Financial Markets with Multidimensional Uncertainty

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Statement of Originality

I certify that this dissertation titled ‘Financial Markets with Multidimensional Uncertainty’ has not previously been submitted for a degree nor has it been submitted as part of requirement for a degree except as fully acknowledged within the text.

I also certify that the dissertation has been written by me. Any help that I have received in my research and the preparation of the dissertation itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the dissertation.

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1. Aliyev, N. and He, X. (2018a), ‘Ambiguous price formation’, Working paper, UTS Business School is based on Chapter 2 of this dissertation and has been presented in the following conferences:
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Abstract

The stability in financial markets is important to promote economic growth. Due to its fundamental importance, the causes of market instability are of broad interest. The purpose of this dissertation is to propose plausible explanations of financial market phenomena related to market stability such as prices, liquidity, volatility, information value, welfare, and market efficiency. The premise in the analysis is that uncertainty in financial markets is multidimensional and information structure is complex. To be more precise, in modern financial markets, forming consistent beliefs about the fundamental values of securities, the composition of market participants, and other market characteristics are complex and uncertain. On the basis of this premise, this dissertation investigates the trading decisions, order sizes, liquidity, security prices, information value, welfare, and market efficiency to shed light on the causes of financial market instability (fragility) and makes a number of empirical predictions some of which provide explanations for results that have been reported in the empirical market microstructure literature and others are yet to be tested. The dissertation also identifies conditions under which markets are vulnerable to instability and thus also has important policy implications.

The first phenomenon investigated in this dissertation is sudden liquidity deteriorations and improvements in financial markets. Chapter 2 presents a security price formation model with ambiguous liquidity provision. The model provides a unified and parsimonious framework to explain the empirically documented features that market liquidity can suddenly deteriorate during market crashes and improve during trading reforms. Consequently, ambiguity in liquidity provision can increase the value of information and social welfare. The ambiguous price formation model helps to understand (i) the dynamics of ambiguity, (ii) the determinants of time-varying ambiguity aversion of liquidity providers, (iii) the price and liquidity dynamics during various order flow patterns, and (iv) the effect of trade size on security prices during ambiguous market episodes.

Chapter 3 develops a model in which traders face uncertainty about the composition of informed and uninformed traders (composition uncertainty) to investigate the “crowded-trade” problem (not being able to know how many others are taking
the same position) in financial markets. This chapter characterizes the equilibrium in the information market where both types of traders are affected by composition uncertainty and in the financial market where only uninformed traders are affected, leading the uninformed traders to be disadvantaged in the face of composition uncertainty. This composition uncertainty distorts traders’ information acquisition, demands, and perceived equity premium, resulting in undervalued (resp. overvalued) stock when traders are sufficiently (resp. insufficiently) uncertainty averse. The model helps to understand a linkage between liquidity and asset prices, proposes plausible explanations for large price swings, and demonstrates how regulations to enhance market efficiency may not work when the composition of traders is uncertain.

Chapter 4 shows that when market participants learn about the level of adverse selection from order flow, a large order imbalance can be destabilizing, causing sharp price movements and evaporation of liquidity, as it signals high “toxicity” (adverse selection). While such effect is consistent with the practitioner view that order flow is informative about toxicity, it contrasts with standard microstructure models in which the level of adverse selection is assumed to be known and thus order imbalance improves liquidity by revealing private information. The model helps to understand when markets are most susceptible to imbalance-induced instability and the dynamic process of how markets digest order imbalance.

Chapter 5 examines the implications of the true complexity of real-world information on market efficiency. Using the literature of decision theories and information sciences, Chapter 5 discusses how accounting different attributes of information can unify two controversial views, efficient markets hypothesis and behavioral finance. The main thesis advanced is that the roots of behavioral anomalies are the imprecision and reliability of information. By exemplifying different decision scenarios, Chapter 5 argues that the decision making is rational with precise and reliable information, whereas becomes more behavioral in nature as the information becomes more imprecise and unreliable.

Overall, the results of this dissertation suggest that multiple dimensions of uncertainty formalized in different languages can illuminate on various aspects of market stability that we otherwise label as anomalies and offer a promising middle ground between efficient markets hypothesis and behavioral finance.
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