Fast Synthesis Algorithm for Uniformly Spaced Circular Array with Low Sidelobe Pattern

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***Abstract*—In this paper, a highly efficient approach is proposed to synthesize the low sidelobe pattern of uniformly spaced circular array. The proposed approach can be generalized to deal with the pattern synthesis for the circular array with directional elements. Numerical examples are given to verify the effectiveness and advantage of this approach.**

***Index Terms*—Uniformly spaced circular array; conformal array; low sidelobe pattern.**

I. INTRODUCTION

A uniform circular array (UCA) offers wide angle coverage in the azimuth plane with either an omnidirectional radiation pattern or an electronically steerable directional beam. By rotationally moving the array excitation distribution, omnidirectional scanning can be obtained easily and flexibly [1], [2]. As the basic structure of rotationally symmetric conformal arrays, UCA has been extremely studied in many literatures. However, because UCA do not have the Vandermonde structure, pattern synthesis for UCA is more computationally expensive than that for traditional linear or planar arrays. Several methods such as the one in [3] have been presented apply phase-mode decomposition of the circular array pattern to produce an approximate Vandermonde structure.

In literature, many stochastic algorithms capable of finding globally optimum have been presented to deal with pattern synthesis problems for UCA. In [4], a simple and flexible genetic algorithm (GA) was proposed by Keen-Keong Yan for the pattern synthesis of antenna array with arbitrary geometry. The GA algorithm was laterly generalized to deal with non-uniform circular antenna arrays for maximal sidelobe level (SLL) reduction with a fixed beam width constraint in [5]. Particle swarm optimization (PSO) was used to minimize the maximum SLL for concentric circular array in [6]. Besides in [7], the biogeography-based optimisation is adopted to find a radiation pattern with maximum side lobe level (SLL) reduction under a fixed mainlobe beamwidth. Despite their success, most of the existing methods are time consuming especially when a kind of stochastic optimization is involved.

In this paper, a novel efficient method is proposed which is developed by building the relationship between the fast convolution and the UCA pattern expression. The FFT can be then utilized to speed up the pattern synthesis process. An example for synthesizing a low sidelobe pattern for 30-element UCA is provided to validate the effectiveness and efficiency of the proposed method.

II. THE GENERAL FORMULATION

A. Fast convolution applied to UCA pattern

Consider a circular array with N elements uniformly arranged along its circumference with the radius r in the horizontal plane ($\vartheta = \pi/2$). The element spacing in the angle can be calculated by $\Delta\phi = 2\pi/N$, the element location angle can be expressed by $\phi_n = 2\pi n/N$ for $n = 0, 1, \cdots, N-1$. Assume that the spatial observation angle in the horizontal plane is uniformly sampled with $\varphi_m = 2\pi m/N$ for $m = 0, 1, \cdots, M-1$. Then the far-field pattern of the circular array at the horizontal plane can be written as

$$E(m) = \sum_{n=0}^{N-1} I_n e^{j\alpha_n} a\left(m\frac{2\pi}{M} - n\frac{2\pi}{N}\right) e^{j\beta r \cos\left(m\frac{2\pi}{M} - n\frac{2\pi}{N}\right)} \tag{1}$$

where I_n and α_n are the excitation amplitude and phase for the nth element. $a(\varphi)$ is the element pattern when the element is placed at $\phi = 0$. Clearly, if $M = N$, we can rewrite (1) as the form of $E(m) = \sum_{n=0}^{N-1} x(n)y(m-n)$ where $x(n) = I_n$ and $y(n) = a(2\pi n/N)e^{j\beta r\cos(2\pi n/N)}$. This is a standard discrete convolution form, and the fast discrete convolution based on forward and inverse FFT can be utilized to speed up the pattern calculation.

B. Segmented Fast Convolution applied to UCA pattern

Usually, the observation spatial angle would be sampled with highly sampling density. That is, we usually have $M \geq N$. In this situation, the fast discrete convolution cannot be directly applicable to the calculation of (1). To tackle this problem, we present a new algorithm called segmented fast convolution. Now, we assume that $M = NL$, and then rewrite (1) as

$$E(k_1, k_2) = \sum_{n=0}^{N-1} a\left\{\frac{2\pi\left[(k_1 - n)L + k_2\right]}{NL}\right\} \cdot I_n e^{j\alpha_n} \cdot e^{j\beta r\cos\left\{\frac{2\pi[(k_1-n)L+k_2]}{NL}\right\}} \tag{2}$$

where $k_1 = 0, 1, ..., N-1$, $k_2 = 0, 1, ..., L-1$. L means the total number of segmentation while N refers to the length

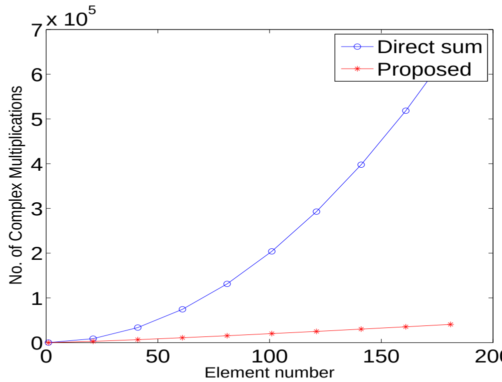

Fig. 1. *The number of complex multiplications for the direct summation method and the proposed method ($L = 20$)*

