A Robust Design Optimization Approach for Electromagnetic Devices Considering Probability Uncertainties

Bo Ma[1], Jing Zheng[2], Gang Lei[1], Jianguo Zhu[1,3]

[1] Faculty of Engineering and Information Technology, University of Technology Sydney, NSW 2007, Australia

[2] State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, School of Mechanical and Vehicle Engineering, Hunan University, Changsha, China

[3] School of Electrical and Information Engineering, University of Sydney, New South Wales, Australia

For the robust optimization problems with probabilistic uncertainties, classical approaches based on Monte Carlo method usually require a huge amount of samples for the robustness estimation of the performance. This paper proposes an efficient robust optimizer based on univariate dimensional reduction method and evolutionary algorithms. The univariate dimensional reduction method is applied for the probabilistic property estimation of the performance functions with significantly reduced sample number. The comparison results of the proposed approach and classical approach based on Monte Carlo method for a numerical example illustrate the feasibility of the presented approach.

***Index Terms*—Monte Carlo method, univariate dimensional reduction method, evolutionary algorithm, probabilistic uncertainties.**

I. Introduction

In the operation of electromagnetic devices, there are various types of unavoidable uncertainties related to material properties, manufacturing errors, loads, etc. Robustness considering these perturbations means acceptable performance fluctuations that do not violate the constraints. In order to prevent the low-reliability design, the influence of the uncertainties should be modeled in the optimization process. For the design problems with probabilistic uncertainties, Monte Carlo method [1] is usually applied to assess the robustness of the objective or constraint functions by sampling according to the known mean and standard deviation of the uncertain variables. Solutions should have acceptable fluctuations under these perturbations under these fluctuations However, thousands of or more function calls are usually required to quantify the uncertainty, the calculation burden will be extremely large especially for high dimensional problems and time-consuming finite element analysis models. Therefore, optimizer with an efficient reliability quantification approach is usually the research motivation for robust optimization.

In this work, the univariate dimension-reduction method [2] is introduced into the optimization process to approximate the mean and variance of the objective and constraints functions with effectively reduced sample numbers. A fast robust optimizer framed on evolutionary algorithms and the dimension-reduction method is proposed. The optimization results of a benchmark design problem of brushless DC motor illustrate the efficiency of the proposed approach.

II. Robust Optimization Model with Uncertainties

A robust design model involving probability uncertainties can be expressed as below

$$\begin{aligned}&\min: f[\mu(\boldsymbol{X}), n_s\sigma(\boldsymbol{X}), \boldsymbol{Y}]\\ &\text{s.t.}:\ g_i[\mu(\boldsymbol{X}), n_s\sigma(\boldsymbol{X}), \boldsymbol{Y}] \le 0,\ i=1,...,m\end{aligned} \tag{1}$$

where $\boldsymbol{X}$ is the vector of uncertain variables, μ and σ are the mean and standard deviation respectively, n_s is the sigma level, $\boldsymbol{Y}$ is the design parameters.

III. Probability Property Estimation with Univariate Dimensional Reduction Method

For an objective function $f = f(\boldsymbol{X})$ with n probability variables, the kth origin moment $E(f^k)$ can be expressed as:

$$\begin{aligned}E(f^k) &= E\left(\left[f(\boldsymbol{X})\right]^k\right)\\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}...\int_{-\infty}^{\infty} f^k(\boldsymbol{X})\, p(X_1, X_2, ..., X_n) dX_1 dX_2 ... dX_n\\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}...\int_{-\infty}^{\infty} f^k(\boldsymbol{X})\, p(X_1)\, p(X_2) ... p(X_n) dX_1 dX_2 ... dX_n\end{aligned} \tag{2}$$

where $p(X_1, X_2, \ldots, X_n)$ is the joint probability distribution function of probability uncertain variables.

Using the univariate dimension reduction method, the multidimensional problem will be transformed into a series of one-dimension problem. As a result, the multidimensional Gauss integral turns to a one-dimension Gauss integral. Then the compliance can be approximated as follows:

$$f(\boldsymbol{X}) \cong \sum_{j=1}^{n} \hat{f}_j(\boldsymbol{X}) - (n-1)\, g(u_1, u_2, \cdots, u_n) \tag{3}$$

where $\hat{f}_j(\boldsymbol{X}) = f(u_1, \cdots, u_{j-1}, X_j, u_{j+1}, \cdots, u_n)$, u_j is the mean value of X_j. Then the kth origin moment of compliance is:

$$E(f^k) = E\left(\left[\sum_{j=1}^{n} \hat{f}_j(\boldsymbol{X}) - (n-1)\, f(u_1, u_2, \cdots, u_n)\right]^k\right) \tag{4}$$

Based on the binomial theorem, we can obtain:

$$E(f^k) = \sum_{i=0}^{k}\binom{k}{i} E\left(\sum_{i=1}^{n}\hat{f}_j(\boldsymbol{X})\right)^i \left[-(n-1)\, f(u_1, u_2, \cdots, u_n)\right]^{k-i} \tag{5}$$

Define

$$S_j^i = E\left[\left\{\sum_{i=1}^{j}\hat{f}_j(\boldsymbol{X})\right\}^i\right] \tag{6}$$

in which $j = 1, \cdots, n$; $i = 1, \cdots, k$. S_j^i can be calculated by the following recursion formula (7). Then the origin moments of the system can be calculated with $3n+1$ samples in total from the origin moments of subsystems with a single variable. Finally, the mean and standard deviation of the objective or constraint functions can be calculated as (8).

