



University of Technology Sydney

Faculty of Engineering and Information Technology

**Robust topology optimization with hybrid uncertainties  
using level set methods**

A thesis submitted for the degree of

**Doctor of Philosophy**

**JING ZHENG**

(2019)

**Title of the thesis:**

Robust topology optimization with hybrid uncertainties using level set methods

**Ph.D. student:**

Jing Zheng

E-mail: [Jing.Zheng-3@student.uts.edu.au](mailto:Jing.Zheng-3@student.uts.edu.au)

**Supervisor:**

A/Prof Zhen Luo

E-mail: [zhen.luo@uts.edu.au](mailto:zhen.luo@uts.edu.au)

**Co-Supervisor and joint-supervisor:**

Prof Nong Zhang

E-mail: [nong.zhang@uts.edu.au](mailto:nong.zhang@uts.edu.au)

Prof Chao Jiang

E-mail: [jiangc@hnu.edu.cn](mailto:jiangc@hnu.edu.cn)

**Address:**

School of Mechanical and Mechatronic Engineering

The University of Technology Sydney, Sydney, NSW 2007, Australia

# Certificate of Original Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

This thesis is the result of a research candidature conducted jointly with another University as part of a collaborative degree. This research is supported by the Australian Government Research Training Program.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Student:

Production Note:  
Signature removed prior to publication.

**JING ZHENG**

Date: 17/10/2018

# Acknowledgments

I would like to take this opportunity to express my deep gratitude to all those who helped me throughout my candidature.

First and foremost, I would like to extend my sincere gratitude to my principal supervisor, A/Prof. Z Luo. During the studying of the course and the writing of the thesis, he had contributed greatly by giving useful suggestions and constructive criticism. He devoted a considerable portion of his time to reading my manuscripts and making suggestions for further revisions. Moreover, he gave me many encouragements and other help in my study and life. Also, I would like to express my heartfelt gratitude to my joint supervisor Prof. C Jiang and co-supervisor Prof. N Zhang for their support and guidance. Their outstanding knowledge, intelligence and wisdom have a profound influence on me.

I also wish to express my gratitude to Dr. Hao Li, Dr. Jinglai Wu, and Dr. Zhiliang Huang. They offered me great help and gave me many valuable suggestions. My sincere thanks should also go to my colleagues Bingyu Ni, Xiangyun Long, Zhe Zhang, Zhicheng Liu, Zhonghua Wang, Bochuan Li, Jie Gao, Huipeng Xue, Xinpeng Wei, Jiachang Tang, Haibo Liu, Teng Fang, Xinyu Jia, for their support.

My last and special thanks would go to my beloved family for their loving considerations and great confidence in me all through these years.

Jing Zheng

Sydney, 2018

# Publications and Conference Contributions

## International scientific journal publications

- [1] **J Zheng**, Z Luo, C Jiang, BY Ni, JL Wu. Non-probabilistic reliability-based topology optimization with multidimensional parallelepiped convex model. *Structural and Multidisciplinary Optimization*, 2018, 57(6): 2205-2221.
- [2] **J Zheng**, Z Luo, H Li, C Jiang. Robust topology optimization for cellular composites with hybrid uncertainties. *International Journal for Numerical Methods in Engineering*, 2018, 115(6): 695-713.
- [3] **J Zheng**, Z Luo, C Jiang, JL Wu. Level-set topology optimization for robust design of structures under hybrid uncertainties. *International Journal for Numerical Methods in Engineering*, 2019, 117(5):523-542.
- [4] **J Zheng**, Z Luo, C Jiang, J Gao. Robust topology optimization for concurrent design of dynamic structures under hybrid uncertainties. *Mechanical Systems and Signal Processing*, 2019, 120: 540-559.
- [5] C Jiang, **J Zheng**, X Han. Probability-interval hybrid uncertainty analysis for structures with both aleatory and epistemic uncertainties: a review. *Structural and Multidisciplinary Optimization*, 2018, 57(6): 2485–2502.
- [6] C Jiang, **J Zheng**, BY Ni, X Han. A probabilistic and interval hybrid reliability analysis method for structures with correlated uncertain parameters. *International*

Journal of Computational Methods, 2015, 12(04): 1540006.

