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"Animal spirits" and bank's lending behaviour, a disequilibrium approach

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Abstract:
The paper analyses from a disequilibrium perspective the role of banks' "animal spirits" and collective behaviour in the creation of credit that, ultimately, determines the credit cycle. In particular, we propose a dynamic model to analyse how the transmission of waves of optimism and pessimism in the supply side of the credit market interacts with the business cycle. We adopt the Weidlich-Haag-Lux approach to model the opinion contagion of bankers. We test different assumptions on banks' behaviour and find that opinion contagion and herding amongst banks play an important role in propagating the credit cycle and destabilizing the real economy. The boom phases trigger banks' optimism that collectively lead the banks to lend excessively, thus reinforcing the credit bubble. Eventually the bubbles collapse due to an over-accumulation of debt, leading to a restrictive phase in the credit cycle.

Keywords: animal spirits, contagion, financial fragility, pro-cyclical credit cycle

JEL classification: E12, E17, E32, G21

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1 Introduction

> *"Banks are much more social creature than most people think. Understanding banks is not about brute facts. The values on a balance sheet are dependent upon confidence, and when an institution is in trouble those values are quite different from the figures when the institution is thought to be doing well. So value is dependent on confidence, which is a social fact rather than a material property."*
>
> Rethel and Sinclair (2012), "The Problem with Banks"

The recent financial crisis has revived the interest in studying the role of financial markets and institutions in amplifying business cycles in the real sector. While models in the Keynesian tradition have always featured an integrated financial sector, mainstream analysis has only recently been applied to study the role of the financial sector beyond its role of generating frictions for the adjustment process. Recent history has shown once again that the financial and the real sector are interconnected, and the boom-bust cycle in the asset market plays an important role in influencing aggregate demand and amplifying the business cycle. It is therefore crucial to build macroeconomic models that take into account factors such as the balance sheet composition of economic agents, and in particular, their credit, debt, and leverage position.

In the traditional banking literature, the commercial bank is often modelled as a passive intermediary that channels funds from the ultimate borrower to the ultimate lender (Allen and Gale, 1998; Bernanke, Gertler and Gilchrist, 1999; Fama, 1980). In reality however, the role of banks goes well beyond the intermediation between supply and demand of savings. A bank functions as an active credit creator (Taylor, 2004; Ryan-Collins et al., 2012). As recently remarked by McLeay, Radia, and Thomas (2014), the creation of a loan simultaneously creates a deposit, which endogenously enlarges the money stock, since deposit is part of the broad money (M3). In other words, the bank's behaviour is not a passive reflection of the conditions of the economy, but is in itself an important factor that influences the economy through the creation of credit. Minsky (1975) puts the role of credit creation of banks at the centre of his framework: it is the relaxing of credit conditions that drives the expansionary phase of the cycle and it is the contraction in credit that exacerbates the downturn.

As Minsky (2008) argues, the fact that banks can create credit puts them at the core of the capitalist economy and, at the same time, makes them "endogenous destabilizer". According to Minsky's narrative, during expansions financial institutions progressively lower they credit standards, eventually leading to a boom. When the

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level of outstanding debt is unsustainable, panic ensues and credit supply contracts creating the conditions of a recession and, possibly, a depression. It is therefore not surprising that the financial regulator has targeted banks (and in particular their loan-to-asset ratios) in the wake of every major financial crisis. It is more surprising that in both traditional banking literature and in the literature of Minskyan inspiration the role of the bank's lending attitude is generally overlooked.[1] An optimistic attitude in the banking sector collectively lowers the lending standard and prompts banks to lend excessively,[2] which potentially leads to the development of a credit bubble. Eventually the bubble bursts due to an unsustainable level of debt (Kindleberger 1989). On the contrary, a collectively pessimistic banking system not only hinders economic growth but also renders expansionary monetary policy ineffective, as we have observed in the recent financial crisis. As it is shown in Figure 1, in the aftermath of the crisis, the money base in the US has grown nearly three-fold due to three rounds of Quantitative Easing (QE), however, it has virtually no effect on the growth of broad money due to the negative outlook (Koo 2011).

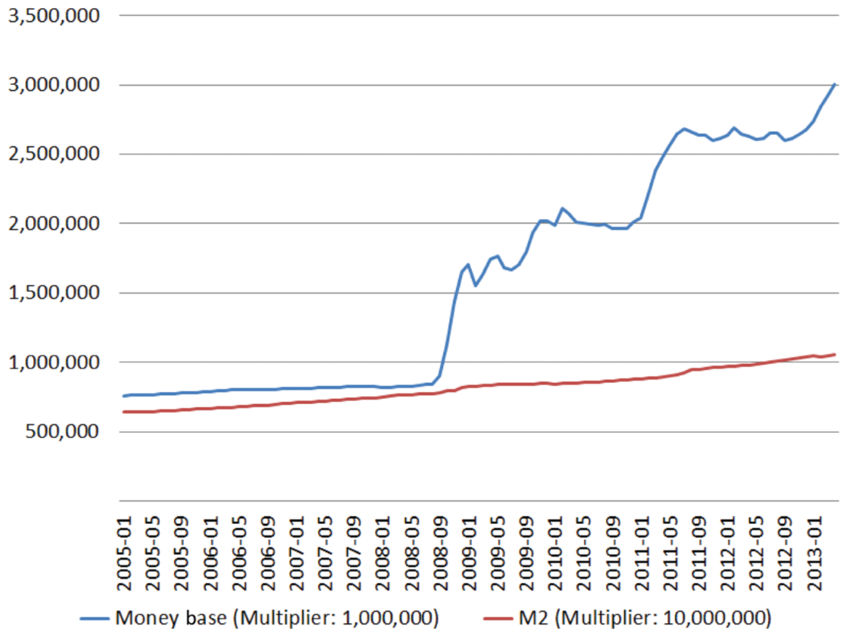

Figure 1: The effect of Quantitative Easing on Money Base and M2.
Source: the Federal Reserve statistics release.

Given the crucial role of banks in fostering exuberance during an upturn and panic during a depression, it is in our opinion a consistent modelling choice to assume that banks behave in a boundedly-rational fashion and are subject to "animal spirits" (Keynes 1936), not differently from other agents in the economy. The concept of "animal spirits" implies two relevant corollaries. First, expectations are self-fulfilling: an optimistic/pessimistic sentiment will bring forth a positive/negative outcome to the market, which further reinforces the optimistic/pessimistic sentiment. Second, market sentiment is contagious: sentiment spreads and it eventually leads to herding amongst agents. The herding behaviour in financial market is well-documented in empirical literature (Sharma and Bikhchandani, 2000; Haiss, 2005). There is also notable amount of literature that finds empirical evidence of herding amongst banks, particularly in the US and Japan (Liu, 2012; Nakagawa and Uchida, 2012). In different papers, Dow relates the Minskyan credit cycle to the psychology of lenders and investors (see for example Dow 2011).

A number of recent finance and macroeconomic studies in the last decades models the "animal spirits" as herding behaviour. This modelling approach is initially proposed by Weidlich and Haag (1983). The basic idea is to model heterogeneous agents that choose and switch between two attitudes in probabilistic terms. A reduced-form Master equation that captures the "average opinion" is applied to simplify the analysis of the stochastic system. Lux (1995) proposes a seminal work that examines the relationship between investors' sentiment and asset price bubble/crash. Franke (2012) terms this approach "Weidlich-Haag-Lux" and extends the Lux model to the context of macroeconomic dynamics. He studies the interplay between firm's sentiment, inflation and output gap, which establishes an alternative microfoundation for macroeconomics in the Keynesian tradition. This model is further extended by Charpe et al. (2012), which proposes a "Dynamic Stochastic General Disequilibrium (DSGD)" model. The DSGD model examines the real-financial interaction by incorporating a speculative financial market populated by heterogeneous investors and it takes a "disequilibrium" approach that models the dynamical adjustment process, instead of assuming immediate equilibrium adjustment.[3]

This paper follows the approach of Franke (2012) but it focuses on the role of "animal spirits" as the determinant of banks' lending decisions. The aim is to assess how the contagious waves of optimism and pessimism contribute to the boom-bust of the credit cycle, via a modification of banks' balance sheet positions and how it amplifies business cycle in the real sector. From this perspective our analysis integrates related contributions in the Minskyan tradition[4] by focusing on the role of banking sector rather than on borrowers.

We present an aggregative model in which banks follow heterogeneous lending strategies and, given their cognitive limits, are assumed to follow herd behaviour. The banks' opinion formation dynamics is modelled in the spirit of Weidlich-Haag-Lux approach. The joint evolution of banks' behaviour, credit supply and aggregate output are analysed in a dynamical system.

In our take of this approach, herding among banks is characterized by two different behaviours. The first is the switching of banks between the two categories of optimistic and pessimistic, which is more intense the bigger is the size of the majority group. The second regards the herding behaviour of banks for what concerns their decision about their loans-to-reserve ratio, which is larger during optimistic phases.[5] In this way we further extend the Weidlich-Haag-Lux approach by including a further dimension of herding.

