Accelerometers used in the measurement of jerk, snap, and crackle

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ABSTRACT

Accelerometers are traditionally used in acoustics for the measurement of vibration. Higher derivatives of motion are rarely considered when measuring vibration. This paper presents a novel usage of a triaxial accelerometer to measure acceleration and use these data to explain jerk and higher derivatives of motion. We are all familiar with the terms displacement, velocity and acceleration but few of us are familiar with the term jerk and even fewer of us are familiar with the terms snap and crackle. We experience velocity when we are displaced with respect to time and acceleration when we change our velocity. We do not feel velocity, but rather the change of velocity i.e. acceleration which is brought about by the force exerted on our body. Similarly, we feel jerk and higher derivatives when the force on our body experiences changes abruptly with respect to time. The results from a gymnastic trampolinist where the acceleration data were collected at 100 Hz using a device attached to the chest are presented and discussed. In particular the higher derivates of the Force $_{total}$ equation are extended and discussed, namely: $i_{jerk} \cdot d^3x/dt^3$, $r_{snap} \cdot d^4x/dt^4$, $n_{crackle} \cdot d^5x/dt^5$ etc.

1 INTRODUCTION

We are exposed to a wide variety of external motion and movement on a daily basis. From driving a car to catching an elevator, our bodies are repeatedly exposed to external forces acting upon us, leading to acceleration.

Being exposed to changes in motion and movement can have significant biomechanical effects on the human body and, in general we try to minimise our exposure to movement. Most transportation devices are designed to reduce acceleration as much as reasonably possible. Amusement rides on the other hand, are unique in that they are deliberately designed to expose us to large and changing types of motion (Eager 2013).

Trampolines are a great testing device that allows the motion to be simplified onto a single axis for analysis.

This paper follows two previous papers by the author. The first bring a paper on the characterisation of trampolines (Pendrill and Eager 2015) and the second being a paper describing jerk and higher deviritives (Eager at al. 2016).

2 DISCUSSION

We are all familiar with the terms displacement, velocity and acceleration. We experience velocity when we move and acceleration when we change the velocity at which we move. Our body does not feel velocity, but only the change of velocity i.e. acceleration, brought about by the force exerted by an object on our body. For example, a passenger in a constantly accelerating car will feel a constant force from the seat on his or her body.

Excluding the force of gravity that we are all exposed to minute by minute the concept of a constant force is rare. The accelerations that we normally experience and thus feel are not constant. When we are in a car and accelerate as the lights change to green, our acceleration is not constant. In this situation our acceleration is changing, so the motion sensation we are feeling is more likely jerk and even snap[1] since there is a change in the jerk.

The terms jerk and snap mean very little to most people, including physicists and engineers. What are jerk and snap? Mathematically jerk is the third derivative of our position with respect to time and snap is the fourth derivative

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[1]   There is no agreement of the names of higher order derivatives. The term snap will be used throughout this paper to denote the fourth derivative of displacement with respect to time. Another name for the fourth derivative is jounce. The fifth and sixth derivatives are crackle and pop [Eager et al. 2016].

of our position with respect to time.

Acceleration without jerk is just a static load. Jerk is felt as the change in force; jerk can be felt as an increasing force on the body or as a decreasing force on the body.

Consider the following. Velocity doesn't suddenly switch on, but instead grows from zero. So, there must be some acceleration involved. Acceleration doesn't suddenly switch on, but instead grows from zero. So, there must be some jerk involved. But the jerk doesn't suddenly switch on, but also grows from zero. So there must be some snap involved. But the snap doesn't suddenly switch on, but also grows from zero. So, there must be some crackle involved. But the crackle doesn't suddenly switch on, but also grows from zero. So, there must be some pop involved.

We know the terms displacement, velocity and acceleration. In 1678 Robert Hooke stated that for relatively small deformations of an object the displacement was directly proportional to the force (Hooke 1678). Mathematically we express that as $F = -k \cdot x$ where k is the spring constant and x is the displacement.

