© 2019. This manuscript version is made available under the CC-BY-NC-ND 4.0 license http://creativecommons.org/licenses/by-nc-nd/4.0/
The definitive publisher version is available online at https://doi.org/10.1016/j.jbankfin.2018.12.016

Predictive Blends: Fundamental Indexing Meets Markowitz

Sergiy Pysarenko[♮,*], Vitali Alexeev[♯,♮], Francis Tapon[♮]

♯ Department of Economics and Finance,
University of Guelph, Ontario N1G2W1, Canada
♮ Finance Discipline Group, UTS Business School, University of Technology Sydney
Sydney, New South Wales 2007, Australia

November 29, 2018

Abstract

When constructing a portfolio of stocks, do you turn a blind eye to the firms' future outlooks based on careful consideration of companies' fundamentals, or do you ignore the stocks' correlation structures which ensure the best diversification? The fundamental indexing (FI) and Markowitz mean-variance optimization (MVO) approaches are complementary but, until now, have been considered separately in the portfolio choice literature. Using data on S&P 500 constituents, we evaluate a novel portfolio construction technique that utilizes the benefits of both approaches. Relying on the idea of forecast averaging, we propose to blend the two previously mentioned techniques to provide investors with a clear *bi*nocular vision. The out-of-sample results of the blended portfolios attest to their superior performance when compared to common market benchmarks, and to portfolios constructed solely based on the FI or MVO methods. In pursuit of the optimal blend between the two distinct portfolio construction techniques, MVO and FI, we find that the ratio of market capitalization to GDP, being a leading indicator for an overpriced market, demonstrates remarkably advantageous properties. Our superior results cannot be explained by classic asset pricing models.

Keywords: fundamental indexing, portfolio optimization, equities, forecast averaging, blended portfolio.
JEL: G11, C58, C63

*Corresponding author: Email: spysaren@uoguelph.ca; Address: Department of Economics and Finance, University of Guelph, 50 Stone Road East, Ontario N1G2W1, Canada; Phone: +1 226 343 9345.

1 Introduction

The following analogy will help motivate our argument. Metallurgy teaches us that blending different metals produces alloys with better properties than their pure constituents. Even if new additions represent a very small percentage of the new alloy, its properties can change dramatically. For instance, duralumin, contains less than 6% of additives to 94% aluminium, but these additives dramatically change the properties of otherwise soft aluminium to an aircraft-grade strong alloy. We show that in composing stock portfolios the same phenomenon exists: blending portfolio construction approaches results in "blended" portfolios that outperform the benchmarks that sole-approach portfolios do not beat.

In this paper, we propose an innovative portfolio blending technique, combining the efficient portfolio selection method of Markowitz [1952] that takes into account the covariance structure of portfolio holdings and the fundamental indexing (FI) approach that favours investments with sound economic, financial, and managerial features.

Markowitz [1952] distinguishes between two stages in the portfolio selection process. The first stage is about forming beliefs about future performance. In practice, this often translates into reliance on historical data in estimating future rates of returns and their correlations. The second stage relies on the beliefs formed in the first stage and involves selecting a portfolio. Focusing only on the second stage, Markowitz [1952] introduces the mean-variance optimization (MVO) method for portfolio selection recommending that the choice of appropriate expected return and variance-covariance matrix "...should combine statistical techniques and the judgment of practical men..." [Markowitz, 1952, p.91]. The conventional approach often ignores the need to develop appropriate beliefs. As Markowitz emphasizes, it is our responsibility to use "observation and experience" to develop "beliefs about the future performances" [Markowitz, 1952, p.77]. While predicting future performance of stocks may be a daunting task, there is strong evidence that fundamental analysis may have some merit (Arnott et al. [2005], Walkshäusl and Lobe [2010], Basu and Forbes [2014]). As discussed in the forecast combination literature [Eklund and Karlsson, 2007, Smith and Wallis, 2009, etc.], we believe that fundamental analysis may improve the out-of-sample performance of MVO portfolios.