This paper presents two main novelties. First, while several other contributions have already treated the effects of herding and bounded rationality of firms and households on the business cycle, to the best of our knowledge this study represents a primer in modelling banking behaviour as influenced by animal spirits in a dynamic setting. The second original aspect concerns the introduction of heterogeneity in the credit sector, which represents a novelty in this stream of aggregative dynamical models. In contrast to the traditional banking literature, we stress the role of the mechanism of credit-creation by banks as a potentially destabilising factor.

We find from the analytical and numerical study of the dynamical systems that (i), sentiment contagion and collective behaviour amongst bankers play an important role in destabilizing the system and propagating boom-bust of credit cycle and business cycle in the real sector; (ii) the irrational behaviour of the banking sector, companied by the speculative behaviour in the financial market, will not only lead to fluctuations in the short run, but also give rise to an unpredictable evolutionary path in the credit cycles, which would eventually propagate macroeconomic volatility.

The remainder of the paper is organized as follows. In the next section, we propose a two-dimensional baseline model, where banks are categorized in optimistic and pessimistic. The dynamical system consists of the average opinion and the output dynamics. This baseline model is assessed analytically in order to identify the main properties of the framework in a simplified setting. In Section 3, the loan-to-reserve ratio of the optimistic and pessimistic banks are endogenised providing some additional insights on the role of heterogeneous lending strategies in destabilizing the real sector by means of a four dimensional dynamical system. In Section 4, the four-dimensional model is further enriched by introducing a nonlinear I-S disequilibrium dynamics in a Kaldorian manner. In such a way we introduce a credit-driven investment function as well as an income-driven saving function and we can identify through numerical simulations the effects of an autonomous dynamics in the real sector. In Section 5 we further extend the model from an alternative perspective by incorporating a speculative financial sector based on the framework of Charpe et al. (2012). Finally, Section 6 offers some concluding remarks.

2 The 2D model

In this first section we present the main behavioural hypotheses of the model. Banks are classified into the two categories of optimistic and pessimistic. Both types of banks are assumed to keep constant their loan-to-reserve ratio. The fluctuations in the supply of credit (and consequently in the real output) are therefore an effect of the switching of banks between the optimistic and the pessimistic state.

Table 1 illustrates the structure of a typical balance sheet of a commercial bank. On the asset side it consists of bank reserves, loans, and other assets such as treasury bonds; on the liability side there are deposits, bank borrowing, and bank equity. When a bank makes loans, it simultaneously creates deposits.

Table 1: A simplified balance Sheet of commercial bank.

Asset	Liability
Reserve [R]	Deposit
Loan [L]	Central Bank Borrowing
Bond	Interbank Borrowing
	Bank Equity
Loan+	Deposit+

2.1 The basic set-up

Following Taylor (2004), we focus on the loan-to-reserve ratio (λ^s), which is defined as the ratio between loans and unborrowed reserves.[6] In our simplified balance sheet the only zero-risk assets are represented by reserve, therefore the loan-to-reserve ratio represents our proxy to measure the financial soundness of financial institutions. The variable λ^s reflects not only the banks' lending attitude, but also the level of debt accumulation due to banks' credit creation. Specifically, we have that

$$L^s = \lambda^s T_c, \tag{1}$$

where L^s is the level of aggregate credit supply; λ^s is the loan-to-reserve ratio of banks; T_c is the total amount of unborrowed reserves, which is assumed to be exogenous.

The total number of banks is $2N$, while n_+ is number of optimistic banks and n_- is the number of pessimistic banks ($2N = n_+ + n_-$). The optimistic banks lend at a loan-to-reserve ratio λ_+ assumed to be larger than the pessimistic banks' one λ_-. We assume that each bank holds the same amount of reserves, and the two loan-to-reserve ratios are initially set as constant as $\bar{\lambda}_+$ and $\bar{\lambda}_-$. Hence equation (1) becomes

$$L^s = R(n_+\bar{\lambda}_+ + n_-\bar{\lambda}_-), \tag{2}$$

$$T_c = 2NR, \tag{3}$$

where R indicates the reserves. Following Lux (1995), the difference in the size of the two groups is quantified by the index x

$$x = (n_+ - n_-)/2N. \tag{4}$$

As $2N = n_+ + n_-$, it is easy to derive that

$$n_+ = N(1 + x), \tag{5}$$

$$n_- = N(1 - x). \tag{6}$$

The index x describes the "average opinion" or, in the context of this chapter, the general lending attitude of banks. When $x = 0$, there are equal number of optimistic and pessimistic banks. When $x = \pm 1$, it implies that all the banks are either optimistic or pessimistic. Since $T_c = 2NR$, equation (2) is therefore modified in terms of x as

$$L^s = \frac{T_c}{2}[(1 + x)\bar{\lambda}_+ + (1 - x)\bar{\lambda}_-]. \tag{7}$$

We postulate that the availability of credit L^s determines the aggregate demand in the real sector. Furthermore, we assume that output (y) is demand-driven and it follows Blanchard (1981) as a stylized AS-AD dynamic multiplier process.[7] That is,

$$y^d = y_0^d + kL^s, \tag{8}$$

$$\dot{y} = \sigma(y^d - y), \tag{9}$$

where y_0^d is the autonomous component of the aggregate demand.

Substituting (7) and (8) into (9) we get

$$\dot{y} = \sigma\{y_0^d + k\frac{T_c}{2}[(1 + x)\bar{\lambda}_+ + (1 - x)\bar{\lambda}_-] - y\}. \tag{10}$$

2.2 Opinion dynamics

In this sub-section we introduce the law of motion for the average opinion x. Let p_{+-} be the transition probability for a pessimistic bank to become optimistic, while p_{-+} is the probability of the opposite transition. Accordingly, the change in the level of x depends on the size of each group multiplied by their transition probability. Thus we have

$$\dot{x} = (1 - x)p_{+-} - (1 + x)p_{-+}. \tag{11}$$

Three factors affect the probability of transition of banks from one group to another: the bankers average opinion x, which captures the contagion effect; the output gap in the real sector $y^d - y$; and a general financial index d, representing the propensity of banks to switch. The relevance of herding behaviour in the banking sector, in particular with reference to the latest major financial crisis has been already stressed in the literature (see Haiss 2010, among others). We consider also the influence of the general macroeconomic conditions on the supply of credit, using as a proxy the output gap. Further, we introduce the constant d in order to account for possible institutional factors that, due to their nature, are assumed to be constant over time. Accordingly, we compose a switching index s as a linear combination of the three factors

$$s(x, y, d) = a_1 x + a_2(y^d - y) + d. \tag{12}$$

The parameter a_1 quantifies the effect of herding and plays an important role in our story. Assuming that the relative changes of p_{+-} and p_{-+} in response to changes in s are symmetric, the probabilities can be written as

$$p_{+-} = v \cdot exp(s), \tag{13}$$

$$p_{-+} = v \cdot exp(-s). \tag{14}$$

Hence

$$\dot{x} = v[(1 - x)\exp(s) - (1 + x)\exp(-s)]. \tag{15}$$

Equations (12) and (15) show that the emergence of a lending behaviour as the most popular is self-strengthening: more and more banks are assumed to follow the strategy adopted by the largest number of banks. An increase in s increases the probability that the pessimistic banks will become optimistic ones. Equation (15) models the lending behaviour of banks in fashion similar to the famous Keynes' beauty contest metaphor. In a situation of uncertainty and less than perfect information, an anchor to the expectations is provided by the behaviour of other agents.

It is therefore possible to represent the dynamics by means of the two-dimensional system composed by equations (9) and (15).

$$\dot{y} = \sigma(y^d - y), \tag{16}$$

$$\dot{x} = v[(1 - x)\exp(s) - (1 + x)\exp(-s)], \tag{17}$$

where

$$y^d = y_0^d + kL^s = y_0^d + k\frac{T_c}{2}[(1 + x)\bar{\lambda}_+ + (1 - x)\bar{\lambda}_-], \tag{18}$$

$$s = a_1 x + a_2(y^d - y) + d. \tag{19}$$

2.3 Analysis of the two-dimensional system

In order to study the properties of the system (16), (17) we first set $LHS = 0$ on both equations and derive the following isoclines

$$y = y_0^d + k\frac{T_c}{2}[(1+x)\bar{\lambda}_+ + (1-x)\bar{\lambda}_-], \tag{20}$$

$$y = \frac{a_1}{a_2}x - \frac{1}{2a_2}\ln\frac{1+x}{1-x} + y^d + d. \tag{21}$$

It is difficult to obtain the close-form solution of equation (20), (21). Yet in a special case when we set $d = 0$, we can easily obtain a neutral opinion equilibrium $(x^\star = 0, y^\star = y^d)$. Furthermore, there is potentially an emergence of other equilibria $(x_+^\star, y_+^\star)$ and $(x_-^\star, y_-^\star)$, depending on the value of contagion parameter a_1 – as shown in Figure 2, which plots the two isoclines.[8] We analyse the local stability of the neutral opinion equilibrium by deriving the Jacobian of the system (16), (17)

$$J = \begin{pmatrix} -\sigma & \sigma k\frac{T_c}{2}(\bar{\lambda}_+ - \bar{\lambda}_-) \\ -2va_2 & 2v[a_1 + a_2 k\frac{T_c}{2}(\bar{\lambda}_+ - \bar{\lambda}_-)] \end{pmatrix}. \tag{22}$$

