Authentic numeracy contexts for proportional reasoning – the case of the seven summits

Shelley Dole
University of the Sunshine Coast
<sdole@usc.edu.au>

Merrilyn Goos
The University of Queensland
<m.goos@uq.edu.au>

Annette Hilton
University of Technology, Sydney
<Annette.Hilton@uts.edu.au>

Geoff Hilton
The University of Queensland
<g.hilton@uq.edu.au>

This paper presents a case study of one primary school teacher's journey of realisation about the importance of proportional reasoning for numeracy. Through immersing students in a rich numeracy investigation to meaningfully compare the world's tallest mountains, this teacher reflects on authentic contexts and hands-on experiences for promoting and enhancing students' multiplicative thinking. This study included analysis of interview data, classroom observations and student artefacts against a rich model of numeracy that served to emphasise the power of meaningful contexts for promoting multiplicative comparison.

Introduction

Proportional reasoning is one of the most commonly applied mathematics concepts in the real world (Lanius & Williams, 2003). Adjusting measures of ingredients in a recipe, adding sugar for the perfect cup of coffee, estimating the time to travel when found in a traffic jam, choosing the right food storage container when saving left-overs, calculating percentage discounts on sale items, are some everyday tasks that require proportional reasoning. As proportional reasoning is required in so many everyday situations, it is essential to numeracy (Dole, Goos, Hilton & Hilton, 2015). However, students' difficulties with ratio and proportion and proportional reasoning tasks are well-documented (e.g., Lamon, 2007).

In the absence of knowledge of ways to promote proportional reasoning, teachers may revert to skill-based approaches that may serve to hamper students' proportional reasoning development and capacity to use proportional reasoning in complex and unfamiliar situations. Tasks requiring proportional reasoning are a continual stumbling block for so many students in many areas of the curriculum, which suggests the need for a broad-spectrum, multi-pronged strategy for action.

Theoretical Framework

The theoretical framework that guided the research reported in this paper draws from two fields of mathematics education research: (1) proportional reasoning, and (2) numeracy.

The essence of proportional reasoning is an awareness of how two quantities are related in a multiplicative sense. The American Association for the Advancement of Science (AAAS) (2001) Atlas of Scientific Literacy identified two key components of proportional reasoning: Ratios and Proportion (parts and wholes, descriptions and comparisons, and computation) and Describing Change (related changes, kinds of change, and invariance). Lamon (2007) outlined central core ideas for proportional reasoning as rational number interpretation, measurement, quantities and covariation, relative thinking, unitizing, sharing and comparing, and reasoning up and down. These two sources highlight the encompassing nature of proportional reasoning and the fact that it is more extensive than simple rules or calculation procedures, and certainly more than promoting multiplication as repeated addition. This theoretical framework has underpinned the design and modification of tools for assessing middle school students' proportional reasoning (see Dole, Clarke, Wright & Hilton (2012); Hilton, Hilton, Dole & Goos, 2016).

A rich model of numeracy has been proposed by Goos (2007) and elaborated elsewhere (e.g., see Goos, Geiger & Dole, 2010). The model highlights five elements of numeracy as comprising mathematics knowledge, use of tools, positive dispositions, a critical orientation, and grounded in context. The model has been found to support teachers in designing rich learning tasks to promote numeracy (Goos, Geiger & Dole, 2013).

Drawing on this theoretical framework, this project aimed to answer the following research question: to what extent can a rich model of numeracy and a broad conceptualisation of proportional reasoning support teachers in building curriculum knowledge for proportional reasoning?

Design and Approach

This paper reports on a single case study of a teacher who participated in a large project that involved middle school teachers from five school clusters over an extended period of three years. In this project, we designed a professional development (PD) program to build teachers' awareness of the pervasiveness of proportional reasoning throughout the curriculum. We drew upon the Loucks-Horsley, Love, Stiles, Mundry and Hewson (2003) framework to guide our PD approach. Teachers were required to devise and trial teaching sequences tailored to their own school context based on ideas and suggestions presented at the PD. The researchers visited project teachers' classrooms in-between the PD sessions and offered support, advice, and encouragement for their designed teaching sequences.

The case study reported here draws from interview data (ID), classroom observations (CO) and student artefacts (SA) to describe one teacher's journey of developing awareness of the pervasiveness of proportional reasoning and how engaging learning experiences can support all learners in developing proportional reasoning capabilities.