In practice, the MVO method relies on past returns to predict expected returns and estimate correlations. Past correlations predict future correlations much better that past returns predict future returns [Cuthbertson and Nitzsche, 2005, p.158]. Moreover, past returns fail to predict future returns in the long-run [Jorion, 1986, Poterba and Summers, 1988]. Given the volatile nature of these underlying processes, the MVO method likely produces superior out-of-sample results only for short-term investments. To mitigate this, frequent portfolio rebalancing based on the latest historical data is recommended for consistent superior results, but leads to high portfolio turnover and increased transaction costs. Transaction costs are of particular concern for funds with long-term performance objectives. Thus, in the industry, long-term investments are often based on "the judgment of practical men", rooted in fundamental analysis. In turn, fundamental analysis focuses on financial statements and the economic health of a company in an attempt to evaluate its long-term economic prospects, assessing its future growth, and investment potential.

Taken separately, both the classical MVO and the FI methods have their own limitations: the FI approach ignores the correlation structure of stocks' returns, while the classic MVO method is silent about the firms' fundamentals, which may well be the driving factors of the stocks' future performance. Berger et al. [2013] have also shown empirically that the MVO technique provides some diversification gains. Our blending technique combines the classical MVO method and the FI approach, by bridging the two stages of portfolio construction mentioned in Markowitz [1952]. Relying on 29 years of historical data we test and analyze the out-of-sample performance of our proposed blending method and show that our blended portfolios are superior to conventional benchmarks as well as portfolios based on each method alone. Heteroskedasticity and autocorrelation (HAC) robust inference tests developed by Ledoit and Wolf [2008] show that our technique delivers statistically significantly higher Sharpe ratios than the (value weighted) S&P 500 Index and the Equally-Weighted S&P 500 Index.

Currently the MVO and the FI literatures are isolated from each other.[1] Each of these literature streams considers stocks through a specific "oculus" described in the next two paragraphs. Up until now stocks have been considered separately through either one of these oculi.

In the first "oculus" considered, the MVO method, the expected returns and the variance-matrix are calculated based on in-sample information. Securities are sorted according to the MVO procedure, by maximizing the expected portfolio return while attaining a specific level of standard deviation. Since the introduction of the MVO by Markowitz [1952], myriad methods have been proposed in an attempt to refine this approach and offer superior out-of-sample performance. Among the most noticeable and practical extensions of the MVO method are those that control for outliers. Outliers often result in biased estimates of sample statistics translating in disproportionate portfolio holding weights. Several prominent robust techniques have been proposed to take this into account. For example, Ledoit and Wolf [2004] introduce a method that shrinks the sample covariance matrix to a well-conditioned parsimonious structure to reduce estimation errors that were shown to bias the classic MVO method. As an alternative to shrinkage methods, limiting portfolio holdings only to long positions, can produce similar results [Jagannathan and Ma, 2003]. However, Jagannathan and Ma [2003] note that such methods might lead to poor diversification, with only 20-25

[1]The FI approach was first proposed in Arnott et al. [2005] for US data; methodological improvement and empirical evidence can be found in Treynor [2005], Dopfel [2008]. Walkshäusl and Lobe [2010] and Basu and Forbes [2014] provide international evidence for the FI approach. Extensions and/or empirical evidence in favour of the MVO approach are too numerous to be listed here, however, for excellent surveys of the literature please refer to Markowitz et al. [2000] and Rubinstein [2002]. In a recent paper, Domowitz and Moghe [2018] consider a case where an exogenously pre-chosen "core" portfolio is complemented with other stocks based on the MVO method, without specifying how the "core" portfolio is constructed, and relying on expected returns of the individual components. To the best of our knowledge, no paper considers a portfolio construction strategy that combines the FI and MVO approaches. In our paper, we also propose the blending methodology based on economic conditions without relying on hard-to-predict expected returns of individual components.

stocks in the portfolio. Thus, to increase diversification and reduce the effect of measurement errors, it is possible to set up an upper bound on weights (e.g., 5-10%).[2] Since the MVO method suffers from the negative effects caused by measurement errors, outliers and *blindness* to firms' fundamentals (which are our second "oculus"), the performance of the classic MVO method, even with adjustments for outlier effects, often does not exceed market benchmarks such as equally- or capitalization-weighted portfolios in out-of-sample tests.[3] Hence, if the blended approach shows statistically significant results, they cannot be attributed to the MVO part of the technique alone.

