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Stochastic Modelling of New Phenomena in Financial Markets

by

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Certificate of Authorship and Originality

I certify that the work in this dissertation has not been previously submitted for a degree nor has it been submitted as a part of the requirements for other degree except as fully acknowledged within the text.

I also certify that this dissertation has been written by me. Any help that I have received in my research and in the preparation of the thesis itself has been fully acknowledged. In addition, I certify that all information sources and literature used are quoted in the thesis.

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ABSTRACT

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The Global Financial Crisis (GFC) has revealed a number of new phenomena in financial markets, to which stochastic models have to be adapted. This dissertation presents two new methodologies, one for modeling the "basis spread", and the other for "rough volatility". The former gained prominence during the GFC and continues to persist, while the latter has become increasingly evident since 2014.

The dissertation commences with a study of the interest rate market. Since 2008, in this market we have observed "basis spreads" added to one side of the single-currency floating-for-floating swaps. The persistence of these spreads indicates that the market is pricing a risk that is not captured by existing models. These risks driving the spreads are closely related to the risks affecting the funding of banks participating in benchmark interest rate panels, specifically "roll-over" risk, this being the risk of not being able to refinance borrowing at the benchmark interest rate. We explicitly model funding liquidity and credit risk, as these are the two components of roll-over risk, developing first a model framework and then considering a specific instance of this framework based on affine term structure models.

Subsequently, another specific instance of the model of roll-over risk is constructed using polynomial processes. Instead of pricing options through closed-form expressions for conditional moments with respect to observed process, the price of a zero-coupon bond is expressed as a polynomial of a finite degree in the sense of Cheng & Tehranchi (2015). A formula for discrete-tenor benchmark interest rates (e.g., LIBOR) under roll-over risk is constructed, which depends on the quotient of polynomial processes. It is shown how such a model can be calibrated to market data for the discount factor bootstrapped from

the overnight index swap (OIS) rate curve.

This is followed by a chapter in which a numerical method for the valuation of financial derivatives with a two-dimensional underlying risk is considered, in particular as applied to the problem of pricing spread options. As is common, analytically closed-form solutions for pricing these payoffs are unavailable, and numerical pricing methods turn out to be non-trivial. We price spread options in a model where asset prices are driven by a multivariate normal inverse Gaussian (NIG) process. We consider a pricing problem in the fixed-income market, specifically, on cross-currency interest rate spreads and on LIBOR-OIS spreads.

The final contribution in this dissertation tackles regime switching in a rough-volatility Heston model, which incorporates two important features. The regime switching is motivated by fundamental economic changes, and a Markov chain to model the switches in the long-term mean of the volatility is proposed. The rough behaviour is a more local property and is motivated by the stylized fact that volatility is less regular than a standard Brownian motion. The instantaneous volatility process is endowed with a kernel that induces rough behaviour in the model. Pricing formulae are derived and implemented for call and put options using the Fourier-inversion formula of Gil-Pelaez (1951).

Dedication

To my beloved late great-grandmother Paulina Kalomho Wanghondeli, 1907-2010

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Mesias Alfeus
Sydney, Australia, 2019.

List of PhD Activities

Journal Papers

J-1. **Alfeus, M., Overbeck, L. and Schlögl, E.**. "Regime Switching Rough Heston Model", *The Journal of Futures Markets, accepted for publication, forthcoming, May 2019*.

Team Challenge

T-1. **Alfeus, M., Korula, F., Soane, A., Lopes, M., and McWalter, T.**. "Rough Volatility". In Taylor, D. and Macrine, A., editors. *Financial Mathematics Team Challenge, number 5 in A collection of the five reports from the 2017 Financial Mathematics Team Challenge*. Pages 182-210. ACQUFRR, University of Cape Town, 18-28 July 2017.

Conference Presentations

C-1. **Alfeus, M.**, and Overbeck, L. and Schlögl, E. "Regime Switching Rough Heston Model", *Quantitative Methods in Finance (QMF 2018)*, QFRC, Sydney, Australia, Dec 11-14, 2018.

C-2. **Alfeus, M.**, and Overbeck, L. and Schlögl, E. "Regime Switching Rough Heston Model", *10th World Congress of The Bachelier Finance Society*, Trinity College Dublin, Ireland, July 16-20, 2018.

C-3 **Alfeus, M.**, and Schlögl, E. "On Numerical methods for Spread Options", *AMSI OPTIMISE 2018 conference*, University of Melbourne, Australia, June 18-22, 2018. **Recipient of AMSI conference grant.**

C-4. **Alfeus, M.**, and Overbeck, L. and Schlögl, E. "Regime Switching Rough Heston Model", *XIX Workshop on Quantitative Finance(QFW 2018)*, University of Rome Tre, Italy, January 24-26, 2018. **Winner of the Young Investigator Training Program Prize (YITP).**

C-5. **Alfeus, M.**, and Schlögl, E. "On Numerical methods for Spread Options", *Quantitative Methods in Finance (QMF 2017)*,, QFRC, Sydney, Australia, Dec 11-14, 2017.