Results and Discussion

Luke (pseudonym) is a teacher of a composite upper primary school class of 27 students. The school is located in a rural community. Prior to commencement in this project, Luke had planned to teach a unit based around the seven summits, drawing on his personal interest in mountain climbing. He commenced this unit with students "undertaking some basic mapping and activities involving coordinates". Initially he felt that his students had a "pretty good understanding of how to use scale, but ratio, they didn't understand" (ID). Luke elaborated that he had provided students with some mathematical exercises where the scale was indicated as 1cm:1km. The students successfully completed conversion exercises to determine the length between particular places based on this scale. Luke's comment was in relation to students' conceptualisation of magnitude of the scale in which they were working. This was evidenced when he referred to a map of Australia and Oceania that was located on the classroom wall. The scale was presented as 1:15 000 000 (CO). Luke reflected on how he attempted to make this scale meaningful to the students: "I explained to them in a very poor way, that according to the scale on that map that Australia is 15 000 000 times bigger than the image of it on the map. Of course no one can visualize that." At the end of the lesson, Luke was left pondering how he might be able to assist students to "visualise" the ratio.

Luke's next (Art) lesson focused on scale drawings of the human body. At the beginning of the lesson, Luke used the word "proportion" and drew students' attention to the structure of the human body. Students paired with a partner and compared the length of their arms, noting where their arms finished, and whether the arms were longer or shorter than other parts of the body. They were then instructed to sit with their partner and draw each other as life-like as they could. Many students expressed frustration with their drawing as the "proportions were wrong" (CO). At the end of the lesson, the drawings were in a rather crude form. Noting students' frustration with their drawings, Luke asked the students to measure the height of the person on their drawing and to make a calculation of the actual size of the person. Students readily determined that the size of the paper was approximately 20 cm. One student stated that the picture would need to be enlarged 20 times to be lifesize. Many other students readily agreed until there was growing realisation that "twenty times twenty - they're not that big" was not an appropriate scale-factor. The mathematical behaviour exhibited by the students was exciting to Luke: "they were estimating a ratio, then calculating it and then refining it…Some kids in my class are not into estimating at all, they won't do it, they just feel that there is too much room for going wrong" (ID). Students then began to spontaneously engage in undertaking multiplicative comparisons: "And from there we looked at their drawings and they actually worked out just looking at the height from head to toe, they worked out an actual correct scale for that drawing. We picked one part of the body and that part of the body was 1cm on the paper, so then it must therefore equal a certain amount in real life" (ID). There was a new sense of industry in the classroom from this point. Students used rulers, tape measures and calculators to take measures of body parts and to then draw them on the page. In efforts to increase the realism of their drawings, many students were seen to sketch and then to erase sections of their work, and then to redraw elements after making further measurements or observations of their partner (CO). Luke reflected on how students would measure each other's noses and then compare this length to the nose drawn on the paper. They quickly saw "for example, the arms might be in proportion but then the nose was almost as big as an arm in real life." Luke recounted how he saw students "slapping their heads" and exclaiming "oh no, the eyes would be this big in real life". Luke described how he saw students taking measurements of different parts of their partner and selecting a scale of 1 to 7. Luke noted the pride in which the students viewed their second sketch compared to their first: "they compared their drawings and talked about their first sketch against their refined drawing and they were quite proud. They were telling me how terrible they are at drawing, but it was almost like the scale and ratio had given them a formula for drawing more accurately." The students were very keen to have their refined drawings displayed around the room.

It was from this experience that Luke directed the learning to the end-goal he had from the start: the seven summits. (The seven summits are the highest mountain peaks in each of the seven continents and summiting all of these mountains is considered an aspirational mountaineering challenge). Luke found that the students had little trouble in representing the mountains to scale. After introducing and discussing the seven summits (and students exploring further via the internet), Luke instructed students to create a triangle from an A4 sheet of paper that would represent Mt Everest. The students measured the height of the triangle as 18.5 cm. "Then they got the height and divided it by the scale to give them the measurement of the height of each mountain. They reasoned that if Mt Everest at 8848 m scaled to 18.5cm, then the scale was 1 cm: 478 m." The result was a graphical representation of the seven summits, each mountain a different coloured triangle, all neatly line up as a sequence of triangles of descending heights.