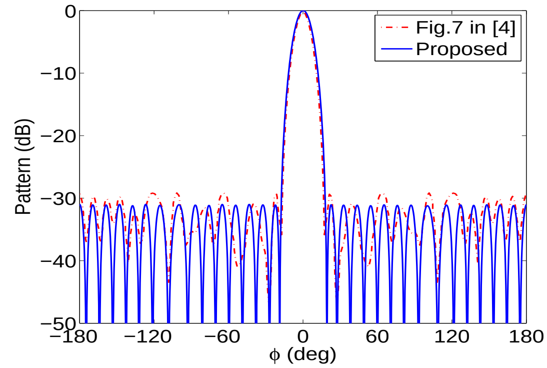

Fig. 2. *The pattern synthesized by the proposed method and the pattern obtained in [4] for the 30-element UCA*

of each one. Here, k_1 represents the samples of azimuth angle of each segment and k_2 means different subsection. When k_2 varies, we get different values of array pattern at different observation angles. By simply sorting these sampling points, UCA pattern with a high calculation resolution can be efficiently obtained. The excitations can be also efficiently obtained from the array pattern by similarly employing FFT and inverse FFT. Therefore, an efficient iterative fast synthesis procedure similar to the alternating projection method [8] can be developed to find the excitation distribution for a desired array pattern performance.

III. SIMULATION EXPERIMENT

Suppose that we have N elements for the UCA, and need to calculate the array pattern at $M = N \times L$ discrete angles. The direct summation of (1) costs $M \times N$ times complex multiplication. If the proposed segmented fast convolution method is used, we need to perform the fast convolution calculation for each segment, and once convolution mainly costs 3 times of N-point FFT. If the radix-2 algorithm is used for the implementation of FFT, each N-point FFT has $L\left(N/2\right)\log_2 N$ complex multiplications. Therefore, the segmented fast convolution costs about $3L\left(N/2\right)\log_2 N$ complex multiplications in total. Fig. 1 shows the required number of complex multiplications for the direct summation method and the proposed method when $L = 20$.

To further verify the effectiveness of the proposed method for the pattern synthesis problem, we consider synthesizing a low sidelobe UCA pattern. Consider a UCA array with 30 isotropic elements. Assume that the desired pattern has the first-null beamwidth (FNBW) of $\pm 18^\circ$ and maximum SLL of -29.16 dB. This pattern was obtained in [4] by taking 22.67 seconds, and the obtained excitation amplitude distribution has the dynamic range ratio (DRR) of 4.76. For the proposed method, we set the parameter $L = 35$. This method take only 0.24 seconds to obtain the pattern with the same FNBW and slightly lower SLL. The obtained pattern as well as the pattern in [4] are both shown in Fig. 2. The obtained DRR for the proposed method is 3.5 which is lower than the one in [4].

IV. CONCLUSIONS

A novel efficient approach is proposed to achieve the low sidelobe pattern synthesis for uniform circular array. Different from conventional synthesis methods, the proposed method applies FFT to speed up the pattern synthesis by employing the relationship between the fast convolution and array pattern of UCA. One synthesis example has been provided to show the effectiveness and superiority of the proposed method.

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REFERENCES

[1] J. A. Tsai and B. Woerner, "The fading correlation function of a circular antenna array in mobile radio environment," in *Proc. IEEE Global Telecommunications Conference*, vol. 3, 2001, pp. 1630–1634.

[2] Q. Shen, E. K. Mao, and W. S. Liang, "The performance analysis of circular array antennas in vhf/uhf band," *Int. Conf. on Radar*, 2006, pp. 1–4.

[3] D. E. N. Davies, "A transformation between the phasing techniques required for linear and circular aerial arrays," *Proc. IEE*, vol. 112, 1965, pp. 2041–2045.

[4] K. K. Yan and Y. L. Lu, "Sidelobe reduction in array-pattern synthesis using genetic algorithm," *IEEE Trans. Antennas Propag.*, vol. 45, no. 7, pp. 1117–1122, Jul. 1997.

[5] M. A. Panduro, A. L. Mendez, R. Dominguez, and G. Romero, "Design of non-uniform circular antenna arrays for side lobe reduction using the method of genetic algorithms," *AEU-Int. J. Electron. Commun.*, vol. 60, no. 10, pp. 713–717, 2006.

[6] G. Ram, D. Mandal, and R. Kar, "Pencil beam pattern synthesis of time-modulated concentric circular antenna array using PSO with aging leader and challenger," *Appl. Spectrosc.*, vol. 29, no. 12, pp. 1610–1629, 2015.

[7] U. Singh and T. S. Kamal, "Design of non-uniform circular antenna arrays using biogeography-based optimisation," *IET Microw. Antenna P.*, vol. 5, no. 11, pp. 1365–1370, Aug. 2011.

[8] W. P. M. N. Keizer, "Fast low-sidelobe synthesis for large planar array antennas utilizing successive fast Fourier transforms of the array factor," *IEEE Trans. Antennas Propag.*, vol. 55, no. 3, pp. 715–722, Mar. 2007.