The trace and determinant of the Jacobian at neutral opinion equilibrium ($x = 0$) is calculated as

$$Tr(J) = 2v\left[a_1 + a_2 k\frac{T_c}{2}(\bar{\lambda}_+ - \bar{\lambda}_-)\right] - \sigma, \tag{23}$$

$$Det(J) = 2v\left\{a_2\sigma k\frac{T_c}{2}(\bar{\lambda}_+ - \bar{\lambda}_-) - \sigma\left[a_1 + a_2 k\frac{T_c}{2}(\bar{\lambda}_+ - \bar{\lambda}_-) - 1\right]\right\}. \tag{24}$$

The necessary and sufficient condition for local stability for this equilibrium is that $Tr(J) < 0$ and $Det(J) > 0$. Otherwise it would become locally unstable in the form of repelling cycle or saddle node. The contagion parameter a_1 plays an important role in determining the local stability: this neutral opinion equilibrium is more likely to be stable when a_1 is relatively small. The results of a representative simulation, together with the bifurcation analysis for the contagion parameter a_1, are provided in Figure 3. All the simulations code are available upon request. The parameters are set as follows: $a_1 = 0.3$, $a_2 = 3.4$, $\sigma = 0.8$, $k = 0.1$, $T_c = 1$, $y_0^d = 10$, $\bar{\lambda}_- = 5$, $\bar{\lambda}_+ = 20$, $v = 0.4$, $d = 0.5$. Note that in the numerical simulation the parameter d takes a non-zero value. As we can see, the neutral opinion equilibrium becomes unstable and a limit cycle emerges as the contagion parameter increases and passes through $a_1 \approx 0.3$. As a_1 further increases, the system becomes stable again, yet it converges to another equilibrium with higher value of x.

3 The 4D model: the convergence and divergence of heterogeneous lending strategies

In this section we relax the assumption of constant loan-to-reserve ratios for both categories of banks (λ_-), them to adjust their ratio according to the state of the economy. In particular, we assume that

$$\dot{\lambda}_+ = \gamma_1(x + g(.)) + \gamma_2\dot{y} + \gamma_3(\bar{\lambda}_+ - \lambda_+), \tag{25}$$

$$\dot{\lambda}_- = \gamma_1(x - g(.)) + \gamma_2\dot{y} + \gamma_3(\bar{\lambda}_- - \lambda_-). \tag{26}$$

where $\gamma_1, \gamma_2, \gamma_3 > 0$ are constants.

The function $g(.)$ on the l.h.s. in (25), following DeGrauwe (2011), captures the reaction gap between optimists and pessimists over the average opinion, which is quantified by

$$g(.) = \xi_0 exp(-\xi_1 x^2). \tag{27}$$

The rationale for (27) is that agents are attracted towards what is the most popular strategy during more unstable period due to the growing uncertainty. On the contrary, in period of stability, banks are assumed to rely more on their internal information and as a consequence the lending strategies will diverge bringing x close to 0. Equation (27) captures a scenario where there is a convergence or divergence of lending strategies during the exuberant/calm period, with agents behaving in a more coordinated manner during a period of increasing optimism or pessimism. However, the gap $g(.)$ narrows and the lending strategies converge when the optimistic or pessimistic sentiment grows over time. The optimistic banks react to $x+g(.)$ while the pessimistic banks react to $x-g(.)$. The parameter ξ_0 determines the magnitude of opinion gap and ξ_1 captures the sensitivity of $g(.)$ relative to the change of average opinion (x).

The second factor $\gamma_2\dot{y}$ represents a passive, accommodative role of banks driven by the growth in the real sector.[9]

The third factor includes the mean-reverting terms $\gamma_3(\bar{\lambda}_+ - \lambda_+)$ and $\gamma_3(\bar{\lambda}_- - \lambda_-)$ and aims to capture the long run adjustments: we assume that optimistic banks tend toward a higher loan-to-reserve ratio while the pessimistic banks tend toward a lower one ($\bar{\lambda}_+ > \bar{\lambda}_-$). The mean reverting process serves the purpose to provide a ceiling (floor) to the growth (decrease) of the lending ratio of optimistic (pessimistic) banks, which is necessary with endogenous λ_+ and λ_-.

Recalling that $L^s = R(n_+\lambda_+ + n_-\lambda_-)$, $n_+ = (1+x)N$, $n_- = (1-x)N$ and $T_c = 2NR$, the quantities L^s and y^d are given by

$$L^s = T_c/2((1+x)\lambda_+ + (1-x)\lambda_-), \tag{28}$$

$$y^d = y_0^d + k[T_c/2((1+x)\lambda_+ + (1-x)\lambda_-)]. \tag{29}$$

Hence the new system is written as

$$\dot{\lambda}_+ = \gamma_1(x+g(.)) + \gamma_2\dot{y} + \gamma_3(\bar{\lambda}_+ - \lambda_+), \tag{30}$$

$$\dot{\lambda}_- = \gamma_1(x-g(.)) + \gamma_2\dot{y} + \gamma_3(\bar{\lambda}_- - \lambda_-), \tag{31}$$

$$\dot{y} = \sigma(y^d - y), \tag{32}$$

$$\dot{x} = v[(1-x)\exp(s) - (1+x)\exp(-s)], \tag{33}$$

where

$$y^d = y_0^d + kL^s = y_0^d + k(T_c/2)[(1+x)\lambda_+ + (1-x)\lambda_-], \tag{34}$$

$$g(.) = \xi_0 exp(-\xi_1 x^2), \tag{35}$$

$$s = a_1 x + a_{2+}\lambda_+ + a_{2-}\lambda_- + a_3(y^d - y) + d. \tag{36}$$

We first analyse the steady state and local stability of the above 4D system. We notice that there is no closed form solution for the steady state condition.[10] Therefore, we consider a special case where the average opinion is neutral at equilibrium ($x^\star = 0$). In this case, the general financial condition index d becomes $d = -a_{2+}\lambda_+^\star - a_{2-}\lambda_-^\star$.[11] In this special case, it is easy to derive the close form solution for $\lambda_+^\star$, $\lambda_-^\star$, and y^*.

Proposition 1

In the special case where $x^\star = 0$, the steady state of the system (30)–(33) is unique and given by

$$\lambda_+^\star = \bar{\lambda}_+ + \frac{\gamma_1}{\gamma_3}\xi_0, \tag{37}$$

$$\lambda_-^\star = \bar{\lambda}_- - \frac{\gamma_1}{\gamma_3}\xi_0, \tag{38}$$

$$y^\star = y^{d\star} = y_0^d + k(T_c/2)[(\lambda_+^\star + \lambda_-^\star)], \tag{39}$$

$$x^\star = 0. \tag{40}$$

Proof.

By setting $LHS = 0$, we have

$$0 = \gamma_1(x + \xi_0 exp(-\xi_1 x^2)) + \gamma_2\dot{y} + \gamma_3(\bar{\lambda}_+ - \lambda_+) \tag{41}$$

$$0 = \gamma_1(x - \xi_0 exp(-\xi_1 x^2)) + \gamma_2\dot{y} + \gamma_3(\bar{\lambda}_- - \lambda_-) \tag{42}$$

$$0 = \sigma(y^d - y) \tag{43}$$

$$0 = v[(1-x)\exp(s) - (1+x)\exp(-s)] \tag{44}$$

Since $x^\star = 0$ and $\dot{y} = 0$, from equation (30) and (31) we derive that $\lambda_+^\star = \bar{\lambda}_+ + \frac{\gamma_1}{\gamma_3}\xi_0$ and $\lambda_-^\star = \bar{\lambda}_- - \frac{\gamma_1}{\gamma_3}\xi_0$. Substituting this result to equation (34) we have $y^\star = y^{d\star} = y_0^d + k(T_c/2)[(\lambda_+^\star + \lambda_-^\star)]$. □

We then analyse the local stability of the system. To make the system analytically tractable, we consider a special case without the real sector by setting $\gamma_2 = 0$, $\sigma = 0$, and $a_3 = 0$. The Jacobian of equation (30), (31), and (33) sub-dynamics at neutral opinion equilibrium without the real sector is derived as

$$\begin{pmatrix} -\gamma_3 & 0 & \gamma_1 \\ 0 & -\gamma_3 & \gamma_1 \\ 2va_{2+} & 2va_{2-} & 2v(a_1 - 1) \end{pmatrix},$$

which has the sign structure

$$\begin{pmatrix} - & 0 & + \\ 0 & - & + \\ - & - & ? \end{pmatrix}.$$

Proposition 2

The sub-dynamical system (30), (31), (33) without the real sector is locally asymptotically stable if a_1, a_3, γ_1, and γ_2 are sufficiently small, and a_2, γ_3 are sufficiently large.