- [7] JL Wu, Z Luo, **J Zheng**, C Jiang. Incremental modeling of a new high-order polynomial surrogate model. Applied Mathematical Modelling, 2016, 40(7-8): 4681-4699.
- [8] ZL Huang, C Jiang, YS Zhou, **J Zheng**, XY Long. Reliability-based design optimization for problems with interval distribution parameters. Structural and Multidisciplinary Optimization, 2017, 55(2), 513-528.
- [9] ZL Huang, YS Zhou, C Jiang, **J Zheng**, X Han. Reliability-based multidisciplinary design optimization using incremental shifting vector strategy and its application in electronic product design. Acta Mechanica Sinica, 2018, 34(2): 285-302.

**International conference publications**

- [10] **J Zheng**, Z Luo. Reliability-Based Topology Optimization for Continuum Structures with Non-probabilistic Uncertainty. The 12<sup>th</sup> World Congress of Structural and Multidisciplinary Optimization (WCSMO12). Springer, Germany, 2017: 390-395.
- [11] **J Zheng**, C Jiang, Z Luo. A probability and interval reliability analysis method for structures considering correlation. The 12<sup>th</sup> World Congress on Computational Mechanics (WCCM XII), 24-29 July, 2016, Seoul, Korea. Paper NO. 151724.

# List of Figures

Figure 3-1 A 2D boundary embedded as the zeros level set of a 3D level set function.....	40
Figure 3-2 Shape of CSRBFs.....	45
Figure 3-3 Design domain and boundary condition for the cantilever beam.....	58
Figure 3-4 The evolution process of the cantilever beam.....	60
Figure 3-5 Objective function iteration history of the cantilever beam.....	61
Figure 3-6 Volume constraint iteration history of the cantilever beam.....	61
Figure 3-7 Design domain and boundary condition for the MBB beam.....	61
Figure 3-8 The evolution process of the MBB beam.....	62
Figure 3-9 Objective function iteration history of the MBB beam.....	63
Figure 3-10 Volume constraint iteration history of the MBB beam.....	63
Figure 3-11 Design domain and boundary condition for the Michell-type structure.....	64
Figure 3-12 The evolution process of the Michell-type structure.....	65
Figure 3-13 Objective function iteration history of the Michell-type structure.....	66
Figure 3-14 Volume constraint iteration history of the Michell-type structure.....	66
Figure 4-1 Multi-index set $\chi_1$ in a two dimensional space.....	72
Figure 4-2 Construction of the sparse grid.....	77
Figure 5-1 Design domain and boundary condition for example 1.....	104
Figure 5-2 Optimized results for deterministic and robust designs for example 1.....	105
Figure 5-3 Design domain and boundary condition for example 2.....	107
Figure 5-4 Optimized results for deterministic and robust designs for example 2.....	108
Figure 5-5 Design domain and boundary condition for example 3.....	110
Figure 5-6 Optimized results for deterministic and robust designs for example 3.....	111
Figure 5-7 Design domain and boundary condition for example 1.....	127
Figure 5-8 Optimized results for deterministic and robust designs for example 1.....	128
Figure 5-9 Convergent histories of example 1.....	129
Figure 5-10 Design domain and boundary condition for example 2.....	130
Figure 5-11 Optimized results for deterministic and robust designs for example 2.....	131
Figure 5-12 Convergent histories of example 2.....	131
Figure 5-13 Design domain and boundary condition for example 3.....	133
Figure 5-14 Optimized results for deterministic and robust designs for example 3.....	134
Figure 5-15 Convergent histories of example 3.....	134



Figure 6-1 The schematic diagram of the homogenization method..... 138  
Figure 6-2 The schematic diagram of the multiscale system ..... 144  
Figure 6-3 The flowchart of the proposed method..... 156  
Figure 6-4 Design domain and boundary condition for example 1 ..... 158  
Figure 6-5 Optimized results in macro and micro scales for example 1 ..... 159  
Figure 6-6 Convergent histories for example 1 ..... 161  
Figure 6-7 Design domain and boundary condition for example 2 ..... 162  
Figure 6-8 Optimized results in macro and micro scales for example 2 ..... 164  
Figure 6-9 Convergent histories for example 2 ..... 165  
Figure 6-10 Design domain and boundary condition for example 3 ..... 166  
Figure 6-11 Optimized results in macro and micro scales for example 3 ..... 168  
Figure 6-12 Convergent histories for example 3 ..... 169  
Figure 6-13 The flowchart of the proposed method..... 182  
Figure 6-14 Design domain and boundary condition for example 1 ..... 183  
Figure 6-15 Optimized results in macro and micro scales for example 1 ..... 185  
Figure 6-16 Convergent histories for example 1 ..... 186  
Figure 6-17 Design domain and boundary condition for example 2 ..... 188  
Figure 6-18 Optimized results in macro and micro scales for example 2 ..... 190  
Figure 6-19 Convergent histories for example 2 ..... 190  
Figure 6-20 Design domain and boundary condition for example 3 ..... 192  
Figure 6-21 Optimized results in macro and micro scales for example 3 ..... 194  
Figure 6-22 Convergent histories for example 3 ..... 194