Similarly, the velocity is directly proportional to the force and we express this mathematically as $F = c \cdot v$ where c is the damping coefficient and v is the velocity.

Children are taught Newton's second law at school and they are shown to express it as $F = m \cdot a$ where m is the mass and a is the acceleration in a linear system. What children and even many university graduates are not taught is the precise wording that Newton used was 'Lex II: Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur' (Newton 1687). In English we state this as 'The acceleration of an object is directly proportional to the magnitude of the force and inversely proportional to the mass of the object'.

Nevertheless Newtown's Latin wording can also be interpreted as the impulse force where the force acts over a period of time Δt. If the force is impulsive it follows that the acceleration cannot be constant (if the mass of the body remains unchanged).

We can express this relationship as $F = i_{jerk} \cdot j$ where i_{jerk} is a new constant and j as the jerk.

Similarly for snap we have a relationship as $F = r_{snap} \cdot s$ where r_{snap} is a new constant and s as the snap, and so on for the higher derivatives.

When we combine the displacement derivatives we obtain the following:

$$F_{total} = k \cdot x + c \cdot v + m \cdot a + i_{jerk} \cdot j + r_{snap} \cdot s + n_{crackle} \cdot t + o_{pop} \cdot p + \ldots \tag{1}$$

This can also be expressed as the following differential equation:

$$F_{total} = k \cdot x + c \cdot dx/dt + m \cdot d^2x/dt^2 + i_{jerk} \cdot d^3x/dt^3 + r_{snap} \cdot d^4x/dt^4 + n_{crackle} \cdot d^5x/dt^5 + o_{pop} \cdot d^6x/dt^6 + \ldots \tag{2}$$

We can physically appreciate and measure the first three constants i.e. the spring constant (k), the damping constant (c) and the mass (m).

Physically k is measured in N/m or kg/s^2; c is measured in N.s/m or kg/s; and m is measured in $N.s^2/m$ or simply kg.

What physically are i_{jerk}, r_{snap}, $n_{crackle}$ and o_{pop}?

Physically i_{jerk} is measured in kg.s; r_{snap} is measured in $kg.s^2$; $n_{crackle}$ is measured in $kg.s^3$; and o_{pop} is measured in $kg.s^4$.

3   TRAMPOLINE ACCELERATION, JERK, SNAP AND CRACKLE

In a previous paper, trampoline jumps were analysed as a combination of free fall and harmonic oscillator motion (Pendrill and Eager 2015), with a maximum acceleration of (N - 1)g (and a maximum force X = Nmg from the trampoline) at the lowest point. From the theoretical model in Pendrill and Eager, combining harmonic motion with free fall, the total time between two jumps can be expressed as:

$$T = 2\ [\arccos\ (1/(N - 1)) + \mathrm{sqr}(N^2 - 2N)]\ /\ w \qquad\qquad (3)$$

where $2\mathrm{pi}\ /\ w$ is the period for small bounces, where the feet don't leave the mat and N is normalized force experienced by the trampolinist while in contact with the trampoline bed.