Proof.

The trace $Tr(J)$, determinant $Det(J)$, and the three principle minors J_1, J_2, and J_3 are derived as follows:

$$Tr(J) = 2[v(a_1 - 1) - \gamma_3], \tag{45}$$

$$Det(J) = 2v[\gamma_3^2(a_1 - 1) - \gamma_1\gamma_3(-a_{2+} - a_{2-})], \tag{46}$$

$$J_1 = -2v[\gamma_3(a_1 - 1) + \gamma_1 a_{2-}], \tag{47}$$

$$J_2 = -2v[\gamma_3(a_1 - 1) + \gamma_1 a_{2+}], \tag{48}$$

$$J_3 = \gamma_3^2. \tag{49}$$

According to the already cited Routh-Hurwitz theorem, the necessary and sufficient condition for the stability of the 3D sub-dynamics is that $tr(J) < 0$, $J_1 + J_2 + J_3 > 0$, $det(J) < 0$, and $-tr(J)(J_1 + J_2 + J_3) + det(J) > 0$. □

The parameter γ_3, which captures the long-run adjustment of λ_+ and λ_-, plays an important role in stabilizing/destabilizing the system. The system tends to be stable if γ_3 is relatively large, as the reversion toward an average loan-to-reserve ratio is more pronounced.

As for the investigation of the global features of the complete 4D system, we turn to numerical simulations. Regarding parameter setting, while most of the parameters presented in the current model are qualitative, the only parameter that bears empirical relevance is the system leverage in terms of loan-to-reserve ratio. In reality, many central banks around the world have already abolished the reserve requirement since the introduction of deposit insurance by FDIC and Basel II-type regulation that emphasizes on capital rather than reserves (Di Giorgio 1999). Yet reserve requirement is still in place in many central banks that not only use it as a protection against deposit loss and bank run, but also proactively manipulate reserve requirement as an effective macroprudential policy instrument (Glocker and Towbin, 2015; Fungáčová, Nuutilainen and Weill, 2016). In the US, the reserve requirement is set at 10% for transaction accounts over \$122.3 million, while for other countries the requirement ranges from none to over 40%. For simplicity, here we set the aggregate loan-to-reserve ratio (when $x = 0$) at 10 in accordance with the Fed regulation, while the loan-to-reserve ratios for over-lending (optimistic) and under-lending (pessimistic) banks are set to be above and below this level. The possibility of off-the-balance sheet activity such as securitization may potentially overstretch the balance sheet of the overlending banks above the legal requirement. Here we set the long-run convergence of λ_+ and λ_- as $\bar{\lambda}_+ = 15$ and $\bar{\lambda}_- = 5$, respectively, while the other parameters are set as follows: $a_1 = 1.5$, $a_{2+} = -0.3$, $a_{2-} = -0.5$, $a_3 = 1.3$, $\sigma = 0.8$, $k = 0.1$, $T_c = 1$, $y_0^d = 11$, $d = 10$, $v = 0.4$, $\gamma_1 = 0.3$, $\gamma_2 = 0.4$, $\gamma_3 = 0.03$, $\xi_0 = 3.4$, $\xi_1 = 5$.

Concerning the behavioural parameters, since one of the main goals of the paper is to test the relevance of herding in the credit cycle, we assume a more than proportional reaction of banks to the relative proportion of agents in each cluster x and therefore set $a_1 > 1$. Different sets of simulations show that we simulate when herding effect is relatively weak ($a_1 < 1$) the system tends toward a stable equilibrium, while when the herding effect is relatively strong the equilibrium becomes unstable. The values of a_{2+}, a_{2-} are set under the reasonable assumption than pessimistic (optimistic) banks will reduce (increase) their loan-to-reserve ratio in order to prevent it from exploding (becoming null). The value of a_3 is set to introduce some (more than proportional) feedback from the real economy in order to account for the borrowers' situation. We postulate that the effect of herding and of the macroeconomic performance have a relative higher weight than the mean-reverting effect. Finally, the other parameters are set in order to smooth the dynamics enough to provide some visual insights from the plots.

Initial conditions are set as follows: $L(1) = 3; x(1) = 0.1; y(1) = 10; y^d(1) = 11; \lambda_+(1) = 10; \lambda_-(1) = 10$. Initial conditions do not seem to significantly affect the qualitative outcomes of simulations.

Figure 4 provides a representative simulation of the extended model. In the top panel we observe that the optimistic/pessimistic banks become increasingly optimistic/pessimistic over time, until they settle down to two distinct and irregular limit cycles with higher value of λ_+ and lower value of λ_-. The mid-left panel shows the dynamics of x characterized by a transition between a sustained period of optimism and a sudden switch to a period of pessimism. The similar dynamics is observed in output y at the buttom-left panel. The mid-right panel shows the cyclical dynamics of $g(.)$, which indicates a constant convergence and divergence of lending strategies between two groups of banks. In this scenario the convergence of the strategies smooths down the cycle, as shown by the milder swings in x compared with the previous settings. Also, the variable g displays a pattern with a "double-peak". Possibly, the first downswing in g during the cycle is not strong enough to revert the pattern of λ_+ (and therefore of y). The bottom-right panel shows a limit cycle with a three-stage dynamics of the Debt-to-GDP ratio.

Figure 5 provides the bifurcation diagrams of a_1, γ_1, γ_2, and γ_3 of the extended 4D model. The bifurcation diagram for a_1 show that a higher contagion value tends to destabilize the system. The bifurcation occurs for the two behavioural parameters γ_1 and γ_3 at $\gamma_1 \approx 0.2$ and $\gamma_3 \approx 0.04$, respectively. It indicates that these two behavioural parameters play a destabilizing role in the 4D system. We do not observe a clear bifurcation range for γ_2. Yet as γ_2 increases, the cycle diminishes in magnitude. This may be interpreted as a stabilizing factor if lending activity is more directed toward the real sector.

We run a simple policy experiment, introducing a ceiling for the loan-to-reserve ratio, in the spirit of bank capital regulation as in Basel II and III. Keeping the same parameter setting and defining a ceiling $\lambda_m = 9.5$ (as the corresponding measure of Basel III), fluctuations in λ_+ and in the level of output are eliminated. Figure 6 show the bifurcation diagram for x, showing that for lower λ_m, all the banks are optimistic although the level of output appears to be negatively affected, as illustrated by Figure 7. Fluctuations arise when the ceiling is 17 (corresponding to a reserve requirement of 5.9%).[12]

4 The Kaldorian Investment-Saving disequilibrium dynamics with credit-driven investment sector

This section relaxes the assumption of a real sector solely driven by the supply of credit. In particular, following Kaldor (1940), Chang and Smyth (1971), and Tu (1992), we include a Kaldorian-Keynesian dynamic multiplier where output is driven by the Investment-Saving disequilibrium. Formally:

$$\dot{y} = \sigma_K(I - S). \tag{50}$$

This assumption allows us to study the banks' behaviour in the presence of an autonomous dynamics of the real sector in a way that is at the same time simple and consistent with the modelling approach of this paper.

4.1 The credit-driven Kaldorian investment function

Kaldor (1940) discusses the non-linearity of investment sector by postulating that $\frac{dI}{dy}$ will be small both for low and for high levels of y relatively to its "normal" level, which can simply be characterized by the hyperbolic function: $I = a + tanh(ay)$ (Tu 1992). In the context of this chapter, we postulate that the investment function is *credit-driven*:

$$I = \dot{K} = \theta_I L^s, \tag{51}$$

$$= \theta_I \frac{T_c}{2}[(1 + x)\lambda_+ + (1 - x)\lambda_-], \tag{52}$$

$$= \theta_I \frac{T_c}{2}[(\lambda_+ + \lambda_-) + (\lambda_+ - \lambda_-)x]. \tag{53}$$

Hence

$$x = AI - B, \tag{54}$$

$$\text{where} \quad A = \frac{1}{\theta_I T_c/2(\lambda_+ - \lambda_-)}, \tag{55}$$

$$B = \frac{\lambda_+ + \lambda_-}{\lambda_+ - \lambda_-}. \tag{56}$$

It is important to relate this investment function to the value of x. We argue that in the extended model, the opinion formation index s is determined by

$$s = a_1 x + a_{2+}\lambda_+ + a_{2-}\lambda_- + a_K(y - y^\star) + d. \tag{57}$$

Now the parameter a_K enters the opinion formation index, which essentially characterizes the non-linear Kaldorian investment function. In a static sense, if we ignore the dynamics of $\lambda_{+\&-}$ and x ($\lambda_+ = \bar{\lambda}_+$, $\lambda_- = \bar{\lambda}_-$, $a_1 = a_{2+} = a_{2-} = d = 0$) then the static I-y relationship becomes Kaldorian hyperbolic, since

$$0 = v[(1 - x)\exp(s) - (1 + x)\exp(-s)], \tag{58}$$

$$\rightarrow 2a_K(y - y^\star) = ln\left(\frac{1 + x}{1 - x}\right). \tag{59}$$

Substituting equation (54) into equation (59) we have

$$2a_K(y - y^\star) = ln\left(\frac{1 + AI - B}{1 - AI + B}\right). \tag{60}$$

Equation (60) essentially captures the non-linear Kaldorian relationship between y and I, which resembles the *tanh*(.) specification of Tu (1992). We note that the level of investment I depends crucially on x, which measures not only the average opinion but also the relative number of optimistic/pessimistic banks. During the period of extreme pessimism where $x = -1$, the aggregate level of investment $I_- = (B - 1)/A$. During the period of extreme optimism, $I_+ = (B + 1)/A$. At the equilibrium of neutrality when $x = 0$, $I_0 = B/A$.