We now shift our focus to the other "oculus", the FI approach, pioneered by Arnott et al. [2005]. In this approach, firms are ranked based on their fundamentals and securities are allocated proportionally to their overall fundamental scores. The fundamentals might include book value, free cash flow, revenue, sales, dividends, total employment, etc. In a recent paper, Asness et al. [2015] argue that Fundamental Indexing is, basically, systematic value investing. Otuteye and Siddiquee [2015] add that the FI approach helps overcome cognitive biases. The FI approach significantly outperforms major benchmarks based on US market data [Arnott et al., 2005]. Walkshäusl and Lobe [2010] apply the FI approach to stocks from 50 countries and find that the FI approach outperforms capitalization-weighted portfolios in most countries. However, after applying the robust-to-fat-tails performance test proposed by Ledoit and Wolf [2008], the FI portfolios in only 6 countries and the global FI portfolio have statistically significant positive differences in Sharpe ratios. Our empirical results confirm that in the US, the FI portfolio outperforms the cap-weighted portfolio, but these results are not statistically significant.[4] Hence, if the blended approach shows statistically significant results in our US-based study, they cannot be attributed to the FI part of the technique alone.

Out of all portfolios constructed with the MVO method, the richest information about the correlation structure is contained in the Global Minimum Variance (GMV) portfolio[5], which is based solely on the variance-covariance matrix and achieves the highest level of diversification. More importantly, construction of the GMV portfolio does not rely on often noisy estimates of individual expected returns, which makes it the portfolio of choice in blending with the FI portfolio. Firms' fundamentals help us detect and concentrate on 'healthy' stocks that are likely to grow in the long-run, while the assessment of the correlation structure allows us to construct well-diversified portfolios.

Before we discus the "how" in our next section, one question remains: In what proportion do we combine the GMV and FI portfolios? Given that the FI approach is relatively new,

[2]Coincidentally, these weight recommendations are in accord with guidelines used by many investment funds that try to avoid excessive dominance of a single security.

[3]The p-value for the tangency MVO portfolio vs. the Equally-Weighted S&P 500 is 0.543; the p-value for the GMV portfolio against the Equally-Weighted S&P 500 is 0.098. We show p-values of all portfolios against the benchmarks in Table 2.

[4]The p-value for the difference in Sharpe ratios of FI portfolio vs the Equally-Weighted S&P 500 is 0.235, which is not statistically significant at conventional levels.

[5]In the GMV portfolio we find mostly low volatility companies. As Walkshäusl [2013] shows, high quality firms exhibit lower volatility than low quality firms, hence we expect the GMV portfolio to include a larger number of high quality firms than the S&P 500.

and is profoundly different from the MVO method, these two approaches have not yet been combined, even though each method offers distinctive benefits for portfolio choice problems. In fact, Hong and Wu [2016] show empirically that information on past returns and on the firms' fundamentals are complementary. They show that in "good times", when volatility is low, past returns provide better information about future returns. However, fundamentals perform better in "bad times", when volatility in the market is high. In such periods, past returns are not that informative and investors are forced to rely on firms' fundamentals. Thus, a portfolio allocation strategy should rely more on past returns (the GMV portfolio) in times of low volatility and rely more on the firms' fundamentals (the FI portfolio) in times of high volatility. It is a daunting task to predict "good" and "bad" times. We, however, use a metric often mentioned by Warren Buffett as a lead indicator of a stock market "bubble" - the market capitalization to nominal GDP ratio.[6] This approach is in the same spirit as Shiller's cyclically adjusted price-to-earnings (CAPE) ratio [Campbell and Shiller, 1988], where earnings per share are averaged over a long period. When this ratio indicates overpricing, and the likelihood of "bad times" is higher, we tilt the blend of our portfolio closer to the FI and away from the GMV portfolio. We discuss this in more detail in the methodology section.