# List of Tables

Table 4-1 Nodes and weights of the Gauss-Hermite integration of one dimensional.....	76
Table 4-2 Multidimensional integration points .....	76
Table 4-3 Distribution types and parameter of variables .....	84
Table 4-4 Results of the performance function with hybrid uncertainties .....	86
Table 4-5 Distribution types and parameter of variables .....	94
Table 4-6 Results of the performance function with hybrid uncertainties .....	95
Table 5-1 Robust objective function of optimized designs for example 1.....	107
Table 5-2 Robust objective function of optimized designs for example 2.....	109
Table 5-3 Robust objective function of optimized designs for example 3.....	112
Table 5-4 Robust objective function of optimized designs for example 1.....	130
Table 5-5 Robust objective function of optimized designs for example 2.....	132
Table 5-6 Robust objective function of optimized designs for example 3.....	135
Table 6-1 Robust objective function of optimized designs for example 1.....	162
Table 6-2 Robust objective function of optimized designs for example 2.....	166
Table 6-3 Robust objective function of optimized designs for example 3.....	169
Table 6-4 Robust objective function of optimized designs for example 1.....	188
Table 6-5 Robust objective function of optimized designs for example 2.....	191
Table 6-6 Robust objective function of optimized designs for example 3.....	195

# Abbreviations

SIMP	Solid Isotropic Material with Penalization
ESO	Evolutionary Structural Optimization
BESO	Bi-directional Evolutionary Structural Optimization
LSM	Level Set Method
LSF	Level Set Function
PLSM	Parameterized Level Set Method
RBF	Radial Basis Function
GSRBF	Globally Supported Radial Basis Function
CSRBF	Compactly Supported Radial Basis Function
HJ-PDE	Hamilton-Jacobian Partial Derivative Equation
ODE	Ordinary Differential Equation
RBDO	Reliability-based Design and Optimization
RBTO	Reliability-based Topology Optimization
RTO	Robust Topology Optimization
KL	Karhunen-Loeve expansion
PCCI	Polynomial Chaos-Chebyshev Interval
SGNI	Sparse Grid Numerical Integration
HUDR	Hybrid Univariate Dimension Reduction
HDR	Hybrid Dimension Reduction
MCS	Monte Carlo Simulation
MR	Model Reduction
QSRV	Quasi-Static Ritz Vector
OC	Optimality Criteria
MMA	Method of Moving Asymptotes

# Abstract

Topology optimization has been experiencing great popularity in a diversity of engineering areas. Parameters involved in most topology optimization problems are under deterministic assumptions. However, in practical applications, uncertainties are inevitably existing due to various reasons, such as manufacturing tolerances, loads, material properties, geometric dimensions and boundary conditions, as well as aging within the whole life circle of structural service. In particular, in the conceptual design by the topology optimization, more reliable results can be expected if uncertainties are taken into account, as the performance of a topological design varies with the uncertainties. In this setting, the deterministic assumption may result in a design that is unfeasible. Hence, it is of great importance to incorporate uncertainties into the topology optimization to account for unavoidable variations.

Probability models have been widely used to describe the uncertainties of parameters in structures, which in general require a sufficient number of samples to completely construct the distributions. However, in engineering, it is very difficult to gain complete information to accurately describe the probability distributions, while it is relatively easy to get their interval bounds for limited information. In practice, it is recognized that a structure often involves uncertainties of multiple sources, in which some uncertain parameters can be regarded as random variables and the others can be modelled as interval variables. Hence, a design problem under random-interval hybrid uncertainties consists of both the aleatory and epistemic uncertainties at the same time. In this thesis, the hybrid uncertainties will be

considered in topology optimization problems to achieve robust designs. The detailed contents are outlined as follows:

Chapter 1 provides a brief introduction for this research. Chapter 2 gives the background and a literature review. Chapter 3 describes the details of a parametric level set method (PLSM) based on compactly supported radial basis functions (CSRBFs). Some efficient random-interval hybrid uncertain analysis methods are developed in Chapter 4. In the following Chapters, the uncertainty analysis methods are then employed to formulate robust topology optimizations for structures with hybrid uncertainties, as follows:

In Chapter 5, robust topology optimization methods based on orthogonal polynomials are developed for both static and dynamic continuum structures with hybrid uncertainties. In Chapter 6, robust topology optimization methods based on dimension reduction methods are developed for the multi-scale design of static and dynamic structures with hybrid uncertainties.