On the other hand, the Kaldorian saving function can simply be adopted from Tu (1992). As Keynes (1936) argues, the level of saving is primarily determined by the level of aggregate income:

$$S = S^\star + \theta_S(y - y^\star)^3. \tag{61}$$

In Equation (61), S^* captures the level of saving at neutral opinion equilibrium where $x = 0$. The value can be derived by equating S^* and I^*. Hence $S^\star = I^\star = \theta_I \frac{T_c}{2}(\bar{\lambda}_+ + \bar{\lambda}_-)$. We plot this Kaldorian I-S relationship in Figure 8.

4.2 The 4D credit-driven investment sector with Kaldorian I-S disequilibrium

The 4D dynamics with non-linear Kaldorian I-S disequilibrium is therefore written as

$$\dot{\lambda}_+ = \gamma_1(x + g(.)) + \gamma_2\dot{y} + \gamma_3(\bar{\lambda}_+ - \lambda_+), \tag{62}$$

$$\dot{\lambda}_- = \gamma_1(x - g(.)) + \gamma_2\dot{y} + \gamma_3(\bar{\lambda}_- - \lambda_-), \tag{63}$$

$$\dot{y} = \sigma_K(I - S), \tag{64}$$

$$\dot{x} = v[(1 - x)\exp(s) - (1 + x)\exp(-s)], \tag{65}$$

where

$$I = \theta_I L^s, \tag{66}$$

$$S = S^{\star} + \theta_S(y - y^{\star})^3, \tag{67}$$

$$s = a_1 x + a_{2+}\lambda_+ + a_{2-}\lambda_- + a_K(y - y^{\star}) + d. \tag{68}$$

By analysing the 2D sub-dynamics ($\lambda_+ = \bar{\lambda}_+, \lambda_- = \bar{\lambda}_-, a_1 = a_{2+} = a_{2-} = d = 0$), we can derive the Jacobian of the 2D sub-dynamics ($x = 0, y = y^{\star}$):

$$J = \begin{pmatrix} 0 & \sigma_K \theta_I \frac{T_c}{2}(\bar{\lambda}_+ - \bar{\lambda}_-) \\ 2va_K & 2v(a_1 - 1) \end{pmatrix}. \tag{69}$$

The Trace and Determinant are calculated as

$$Tr(J) = 2v(a_1 - 1), \tag{70}$$

$$Det(J) = -2va_K \sigma_K \theta_I \frac{T_c}{2}(\bar{\lambda}_+ - \bar{\lambda}_-). \tag{71}$$

Clearly, the Determinant *Det*(*J*) is always negative, implying that the neutral opinion equilibrium is always locally unstable.

As for the discussion of the global characteristics of the full 4D Kaldorian dynamics with credit-driven investment sector, we resort to numerical simulation. The parameters are set as follows: $a_1 = 2.5$, $\theta_I = 0.3$, $\theta_S = 0.4$, $y^{\star} = 10$, *ceteris paribus* for other parameters from the representative simulation of the baseline 4D CDGZ model discussed in previous section. As is shown in Figure 9, this simulation takes one step closer to a realistic scenario in the sense that the leverage cycle becomes asymmetric: during the boom period the leverage rises gradually for both optimistic and pessimistic banking sectors, which is then followed by a sudden slump during the period of recession (top-left). The real sector, on the other hand, moves in tandem with the banking sector (bottom-left): we observe the endogenous switching between the regime of stability and instability (characterized by the sudden falls of output). We also observe the *S*-shaped saving function and a nonlinear credit-driven investment sector characterized by a regular, 4-stage limit cycle (bottom-right).

5 The 7D model: the interaction between banks and a speculative financial sector

The previous section provides an analysis of the interaction between the credit sector and the real sector. Let us further enlarge the picture including a financial sector that invests in capital assets. The rationale for this extension is to study the possibly destabilising effects of the interaction between the real sector and the financial market according to Minsky's investment theory (Minsky 1975).

In order to model this interaction, we adopt the the framework of Charpe et al. (2012), which is very much in the spirit of the present paper and can be easily integrated in our model in order. Specifically, we postulate the following law of motion for output (y), replacing Equation (9):

$$\dot{y} = \beta_y[(a_y - 1)y + a_k(p_k - p_{k0})K + \bar{A}], \tag{72}$$

where Ā is autonomous expenditure; K is the total capital stock; $a_y \in (0, 1)$ is the propensity to spend; $a_k > 0$ captures the sensitivity of investment and consumption demand to deviations between the actual and the equilibrium level of capital stock.

As is pointed out by Charpe et al. (2012), the market for K is imperfect due to information asymmetries, adjustment costs, or institutional constraints, hence price do not move instantaneously to clear markets. We assume that the price of K moves according to the expected rate of return on the capital stock, ρ_k^e. The law of motion for capital price ($\hat{p_k}$) and the expected rate of return ($\dot{\pi_k^e}$) is given by

$$\hat{p_k} = \theta_S L^s(\rho_k^e - \rho_{k0}), \tag{73}$$

$$\dot{\pi}_k^e = \beta_{\pi_k^e}\left[\frac{1+x_p}{2}\hat{p_k} - \pi_k^e\right], \tag{74}$$

$$\rho_k^e = \frac{by}{p_k K} + \pi_k^e, \tag{75}$$

where b is the profit share; ρ_{k0} denotes the equilibrium level of expected rate of profit. here we assume that the profits are completely distributed as dividends. Furthermore, the aggregate level of loan issued by banking sector (L^s) enters equation (73) in the form of an adjustment parameter ($\theta_S L^s$), since credit expansion will accelerate the rise or fall of asset prices.

We further adopts the Weidlich-Haag-Lux approach to model the opinion formation dynamics of speculative investors. Following Charpe et al. (2012), we assume that there are $2M$ number of investors. Of these, M_c are chartists and M_f are fundamentalists so that $M_c + M_f = 2M$. Let $m = \frac{M_c - M_f}{2}$ and $x = \frac{m}{M}$. We focus on the difference in the size of the two groups (normalised by M). Let $p^{f\to c}$ be the transition probability that a fundamentalist becomes a chartist, and similarly for $p^{c\to f}$. The change in x_p depends on the relative size of each population multiplied by the relevant transition probability ($\exp(s_p)$ and $\exp(-s_p)$). To be precise, we define

$$\dot{x_p} = v_p[(1-x_p)\exp(s_p) - (1+x_p)\exp(-s_p)], \tag{76}$$

$$s_p = s_{xp}x_p + s_x x - s_{pk}(p_k - p_{k0})^2 - s_{\pi_k^e}(\pi_k^e)^2, \tag{77}$$

$$s = a_1 x + a2_+\lambda_+ + a2_-\lambda_- \tag{78}$$

$$+ a_3((a_y - 1)y + a_k(p_k - p_{k0})K + \bar{A}) + a_{sp}x_p + d, \tag{79}$$

where s_{xp}, s_{pk}, and $s_{\pi_k^e}$ are the cognitive parameters that determine the average opinion of investors, s_x and a_{sp} capture the cognitive interactions between banks and the speculative financial sector.