The rest of the paper is organized as follows. We introduce the method of blended portfolios in Section 2. We summarize our data in Sections 3 and our empirical findings in Section 4. Finally, Section 5 concludes.

2 Methodology

The FI and the GMV portfolios are depicted in Figure 1, which illustrates our proposed technique of blending these two portfolios into one. First, the FI portfolio is constructed based on firms' fundamentals using the FI approach. Second, the GMV portfolio is identified on the mean-variance portfolio frontier. We construct 101 blended combinations (in one percent increments) of these two portfolios, which generate the new, blended GMV/FI mean-variance frontier (in red). On the blended GMV/FI portfolio frontier, we select a portfolio depending on prediction of stock market correction (captured by the Buffett Indicator Index, which is discussed in more detail in Subsection 2.3). This Predictive Blended (PB) portfolio is the final outcome of our blended GMV/FI technique. It is the performance of this portfolio that we compare to our benchmarks, the S&P 500 index and the S&P 500 Equally-Weighted index. Next, we describe several desirable features of our proposed technique.

First, the two initial portfolios are formed using profoundly different methods, that should result in better performance of the combined model. Since we are concerned with out-of-sample performance of our portfolios in mean-variance space, our blended approach is inspired by methods proposed in the forecast combination literature. Models with combined forecasts have been shown to outperform individual forecasts [Bates and Granger, 1969, Ericsson, 2017].[7]

[6] We use nominal GDP since we employ nominal market capitalization.
[7] For an excellent survey of the literature, see Hamilton [1994].

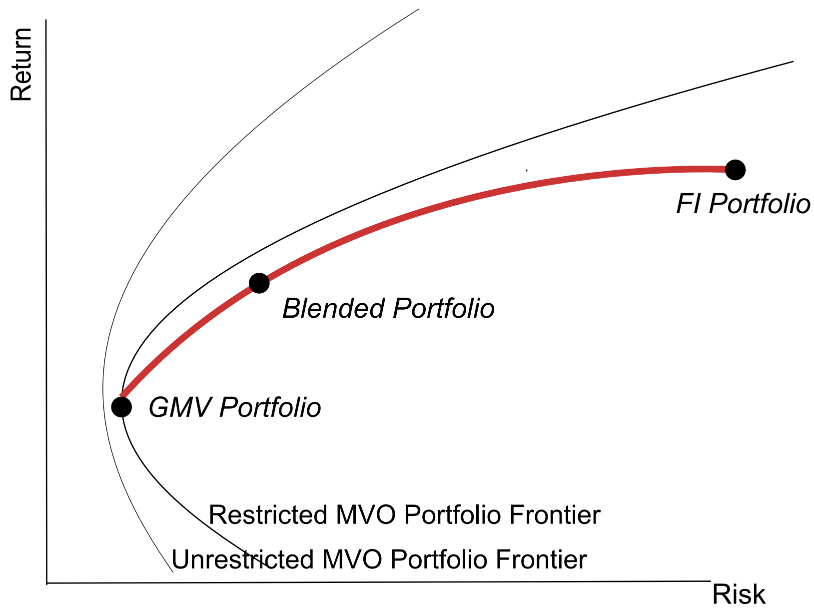


Figure 1: BRIDGING MVO AND FI APPROACHES. The figure illustrates hypothetical unrestricted and restricted minimum variance sets (MVS) based on Markowitz mean–variance optimization, incorporating short-sale and no short-sale constraints, respectively. The FI portfolios are constructed with long positions only, thus appearing in the interior of the restricted MVS. Typically, construction of the GMV and FI portfolios result in conceptually different asset allocation which allows for nontrivial correlation, and results in the MVS being located between these two portfolios, as depicted by the bold red curve.