Finally, conclusions and prospects are given in Chapter 7.

**Key words:** Topology optimization; level set method; hybrid uncertainty; orthogonal polynomial; dimension reduction method.

# Contents

<b>Certificate of Original Authorship .....</b>	<b>I</b>
<b>Acknowledgments .....</b>	<b>II</b>
<b>Publications and Conference Contributions .....</b>	<b>III</b>
<b>List of Figures .....</b>	<b>VI</b>
<b>List of Tables .....</b>	<b>VIII</b>
<b>Abbreviations.....</b>	<b>IX</b>
<b>Abstract.....</b>	<b>X</b>
<b>Chapter 1 Introduction.....</b>	<b>4</b>
1.1 Overview of the project.....	4
1.2 Research contribution.....	7
1.3 Outline of the thesis.....	8
<b>Chapter 2 Background and literature review .....</b>	<b>10</b>
2.1 Literature review of topology optimization.....	10
2.1.1 Topology optimization methods .....	10
2.1.2 Topological optimization of microstructural metamaterials .....	14
2.1.3 Multi-scale design of cellular composites .....	17
2.2 Literature review of uncertain analysis .....	19
2.2.1 Random uncertain analysis .....	19
2.2.2 Interval uncertain analysis .....	22
2.2.3 Random and interval hybrid uncertain analysis.....	26
2.3 Literature review of uncertain topology optimization.....	33
<b>Chapter 3 A level set method for topology optimization .....</b>	<b>39</b>
3.1 The level set method.....	39
3.1.1 Representation of the boundary via LSM.....	39
3.1.2 Hamilton-Jacobi Equation .....	40

3.2 The parametric level set method.....	42
3.2.1 Compactly Supported Radial Basis Functions .....	42
3.2.2 Parameterization of the level set function.....	46
3.3 Topology optimization based on the PLSM .....	48
3.3.1 Topology optimization formulation .....	48
3.3.2 Sensitivity analysis .....	50
3.4 Optimization method.....	53
3.4.1 Optimality Criteria Method .....	53
3.4.2 The Method of Moving Asymptotes .....	56
3.5 Numerical examples .....	58
<b>Chapter 4 Random-interval hybrid uncertainty analysis .....</b>	<b>68</b>
4.1 Hybrid uncertainty analysis method based on orthogonal polynomials.....	69
4.1.1 A hyperbolic Polynomial Chaos-Chebyshev Interval method (hPCCI) .....	69
4.1.2 An improved Polynomial Chaos-Chebyshev Interval method (iPCCI).....	75
4.1.3 Numerical example .....	84
4.2 Hybrid uncertainty analysis method based on dimension reduction method .....	86
4.2.1 A Hybrid univariate dimension reduction method (HUDR).....	87
4.2.2 A Hybrid dimension reduction method (HDR).....	90
4.2.3 Numerical example .....	94
<b>Chapter 5 Robust topology optimization for structures based on orthogonal polynomials .....</b>	<b>96</b>
5.1 Robust topology optimization for static structures with hybrid uncertainties.....	96
5.1.1 Random field approximation by KL expansion.....	97
5.1.2 Robust topology optimization.....	100
5.1.3 Numerical examples .....	104
5.2 Robust topology optimization for dynamic structures with hybrid uncertainties .....	112
5.2.1 Dynamic topology optimization .....	113
5.2.2 Robust dynamic topology optimization.....	124
5.2.3 Numerical examples .....	127

5.3 Summary .....	135
<b>Chapter 6 Robust topology optimization for cellular composites based on dimension reduction methods .....</b>	<b>137</b>
6.1 Homogenization method .....	137
6.2 Parametric level sets for the multi-scale system .....	143
6.3 Robust topology optimization for multi-scale design of static structures with hybrid uncertainties.....	147
6.3.1 Deterministic multi-scale topology optimization .....	147
6.3.2 Robust topology optimization.....	152
6.3.3 Numerical examples .....	157
6.4 Robust topology optimization for multi-scale design of dynamic structures with hybrid uncertainties .....	170
6.4.1 Deterministic multi-scale dynamic topology optimization.....	170
6.4.2 Robust dynamic topology optimization.....	177
6.4.3 Numerical examples .....	183
6.5 Summary .....	196
<b>Chapter 7 Summary and prospect.....</b>	<b>197</b>
7.1 Summary .....	197
7.2 Perspective for future work .....	198
<b>References .....</b>	<b>201</b>