Problems . . . . .	6
2.1.3	Complexity Classifications . . . . .	7
2.1.4	Approximation Algorithms . . . . .	13
2.2	Quantum Computation . . . . .	14
2.2.1	Quantum States . . . . .	14
2.2.2	Quantum Circuits . . . . .	15
2.2.3	Quantum Algorithms . . . . .	16
2.2.4	The Hadamard Test . . . . .	17
<b>3</b>	<b>Combinatorial Structures</b>	<b>19</b>
3.1	The Tutte Polynomial . . . . .	19
3.2	The Jones Polynomial . . . . .	22
3.3	The Ising Model . . . . .	28
<b>4</b>	<b>The Complexity of Random Quantum Computations</b>	<b>30</b>

4.1	Introduction . . . . .	31
4.2	Random Quantum Computations . . . . .	34
4.3	Proof of Theorem 4.6 . . . . .	37
4.4	The Jones Polynomial and Quantum Computing . . . . .	40
4.5	Random Jones Polynomials . . . . .	41
4.6	Parallelisation of Random Braids . . . . .	44
4.7	Conclusion & Outlook . . . . .	45
<b>5</b>	<b>Approximation Algorithms for Complex-Valued Ising Models</b>	<b>47</b>
5.1	Introduction . . . . .	48
5.2	Graph Homomorphism Partition Functions . . . . .	52
5.3	Proof of Lemma 5.9 . . . . .	57
5.4	Proof of Lemma 5.11 . . . . .	61
5.5	Ising Model Partition Functions . . . . .	63
5.6	Quantum Simulation . . . . .	66
5.7	Conclusion & Outlook . . . . .	69
<b>II</b>	<b>Other Results</b>	<b>70</b>
<b>6</b>	<b>Efficient Preparation of Fock States from Single-Photon Sources</b>	<b>71</b>
6.1	Introduction . . . . .	72
6.2	Spontaneous Parametric Down-Conversion with Postselection . . . . .	73
6.3	Single-Shot Linear Optics with Postselection . . . . .	74
6.4	Bootstrapped Linear Optics with Postselection . . . . .	75
6.5	Fusion . . . . .	77
6.5.1	Generalised Fusion Protocol . . . . .	77
6.5.2	Analytic Approximations . . . . .	78
6.5.3	Fusion Schemes . . . . .	81
6.5.4	Hybrid Schemes . . . . .	82

6.6	Fock State Reduction . . . . .	82
6.7	Results . . . . .	83
6.8	Conclusion & Outlook . . . . .	85
7	<b>Conclusion &amp; Outlook</b>	<b>88</b>
	<b>Bibliography</b>	<b>90</b>