The full 7D dynamics with a speculative financial sector thus becomes:

$$\dot{\lambda}_+ = \gamma_1(x + g(.)) + \gamma_2\dot{y} + \gamma_3(\bar{\lambda}_+ - \lambda_+), \tag{80}$$

$$\dot{\lambda}_- = \gamma_1(x - g(.)) + \gamma_2\dot{y} + \gamma_3(\bar{\lambda}_- - \lambda_-), \tag{81}$$

$$\dot{y} = \beta_y[(a_y - 1)y + a_k(p_k - p_{k0})K + \bar{A}], \tag{82}$$

$$\dot{x} = v[(1-x)\exp(s) - (1+x)\exp(-s)], \tag{83}$$

$$\hat{p_k} = \theta_S L^s(\rho_k^e - \rho_{k0}), \tag{84}$$

$$\dot{\pi}_k^e = \beta_{\pi_k^e}\left[\frac{1+x_p}{2}\hat{p_k} - \pi_k^e\right], \tag{85}$$

$$\dot{x_p} = v_p[(1-x_p)\exp(s_p) - (1+x_p)\exp(-s_p)]. \tag{86}$$

We conduct numerical simulations to investigate the global properties of system (80–86). The parameters are set as follows: $a_1 = 1$, $a_{2+} = -0.3$, $a_{2-} = -0.5$, $a_3 = 1$, $d = 10$, $\sigma = 0.8$, $k = 0.1$, $T_c = 1$ $v = 0.4$, $\gamma_1 = 0.3$, $\gamma_2 = 0.4$, $\gamma_3 = 0.03$, $\bar{\lambda}_+ = 15$, $\bar{\lambda}_- = 5$, $\xi_0 = 0.2$, $\xi_1 = 3$, $s_{xp} = 0.9$, $s_{pk} = 2$, $s_{\pi_k^e} = 2$, $b = 0.1$, $K = 1$, $\beta_y = 0.4$, $a_y = 0.3$, $a_k = 0.5$, $p_{k0} = 0.8$, $A = 2$, $\theta = 0.1$, $\rho_{k0} = 0.7$, $\beta_{p_k^e} = 0.2$, $v_p = 0.3$, $s_x = 0.2$, $a_{xp} = 0.4$. Initial conditions are set as in previous simulations and $x_p(1) = 0.1; p_k(1) = 0.5; \pi_k^e(1) = 0.1$;

Figure 10 provides the simulation results. Again we stress that this is just one particular example, providing results that are interesting from the perspective of the 7D full dynamics of real-financial interaction. The result would be highly sensitive with different parameter settings. Yet it generates some interesting results that capture the real-financial market interaction in a stylized manner. The top-left panel shows a turbulent, chaotic swings of asset price in the financial sector generated from this deterministic framework. The top-mid/right panel shows the dynamics of "animal spirits" in both the banking sector (x) and the speculative financial sector (x_p). We observe that x and x_p generally move in tandem. Furthermore, we observe that the volatility of the mood swing in the financial market increases when the average opinion of the banking sector becomes more optimistic. This is consistent with the observed positive relationship between x and the volatility of p_k in the mid-right panel. In other words, as the sentiment in banking sector becomes increasingly optimistic, more speculative behaviours start to emerge in the stock market in the form of a larger swings of both x_p and p_k. The mid-left panel shows the irregular dynamics of real sector, as well as the positive relationship between output and stock price (mid-mid), which is similar to the simulation results of Charpe et al. (2012). Finally, the bottom-mid/right panel shows the dynamics of λ_+ and λ_-. It indicates that the loan-to-reserve ratio of the pessimistic bank tends to be more volatile in the long run. Overall, This set of simulations captures the crucial role of banks in propagating financial instability, which eventually transmits into the macroeconomic fluctuations in the real sector. It indicates that the interplay between "animal spirits"-driven banking sector, coupled by a turbulent speculative financial sector will not only lead to a chaotic swings of asset prices, but also give rise to an unpredictable evolutionary path in both credit sector and real sector in the long run.

We also consider the efficacy of policy interventions in the form of Tobin-type tax (at the rate τ) in line with Charpe et al. (2012), where the law of motion for capital gain expectations ($\dot{\pi}_k^e$) is modified as

$$\dot{\pi}_k^e = \beta_{\pi_k^e}\left[\frac{1 + x_p}{2}(1 - \tau)\hat{p}_k - \pi_k^e\right]. \tag{87}$$

Figure 11 provides the simulation results augmented by a Tobin-type tax parameter.[13] From this bifurcation plot we observe that the Tobin tax parameter τ has a stabilizing effect over the system. When τ increases and passes through $\tau \approx 0.24$, the system switches from a regime of instability to stability with the disappearance of limit cycle. It is arguable that Tobin tax is able to stabilize the system by altering the adjustment speed of capital gain expectations.

6 Concluding remarks

The inherent instability of the credit cycle lies at the centre of financial crises (Minsky, 1975; Kindleberger, 1989). In particular, the commercial banks' coordination failure, driven by waves of optimism and pessimism, ultimately leads to sub-optimal credit provision and amplifies the credit cycle.

This paper presents a simple model to investigate this issue. The analysis of the model shows that the self-fulfilling and contagious waves of optimism and pessimism amongst bankers lead to a boom-bust pattern for the credit cycle.

Summarising the results, we find that the switching and the herding behaviours of banks are destabilising for the economy. The study of the two scenarios with a constant or variable loan-to-reserve ratio for banks highlights the different destabilizing effects of herding and switching. Switching of banks between the two clusters with different levels in the supply of loan is able by itself to generate fluctuations in the aggregate output. Introducing herding in the banks' decisions about their loan-to-reserve adds to the instability and amplifies the fluctuations. Instability and amplitude of fluctuations grow with the intensity of herding. A larger value in the herding parameter also causes higher values of the aggregate leverage ratio. The study of the stability and the numerical analysis demonstrate that the parameters that measure the interaction of banks prove to be more relevant than other behavioural parameters, such as the targeted loan-to-reserve ratio.

The baseline 4D model is further extended in two different scenarios. The first extension involves a Kaldorian I-S disequilibrium with credit-driven investment sector. We keep the system remaining in 4 dimensions, yet the nonlinear credit-driven investment function can be naturally derived from the isocline of x. The local stability analysis indicates that the neutral equilibrium is always unstable, yet the global stability of the 4D system is insured due to the self-stabilizing long run adjustment of loan-to-reserve ratio (λ_+ and λ_-) both in cognitive (through influencing the average opinion index) and in behavioural terms (through influencing the behavioural rules of bank's lending activity). The representative simulation is characterized by asymmetric leverage cycles with gradual boom and sudden bust, as well as an endogenous switching from stability to instability in the real sector. The next extension of the present framework involves a properly modelled speculative financial sector, adopting from the Dynamic Stochastic General Disequilibrium framework proposed by Charpe et al. (2012). The simulation captures a positive correlation between bank's sentiment and the degree of exuberance in the speculative market, in the form of volatility of asset prices and mood swing of speculative investors. Besides providing a better assessment of the destabilising effects of the credit cycle on the real sector, the interaction between the banking sector and a speculative financial sector can open interesting perspective for policy analysis in the present setting. The complete model is useful to study the feedback effect on the credit cycle of the phenomena of over-investment and over-indebtedness.

Future developments of the present framework will first consider the effects of irrational exuberance and herding of the interbank market, by allowing banks to borrow and lend reserves. Second, other relevant variables, such as interest rate and asset price, can be included to provide further policy indication and allow to model a more realistic behaviour of financial agents. Third, a more refined modelling of the behaviour of the real sector is necessary in order to study the interaction of animal spirits on both sides of the credit market.

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A Simulation results

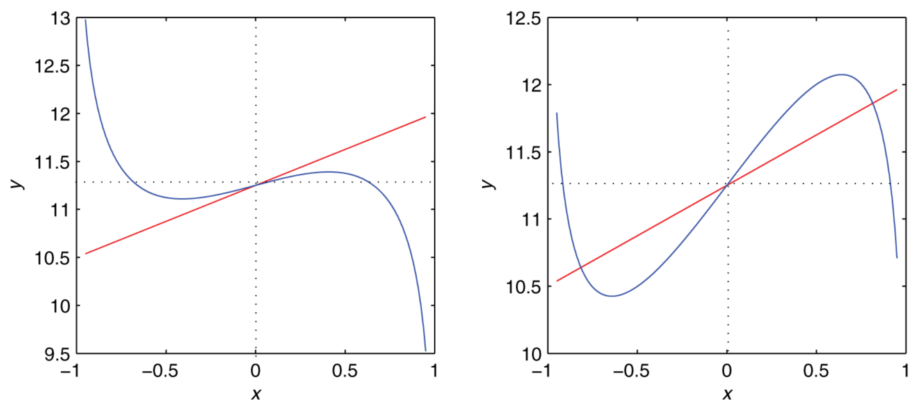

Figure 2: The isocline of 2D model with $\dot{y} = 0$ (straight line) and $\dot{x} = 0$.

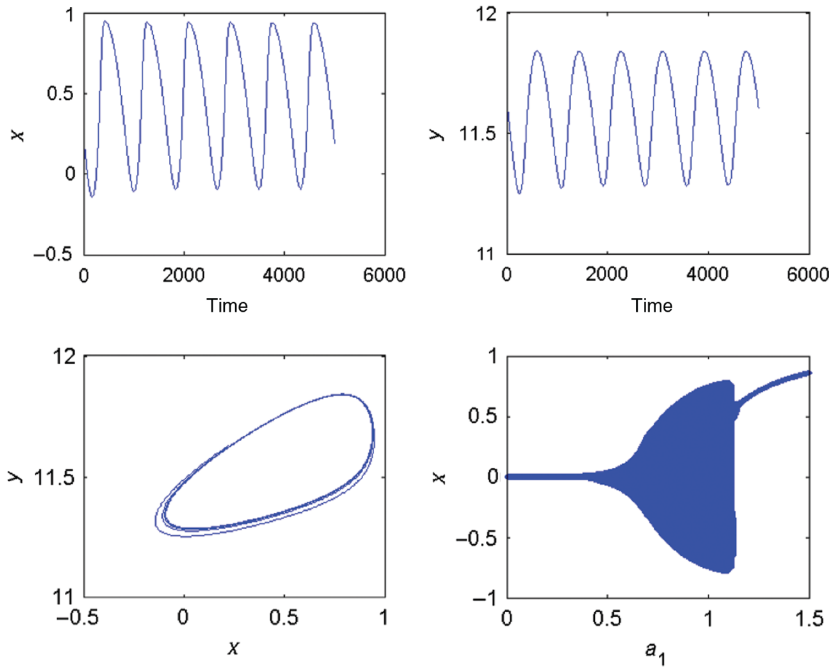

Figure 3: Simulations of the baseline model.