As the trampolinist's feet reach the mat, the acceleration increases as part of a sine function which passes zero, as the jumper passes the equilibrium position (at the moment where the acceleration is 0 and the force from the mat is mg, compensating for gravity).
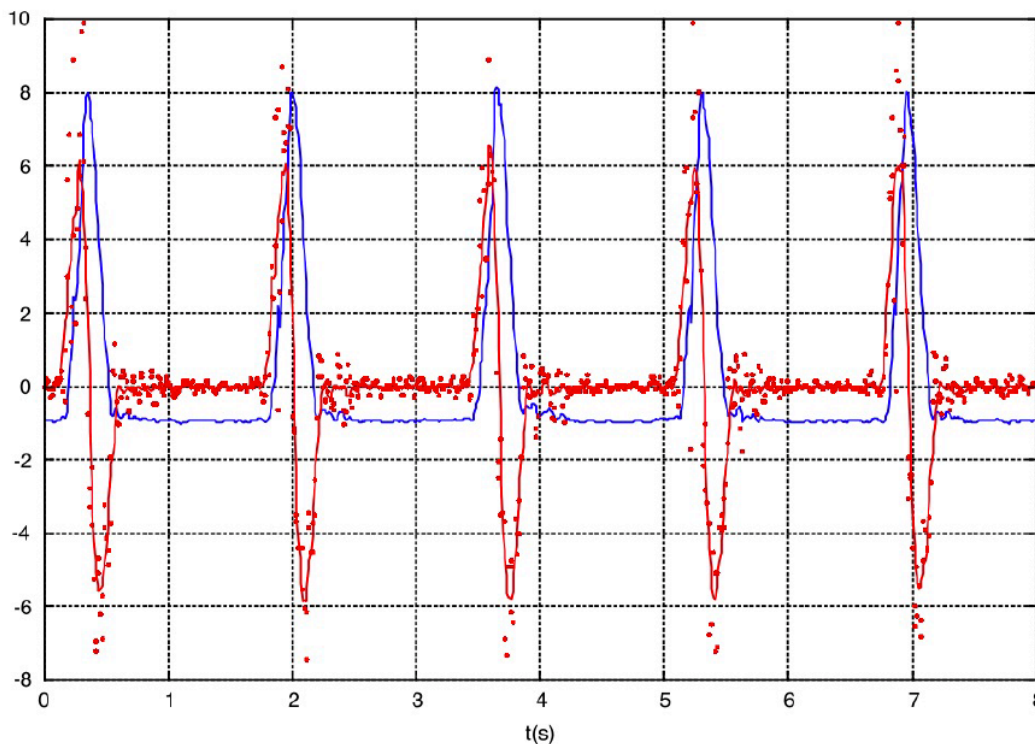

Figure 1: Acceleration in units of g (blue) and jerk in g/10s (red) during five consecutive
bounces on a gymnastic trampoline. The red dots show the raw jerk data.

Figure 1 shows accelerometer data, together with its derivative, jerk, for five consecutive jumps on a gymnastic trampoline where force on the jumper reaches 9.1mg (and the acceleration reaches 8.1g).

From Equation 1 we find that a maximum force of around 9mg corresponds to a ratio of about 3 between cycle time and the period for small oscillations. Figure 1 shows four periods in about 6.5 s, giving a period of 0.53 s for small bounces or w ~ 12/s, which is also the factor added for each successive derivative.

The accelerometer data were collected with a sampling frequency of 100 Hz and the derivative at a given time was obtained from the data points collected just before and after that time. These raw jerk data are shown as points in Figure 1 and show large fluctuations, whereas the smoother solid line shows a 10 point running average.

Two jerk maxima were observed per jump cycle. The contacting or leading edge maximum where jerk maximum was positive and occurred nearly immediately after the trampolinist's feet contacted the mat, and the separating or trailing edge maximum where the jerk maximum was negative and occurred prior to the trampolinist's feet separating from the mat. The averaging leads to slower rise than in the raw data, and also to smaller peak values for the jerk.

Figure 2 shows the 10-point running averages for the higher derivatives snap and crackle for one of the jumps. As expected, each derivative brings an additional factor to the maximum of around w: a snap minimum is of approximately $-1,100g/s^2$, is observed when the acceleration is at a maximum, whereas the crackle has minimum and maximum values around $\pm13,000g/s^3$.