$$
\begin{aligned}
&S_1^i = E\left[\left(\hat{f}_1(\boldsymbol{X})\right)^i\right]; && i = 1,\cdots,k \\
&S_2^i = \sum_{l=0}^{i}\binom{i}{k} S_1^l E\left[\left(\hat{f}_2(\boldsymbol{X})\right)^{i-l}\right]; && i = 1,\cdots,k \\
&\vdots \\
&S_j^i = \sum_{l=0}^{i}\binom{i}{l} S_{j-1}^l E\left[\left(\hat{f}_j(\boldsymbol{X})\right)^{i-l}\right]; && i = 1,\cdots,k \\
&\vdots \\
&S_n^i = \sum_{l=0}^{i}\binom{i}{l} S_{n-1}^l E\left[\left(\hat{f}_n(\boldsymbol{X})\right)^{i-l}\right]; && i = 1,\cdots,k
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
&\mu(f(\boldsymbol{X})) = E(f(\boldsymbol{X})) \\
&\sigma(f(\boldsymbol{X})) = \sqrt{E(f^2(\boldsymbol{X})) - E^2(f(\boldsymbol{X}))}
\end{aligned} \tag{8}
$$

IV. Design Optimization Framework

The robust optimization framework can be easily established combined with general global optimization algorithm such as differential evolution algorithm. To reduce computation burden further, the application of the dimension reduction approach is conducted after the deterministic constraint value estimation in the upper step. For the population which violates the constraints, there is no need for the robustness assessment. Then the computation time can be reduced. The flowchart of the framework will be illustrated in the full paper.

V. Design Example

A brushless DC wheel motor benchmark is investigated in this work as mono-objective and multi-objective cases [3]. The design parameters and their optimization intervals are listed in Table I. The mean and standard deviation of the uncertain variables are presented in Table II. The deterministic mono-objective problem of the benchmark aims to have the best efficiency. For the bio-objective case, the efficiency objective is kept while reducing the mass respecting the same technical constraints which can be written as

$$
\begin{aligned}
&obj. \quad \begin{cases} \max \ \eta \\ \min \ M_{tot} \end{cases} \\
&s.t. \quad \begin{cases} M_{tot} \le 15kg, D_{\text{int}} \ge 76mm, I_{\max} \ge 125A \\ D_{ext} \le 340mm, T_a \le 120°C, discr \ge 0 \end{cases}
\end{aligned} \tag{9}
$$

where η, M_{tot}, I_{max}, D_{int}, D_{ext}, T_a, and *discr* are the efficiency, total mass of the active parts, demagnetization phase current, inner and outer diameter, the temperature of the magnets, and the determinant used for the calculation of the slot height respectively. Their robust function can be transferred to the formation as expressed in equation (1), in which the sigma level n is defined as 6 for the ensuring the robustness.

TABLE I
Design Parameters of the Brushless DC

Par.	Description	Unit	lower	upper
B_e	Maximum magnetic induction in the air gap	T	0.5	0.76
B_d	Average magnetic induction in the teeth	T	0.9	1.8
B_{cs}	Average magnetic induction stator back iron	T	0.6	1.6
Ds	Stator out diameter	mm	150	330
J	Current density	A/mm^2	2	5

TABLE II
Uncertain Parameters of the Brushless DC

Par.	Description	Unit	μ	σ
α	Width of the stator tooth	deg	30	0.08
e	Length of air gap	mm	0.8	0.02
L_m	Length of the motor	mm	45	0.1
β	Width of the intermediate tooth	deg	6	0.08

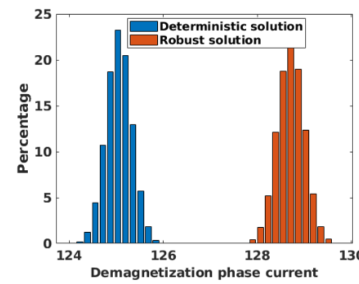

Fig. 1. Comparison of the mono-objective optimization results

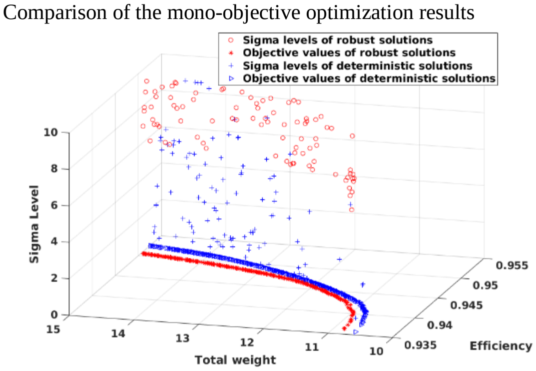

Fig. 2. Comparison of the bi-objective optimization results

For the mono objective case, Fig. 1 illustrates the demagnetization phase current distribution of the deterministic and robust solution considering the uncertainties verified by Monte Carlo analysis. Fig. 2 illustrates the Pareto results of the deterministic and proposed robust approach. Their sigma levels achieved by Monte Carlo analysis are also presented in the figure. For both mono objective and bi-objective cases, the sigma levels of the robust results are no smaller than preset value which proves the accuracy of the dimension reduction approach. Meanwhile, only 13 samples are required for the sigma level assessment of each population which means much higher efficiency than the Monte Carlo method.

References

[1] G. Lei, J. Zhu, and Y. Guo, “Multidisciplinary Design Optimisation Methods for Electrical Machines and Drive Systems,” Springer-Verlag Berlin Heidelberg, 2016.

[2] S. Rahman, H. Xu, “A univariate dimension-reduction method for multi-dimensional integration in stochastic mechanics,” Probabilistic Engineering Mechanics, vol.19, pp.393-408, 2004.

[3] S. Brisset, and P. Brochet, “Analytical model for the optimal design of a brushless DC wheel motor,” COMPEL-The international journal for computation and mathematics in electrical and electronic engineering, vol. 24, no.3, pp. 829-848, 2005.