Luke could see the rich numeracy experiences that he had created for the students and how the activities built on one another to continue to fuel students' interests and learning:

> "yeah so initially it started out as our SOSE and Science, but then health because we started talking about pulmonary oedema and hypoxia the kids had no understanding of what these meant but just this week the kids finished off a dictionary of all of the different mountaineering terminology, equipment and the conditions or ailments that affect them. So it ended up going into our health curriculum because the kids would then argue about whether hypoxia and pulmonary oedema were diseases, because you can't catch

it, so how are they diseases? So it's been a really good term, when I first walked in and told them that we were going to be doing it for a term, none of them cared."

In analysing the lesson sequence from a numeracy perspective, we can see the richness of the experiences provided to these upper primary school students. The sequence was grounded in *context*, with Luke's personal interest in mountain climbing fuelling and generating continued student interest in natural phenomena. The students drew upon and applied their *mathematical knowledge* to the context, making mathematical estimations and reasoning and justifying their calculations. They used *tools*, not only via the use of measuring instruments and calculating devices, but also through the creation of the visual representational tool of the seven summits. They were developing positive *dispositions*. They were clearly enjoying the learning in which they were engaged. They also took risks in calculating and estimating and sharing their ideas, rather than seeking confirmation from the teacher at every step. They were developing a *critical orientation*, not only through their growing awareness of health issues and mountain climbing, but on reviewing the reasonableness of their results. This lesson sequence aligns with all the elements of the rich model of numeracy proposed by Goos (2007).

The lesson sequence was also a multi-directional approach to developing ratio and scale and proportional reasoning. The purpose for scale and ratio emanated from the task, and all students were seen to build their confidence in relation to dealing with scale and large numbers. When discussing their mountain representation pictures, the students would readily discuss how they approached the calculations and could confidently discuss the magnitude of the mountains. Of most interest was the complex scale factor of 1 cm : 478 m that was confidently discussed by all students (CO).

Concluding comments

Multiplicative thinking was fundamental to this lesson sequence. The sequencing of the learning experiences contributed to students developing multiplicative thinking in context. This case study has presented one teachers' journey of realisation about the power of a multi-dimensional approach to proportional reasoning through a rich numeracy investigation. With the end product in mind - the seven summits - the teacher did not revert to a skill and drill lesson of scale and ratio. In fact, early lessons of this type were regarded to be of minimal value for the end-goal: an appreciation of the size of the largest mountains in each continent. This case study serves to remind us of the value of non-sequential approaches to developing multiplicative thinking and proportional reasoning, through learning experiences that are inclusive of all learners.

References

American Association for the Advancement of Science (AAAS). (2001). Atlas of Science Literacy: Project 2061. AAAS.

Dole, S., Clarke, D., Wright, T., & Hilton, G. (2012). Students' proportional reasoning in mathematics and science. In T. Tso (Ed.) Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education (Vol. 2, pp. 195-202). Taipei, Taiwan, PME.

Goos, M. (2007).Developing numeracy in the learning areas (middle years). Keynote address delivered at the South Australian Literacy and Numeracy Expo, Adelaide.

Goos, M., Geiger, V., & Dole, S. (2013). Designing rich numeracy tasks. In C. Margolinas (Ed.), Task Design in Mathematics Education (Proceedings of the 22nd annual conference of the International Commission on Mathematical Instruction, Oxford, pp. 589-598). ICME.

Goos, M., Geiger, V., & Dole, S. (2010). Auditing the Numeracy Demands of the Middle Years Curriculum. In L. Sparrow, B. Kissane & C. Hurst (Eds.), Shaping the Future of Mathematics Education (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia (pp. 210-217). Fremantle, Australia: MERGA.

Hilton, A., Hilton, G., Dole, S., & Goos, M. (2016). Promoting students' proportional reasoning skills through an ongoing professional development program for teachers. Educational Studies in Mathematics, 92(2), 193-219.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In F. K. Lester Jr. (Ed.), Second handbook of research on mathematics teaching and learning (pp. 629-668). Charlotte, NC: Information Age Publishing.

Lanius, C. S., & Williams, S. E. (2003). Proportionality: A unifying theme for the middle grades. Mathematics Teaching in the Middle School, 8(8), 392-396.

Loucks-Horsley, S., Stiles, K. E., Mundry, S., Love, N., & Hewson, P. W. (2010). Designing professional development for teachers of science and mathematics (3rd ed.). Thousand Oaks, CA: Corwin.