Second, since portfolios constructed based on the classic MVO (e.g., GMV) and FI approaches (e.g., Arnott FI), are most likely not perfectly correlated, the mean-variance optimal frontier (red curve in Figure 1) will not result in a straight line. This "second-stage" (blended GMV/FI) mean-variance frontier offers further refinement combining the weights of the GMV and the FI portfolios proportionally as in Figure 1. Since the FI portfolio brings additional forward-looking information which was not included in the estimated mean-variance frontier, the new blended portfolio may generate a frontier that outperforms the MVO efficient frontier in out-of-sample tests.

Third, construction of the GMV and FI portfolios does not depend on individual stocks' expected returns, which, as we mentioned earlier, is a major source of error in portfolio optimization problems. Blending the GMV and FI portfolios together also does not depend on their expected returns. We employ the Buffett Indicator Index discussed below to decide on the Predictive Blend portfolio allocation.

2.1 *Construction of the Global Minimum Variance (GMV) portfolio*

The GMV portfolio carries the most information about the diversification structure. In general, it is obtained from the optimization problem:

$$w^{GMV} = \arg\min_{w} w'\Omega w \qquad \text{s.t.} \qquad w'e = 1, \tag{1}$$

where, Ω is the $N \times N$ variance-covariance matrix of stocks' returns, N is the number of assets, e is the $N \times 1$ column vector of ones, and w is the $N \times 1$ vector of weights, w^{GMV} is a vector of individual asset weights in the GMV portfolio.

Note, that we calculate weight-restricted portfolios, with no short sales and a maximum weight of 10%. The restricted GMV portfolio is obtained by solving the optimization problem (1) with the added constraint of $0 \leq w_i \leq 0.1$ for $i = 1, ..., N$.

2.2 *Construction of the Fundamental Indexing (FI) portfolio*

Previous literature [Arnott et al., 2005, Walkshäusl and Lobe, 2010] considers fundamental indexes based on a single metric or an average of a number of fundamental factors. A single metric fundamental index can be calculated as:[8]

$$FI_i^X = \frac{max\{0, X_i\}}{\sum_{j=1}^{n} max\{0, X_j\}}, \tag{2}$$

where X_i is a numeric value for the considered fundamentals for stock i, e.g., book value *(BV)*, dividends paid *(D)*, free cash flows *(FCF)*, revenues *(REV)*, among others.[9] We side with Arnott et al. [2005]'s composite approach in constructing our FI portfolios as follows:

$$FI_i^{COMP} = \begin{cases} \frac{1}{4}(FI_i^{BV} + FI_i^{D} + FI_i^{FCF} + FI_i^{REV}), & \text{in the presence of dividends for } i; \\ \frac{1}{3}(FI_i^{BV} + FI_i^{FCF} + FI_i^{REV}), & \text{otherwise.} \end{cases} \tag{3}$$

where the superscripts of FI_i^X denote the same set of fundamentals mentioned above. Then, the weights in the FI portfolio are normalized values of the fundamental index constructed above. Note, that we calculate weight-restricted portfolios, with no short sales and a maximum weight of 10%.[10]

$$w_i^{FI} = \frac{FI_i}{\sum_{j=1}^{n} FI_j}. \tag{4}$$

Similarly to Arnott et al. [2005], we use book value for the preceding fiscal year, and trailing five-year averages of free cash flows, revenues and dividends. Combined with equation (2), equation (4) ensures no-short sales, full investment and under-weighting of stocks with non-positive fundamentals.

Arnott's portfolio consists of 1000 stocks; in Walkshäusl and Lobe [2010] portfolio sizes vary. To make sure that the performance of our blending method compared with the S&P 500

[8] The use of $max()$ in equation (2) ensures no short sales in the FI portfolios.