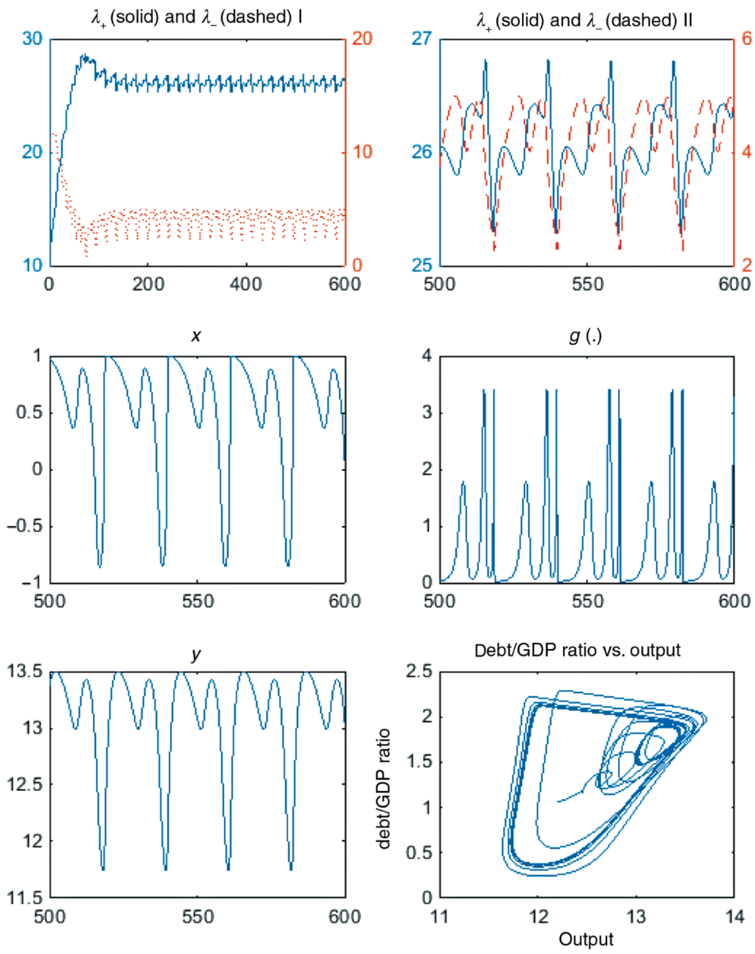

Figure 4: The 4D model: representative simulation (top left panel: simulation over 600 periods; other panels: magnification over 100 periods.)

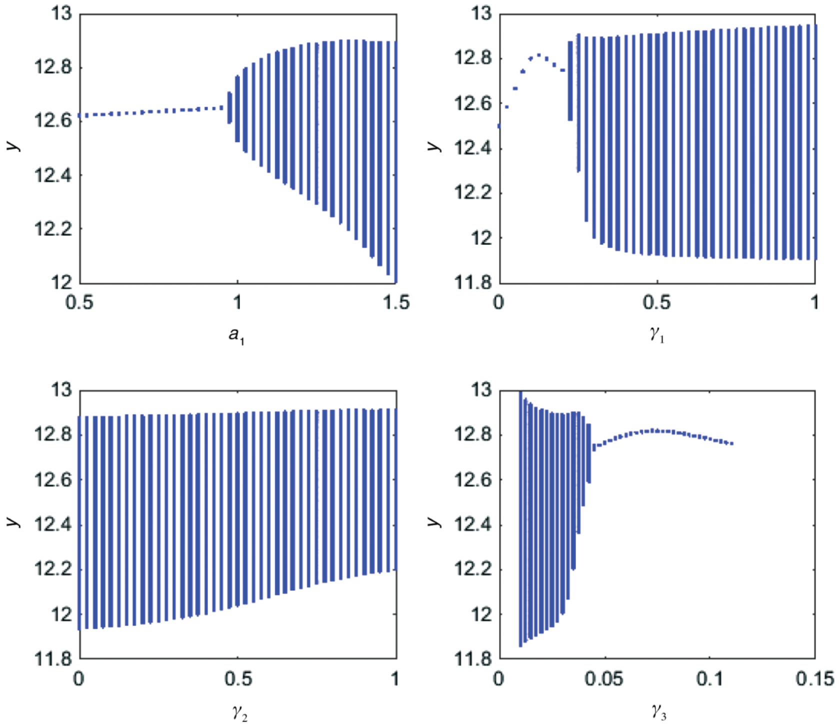

Figure 5: The 4D model: bifurcation diagrams.

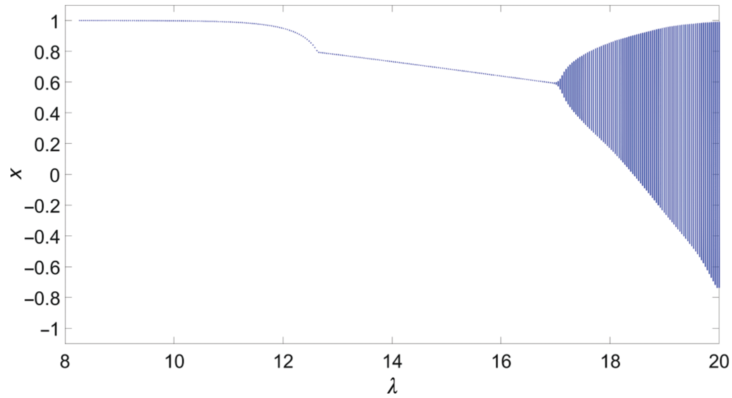

Figure 6: The 4D model: bifurcation diagram for λ_m and x.

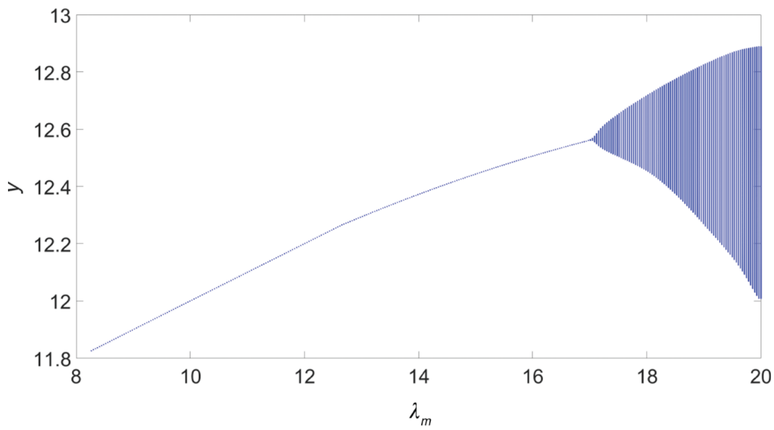

Figure 7: The 4D model: bifurcation diagram for λ_m and y.

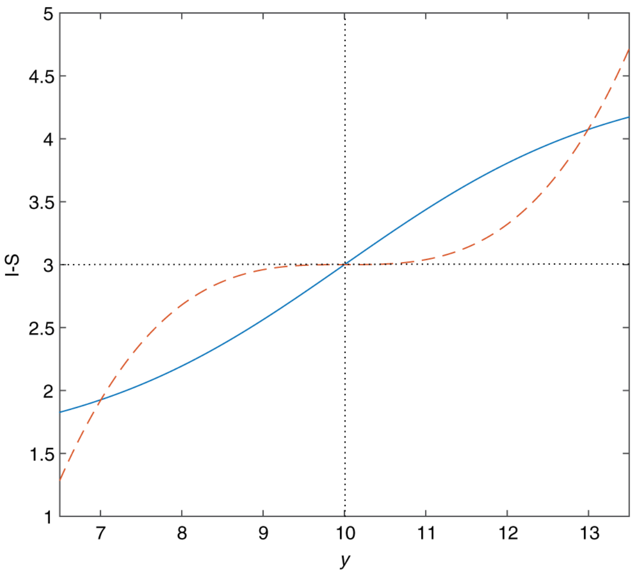

Figure 8: The Kaldorian I-S disequilibrium.