In addition to these expected maxima and minima, we find two side maxima for the snap around the transition from air to mat or vice versa, and one snap minimum around the time the feet make contact with the mat and a maximum as the feet leave the mat. The averaging procedure makes the jerk and higher derivatives "spill over" into the time interval when the trampolinist is in the air. The values outside the contact time are results of the numerical procedures for derivation and averaging and can also be seen as approximations of the Dirac delta function (for snap) and its derivative (for crackle).
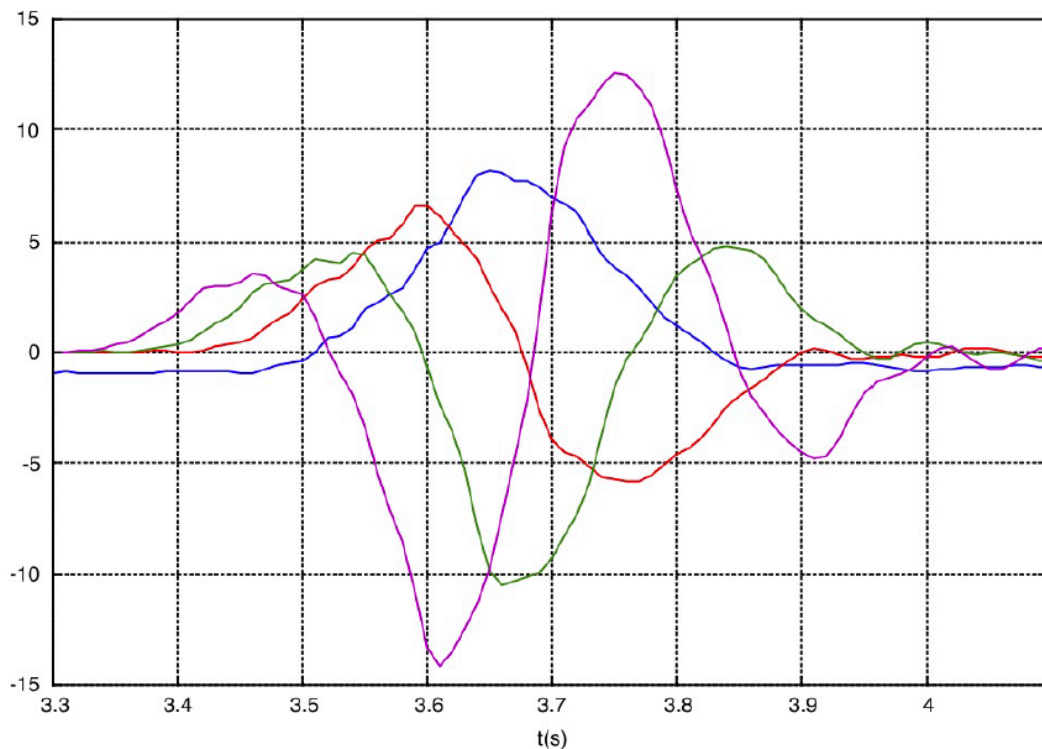

Figure 2: Acceleration in units of g (blue), jerk in $g/10s$ (red), snap $g/100s^2$ (green)
and crackle $g/1000^3$ (magenta) during a single bounce on a gymnastic trampoline.

4   CONCLUSION

The human tolerance to jerk, snap and crackle is not well understood. What is also not understood is the coefficient for each of these terms.

The higher order coefficients for jerk, snap and crackle have the SI unit of: i_{jerk} is kg.s; r_{snap} is $kg.s^2$; and $n_{crackle}$ is $kg.s^3$.

For the part of the jumping cycle where the trampolinist is in the air, acceleration is constant, -g, and the jerk and all higher derivatives are zero. While the trampolinist is in contact with the trampoline, the jerk can be described by a harmonic function, and each additional derivative adds a factor of w and shifts sine to cosine or cosine to negative sine, respectively. The value of the jerk is zero when the acceleration is maximum. As the acceleration passes zero, the jerk has its maximum value $(N - 1)gw$ as the trampolinist is on the way down, and a maximum negative value $-(N - 1)gw$ when the trampolinist is on the way up. There may be a very small lag due to the inherent damping within the trampoline system.

ACKNOWLEDGEMENTS

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