[9] Other fundamentals might include employment, income, sales [see Arnott et al., 2005, Basu and Forbes, 2014]. However, evidence on outperformance of these alternative FI portfolios relative to the originally proposed baseline, FI by Arnott et al. [2005], is mixed.

[10] The maximum-holding-weight constraint, $0 \leq w_i \leq 0.1$, is not binding given equation (4) and our universe of S&P 500 constituents. Rearranging equation (4) we observe that $FI_i = \frac{w_i^{FI}}{1-w_i^{FI}} \sum_{j=1, j\neq i}^{n} FI_j$. To violate the maximum-holding-weight constraint the fundamental value of stock i must be at least one-$\frac{1-w_i^{FI}}{w_i^{FI}}$-th of the sum of fundamental values of all other stocks, e.g., with a maximum weight of 10%, a single stock must have more than one-ninths of fundamental value compared to the sum of fundamentals of all other stocks. Our set of stocks does not contain assets with such outlying fundamentals in any of the periods considered.

is not driven by mid- or small-cap stocks, we include only the top 500 stocks ranked by their market capitalization.[11]

Arnott et al. [2005] rebalance portfolios on January 1st. Since the fundamentals of the preceding fiscal year might be unavailable by January 1st, we follow the Walkshäusl and Lobe [2010] methodology to rebalance portfolios on July 1st, using the data on fundamentals for the preceding fiscal year.

2.3 *Construction of Predictive Blended portfolios*

We define our blended portfolios as the portfolios based on the two risky assets - the GMV and FI portfolios. We consider 101 combinations of GMV and FI portfolios: (0% FI & 100% GMV), (1% FI & 99% GMV), ... , (100% FI & 0% GMV).

Our in-sample results suggest that the optimal blend depends on whether or not financial markets are in turmoil. To avoid look-ahead bias but incorporate this feature, as a proxy for a looming crisis, we use a metric often mentioned by Warren Buffett: Total Market Capitalization divided by GDP. Buffett and Loomis [2001, p.93] argue that this is "the best single measure of where valuations stand at any given moment". We will refer to this ratio as the Buffett Indicator (*BI*):

$$BI_t = \frac{\text{Wilshire } 5000_t}{\sum_{\tau=t-4}^{t} GDP_\tau/5} \,, \tag{5}$$

where, the Wilshire 5000 is a market capitalization-weighted index of the market value of all stocks actively traded in the US (the actual number of stocks in the index may vary), and GDP is annualized US nominal GDP in the last five years. Similar to recent literature we favor GDP over GNP.[12] Nominal GDP is chosen because the Wilshire 5000 is also nominal. The Wilshire 5000 is highly correlated with the S&P 500 but more commonly used in the literature for calculating Market Capitalization-to-GDP ratio.

To adjust BI for cycles, in the spirit of Campbell and Shiller [1988], we test the BI ratio, taking ten-, five-, and one- year US GDP. The time horizon for the GDP average in BI calculations does not play a crucial role, producing similar results. Thus, we take the average GDP over a time span of five years.

We propose to use the Buffett Indicator Index:[13]

$$BII_t = \frac{BI_t - \min\{BI_\tau\}_{\tau=t-4}^{t}}{\max\{BI_\tau\}_{\tau=t-4}^{t} - \min\{BI_\tau\}_{\tau=t-4}^{t}} * 100\% \tag{6}$$

[11] Although the list of top-500 stocks by market capitalization is not identical to the list of the S&P 500, it mimics it closely.

[12] The appropriateness of GDP vs GNP in equation (5) is contentious. Some implementations with GDP can be found in the World Bank and World Federation of Exchanges databases as well as among the Corporate Finance Institute (CFI)'s resources at `https://corporatefinanceinstitute.com/resources/knowledge/valuation/market-cap-to-gdp-buffett-indicator/`.