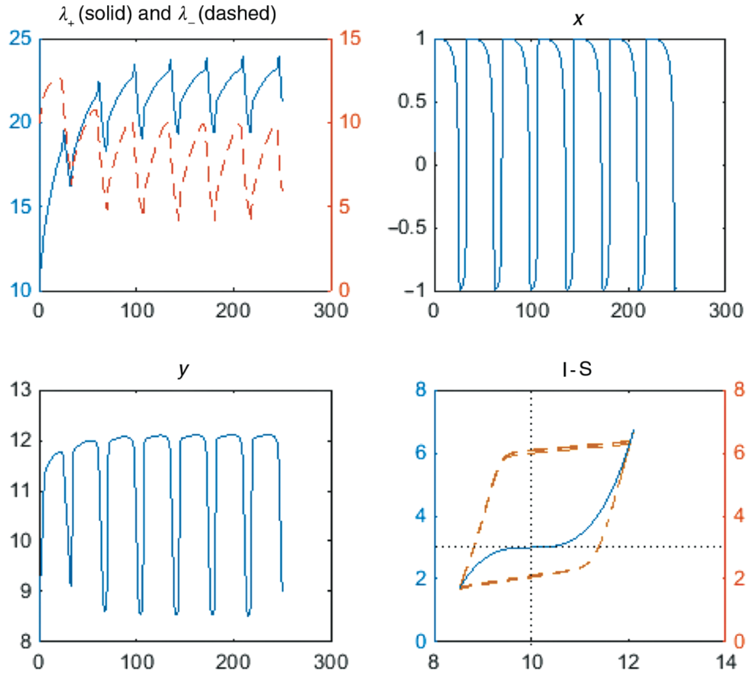

Figure 9: The 5D model with Kaldorian I-S Disequilibrium.

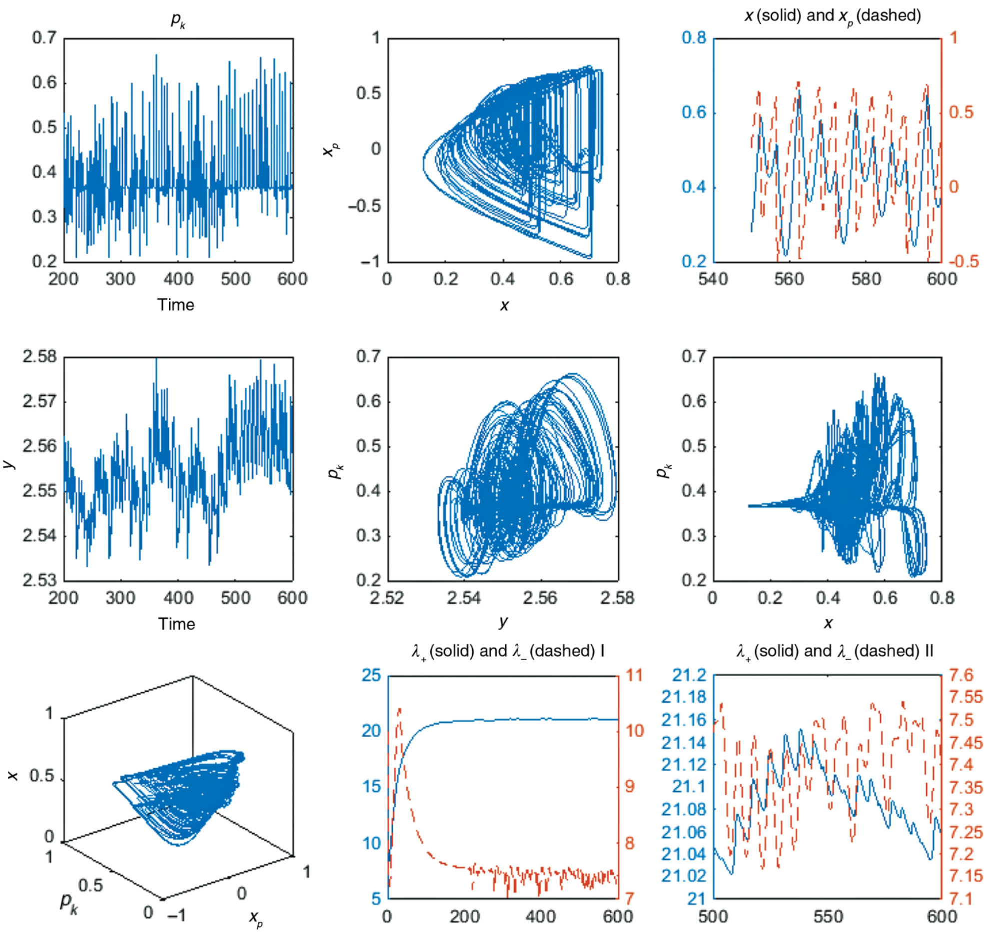

Figure 10: The extended 7D model with a speculative financial sector.

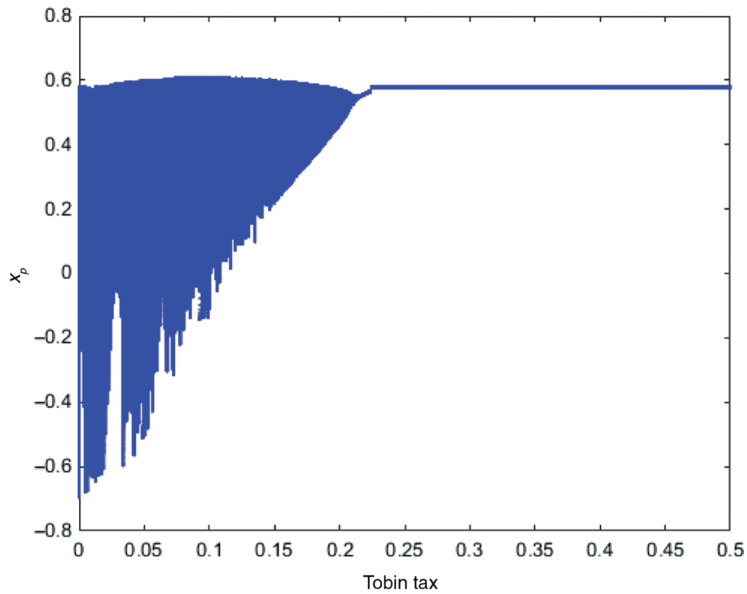

Figure 11: Bifurcation diagram: the effect of Tobin-type tax.

Notes

1 Exceptions are Asanuma (2013) and Berger and Udell (2004).
2 The lending attitude also involves the composition of the credit portfolio and the decision of lending in increasing proportion to a particular sector, as the latest crisis dramatically showed. On the pros and cons of diversification see for example Battiston et al. (2007).

3 In addition to this strand of literature that models the "animal spirits" in disequilibrium process, there are numerous papers that take a more standard equilibrium approach. For example, DeGrauwe (2011) develops a DSGE model that features waves of optimism and pessimism by incorporating agents' cognitive limitations. For a study of the banks lending attitude in an agent-based model see Asanuma (2013).
4 See Charpe et al. (2011), Chiarella and Di Guilmi (2011), and Delli Gatti, Gallegati, and Gardini (1993).
5 For an analytical investigation of the separate effects of switching and herding on market volatility see Di Guilmi, He, and Li (2014).
6 The modelling of an inter-banks market is beyond the scope of the present paper and consequently we assume it away. Reserves therefore can only be unborrowed.
7 Given that the focus of the present paper is the effect of bank's herding behaviour, it is convenient to assume a *passive* real sector. A more refined firms' investment behaviour is proposed in Section 4.
8 The following parameters are set for the isoclines: $a_1 = 0.9$ (left), $a_1 = 1.4$ (right), $a_2 = 0.4$, $T_c = 1$, $k = 0.1$, $d = 0$, $v = 0.7$, $y_0^d = 10$, $\bar{\lambda}_- = 5$, $\bar{\lambda}_+ = 20$. $\sigma = 0.8$.
9 This is consistent with the literature regarding horizontalist & structuralist views of banking (Moore, 1988; Goodhart, 1989), as well as the evidence a positive correlation between aggregate output and private debt presented in the literature on financial fragility and business cycle.
10 By setting the $LHS = 0$, we derive that $a_{2+}[\frac{\gamma_1}{\gamma_3}(x + \xi_0 e^{-\xi_1 x^2}) + \bar{\lambda}_+] + a_{2-}[\frac{\gamma_1}{\gamma_3}(x - \xi_0 e^{-\xi_1 x^2}) + \bar{\lambda}_-] = \frac{1}{2} ln[\frac{1+x}{1-x} e^{-2(a_1 x + d)}]$. This equation has no closed form solution.
11 Setting the $LHS = 0$ in equation (42), we find that $s = 0$ when $x^\star = 0$, hence $d = -a_{2+}\lambda_+^\star - a_{2-}\lambda_-^\star$.
12 Qualitatively similar results are obtained introducing the same type of constraint in the further extensions of the model presented below.
13 Here the Tobin-type tax is referred to as a tax over the capital gains, which is different from Tobin tax that is levied on financial transactions in currency market.

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