C-6. **Alfeus, M.**, and Grasselli, M. and Schlögl, E. "A Consistent Stochastic Model of the Term Structure of Interest Rates for Multiple Tenors", *UTS Business School PhD conference*, UTS Business School, Sydney, Australia, November 14, 2017.

C-7. **Alfeus, M.**, and Grasselli, M. and Schlögl, E. "A Consistent Stochastic Model of the Term Structure of Interest Rates for Multiple Tenors", *Mathematics in Finance(MIF 2017)*, Kruger National Park, South Africa, August 08-12, 2017.

Invited Seminars

S-1. **Alfeus, M.**, and Grasselli, M. and Schlögl, E. "A Consistent Stochastic Model of the Term Structure of Interest Rates for Multiple Tenors", *Reserve Bank of Australia, research seminars series*, 65 Martin Place, Sydney NSW 2000, Australia, February 19, 2019.

S-2. **Alfeus, M.**, and Grasselli, M. and Schlögl, E. "A Consistent Stochastic Model of the Term Structure of Interest Rates for Multiple Tenors", *Sydney Financial Mathematics Workshop (SFMW), Commonwealth Bank*, 201 Sussex Street, Sydney NSW 2000, Australia, October 23, 2018.

S-3. **Alfeus, M.**, and Schlögl, E. "The Term structure of Roll-Over Risk: A Gentle Model Calibration Approach", *Analysis of Variations, Optimal Control, and Applications to Design and Operations (AVOCADO)*, University of Newcastle, Australia, August 24-26, 2018.

S-4. **Alfeus, M.**, and Grasselli, M. and Schlögl, E. "A Consistent Stochastic Model of the Term Structure of Interest Rates for Multiple Tenors", *Centre of Financial Mathematics (CFM), University of Wollongong*, Wollongong, Australia, May 10, 2018.

S-5. **Alfeus, M.**, and Overbeck, L. and Schlögl, E. "Regime Switching Rough Heston Model", *Stochastic methods in quantitative finance and statistics, University of Technology Sydney (UTS)*, Sydney, April 11, 2018.

S-6. **Alfeus, M.**, and Overbeck, L. and Schlögl, E. "Regime Switching Rough Heston Model", *Probability Group, Dipartimento di Matematica, University of Padua*, Padua, Italy, March 11, 2018.

S-7. **Alfeus, M.**, and Schlögl, E. "On Numerical methods for Spread Options", *Department of Mathematics, University of Namibia*, Windhoek, Namibia, August 04, 2017.

S-8. **Alfeus, M.**, and Grasselli, M. and Schlögl, E. "A Consistent Stochastic Model of the Term Structure of Interest Rates for Multiple Tenors", *Reserve Bank of Namibia*, Windhoek, Namibia, August 03, 2017. **Received Bank of Namibia grant**.

Contents

List of Figures

List of Tables

Abbreviation

ASA - Adaptive Simulated Annealing

GFC - Global financial Crisis

GBM - Geometric Brownian Motion

2-D - Two-dimensional

fBm - fractional Brownian motion

FFTW - Fastest Fourier Transform in the West

RFV - Rough fractional Volatility

FRDE - Fractional Ricatti Differential Equation

MODE - Matrix Ordinary Differential Equation

IRS - Interest Rate Swaps

OIS - Overnight Index Swaps

BS - Basis Swaps

CDS - Credit Default Swaps

FRA - Forward Rate Agreement

LIBOR - London Interbank Offered Rate

EONIA - Euro Over Night Index Average

FF - Fed Funds

IBOR - Interbank Offered Rate

CVA - Credit Valuation Adjustment

FVA - Funding Valuation Adjustment

DVA - Debt Valuation Adjustment

RMSE - Root Means Square error

OTC - Over-The-Counter

CSA - Credit Support Annex

ISDA - International Swaps and Derivatives Association

H - Hurst parameter

CBI - Continuously Branching process with Immigration.

OU - Ornstein–Uhlenbeck

CIR - Cox–Ingersoll–Ross

FFT - Fast Fourier Transform

USD - United States Dollar

US - United States

MC - Monte Carlo

1m/3m : 1-month vs 3-month basis

3m/6m : 3-month vs 6-month basis

Nomenclature and Notation

Throughout the thesis we adopt the following convention.

$q = 0.6$ a scaling parameter.

$(.)^T$ denotes the transpose operation.

$\langle \cdot \rangle$ is the inner product.

$x \vee y = \max\{x, y\}$

$\mathbb{Q}$ is the risk neutral measure.

$\mathbb{Q}^T$ is the T-forward measure.

$\mathbb{E}_t^{\mathbb{Q}}[\cdot]$ is the short hand for $\mathbb{E}_t^{\mathbb{Q}}[\cdot \mid \mathcal{F}_t]$ risk neutral measure.

$\mathbf{1}$ denotes an indicator function.

I_n is the identity matrix of dimension $n \times n$.

$T_n^{(y)}$ denotes the tenor structure corresponds to a y-month frequency with n payments.

0_n is the zero matrix of dimension $n \times n$.

$\mathbb{R}$, $\mathbb{R}^*$ denote the field of real numbers, and the set of positive reals, respectively.