[13] Note, this formula is similar to the Dimension Index (attainment levels) in the Human Development Index [Sen, 1994, p.8]

We propose to choose the optimal blend proportionally[14] to BII:

$$w_t^{PB} = BII_t w_t^{FI} + (1 - BII_t)\, w_t^{GMV}. \tag{7}$$

When the market is likely to be undervalued, and the likelihood of steady growth increases, it is prudent to invest in a well-diversified portfolio, best captured by the GMV portfolio. If the current BI is at its lowest point ($BII = 0\%$), we suggest that an investor should invest fully in the GMV portfolio.

When the market is likely to be overvalued, and the likelihood of a market crash increases, it is prudent to invest based on the economic footprint of companies, which is best captured by the FI portfolio. If the current BI is at its highest point ($BII = 100\%$), we suggest that an investor should invest fully in the FI portfolio.

When the market is neither undervalued nor overvalued, the likelihood of a crash or expected boom are unclear. This situation is somewhere between the two extremes, expected crash or expected boom. Thus, a blended portfolio constructed from the GMV and FI should be proportional to how close to either extremes the market happens to be.

For example, on July 3rd, 2017[15] the BI metric was 141%; in the preceding five years the minimum BI was 109%, the maximum BI was 141%, thus according to equation (6), the Buffett Indicator Index is equal to 100%. In such a case, we argue that the PB portfolio should be the 100% FI portfolio.

In this section, we analyzed stocks in-sample and constructed the GMV, FI and PB portfolios out-of-sample. Before we perform the empirical investigation of our technique in Section 4, we describe our data and data preparation procedures in the following section.

3 Data Description and Preparation

3.1 Data Description

Our investable universe consists of the S&P 500 constituents listed on the NYSE, NASDAQ and AMEX from January 1990 to January 2018. To avoid survivorship bias we include delisted stocks in our analysis (see Brown et al., 1992). We obtain daily market values (MV) and return indices (RI), which are price index plus dividend disbursements. We collect annual data on book values (BV), dividends (Div), free cash flows (FCF) and revenues (Rev). We also consider the Wilshire 5000[16] (daily) and nominal GDP (annual) data from 1971 to 2018 to construct the Buffett Indicator. The data on ETFs and Indices relevant for the implementation of the practitioners' portfolios are downloaded for the period since 1990 or since data became

[14]Note that in this paper we round the exact value of BII to the nearest percentage point to improve calculation speed. We obtain: $w_t^{PB} = \alpha *_t w_t^{FI} + (1 - \alpha *_t)\, w_t^{GMV}$, where $\alpha = round(BII)$, i.e. rounding to the nearest integer percent, is the proportion of the FI portfolio in a blended portfolio strategy. We focus on the linear relation between BII and the optimal blending proportion. In our future research we will consider alternatives for $\alpha = f\,(BII)$, e.g., sigmoid functions for $f()$ as a smoothing alternative.

[15]Since scheduled rebalancing day July 1st, 2017 was a Saturday, the actual rebalancing day was the first following trading day, Monday, July 3rd, 2017

[16]The Wilshire 5000 is a market capitalization index.

Table 1: DESCRIPTIVE STATISTICS for the period from July 1, 1990 to June 30, 2017. All values are in billions of USD.

$bn	Mean	StDev	5%	50%	95%	Skew	Kurt
Market Value (MV)	10.20	29.16	0.08	2.36	41.42	8.00	97.91
Book Value (BV)	3.99	12.93	0.03	0.97	15.30	10.28	146.70
Total Dividends (Div)	0.20	0.77	0.00	0.02	0.84	11.34	260.83
Free Cash Flows (FCF)	0.99	3.84	-0.01	0.21	3.97	9.34	256.54
Revenue (Rev)	7.25	20.62	0.06	1.72	29.50	9.59	147.86

available, whichever comes first. These data are sourced from Thomson Reuters Datastream. We collect the data for Shiller's CAPE from Robert Shiller's website.[17] We collect Fama-French factors from Kenneth French's web site for July 1995 to June 2017.[18] The data on the misvaluation factor (UMO) for Walkshäusl's two-factor model is obtained from Danling Jiang's website for the period from July 1995 to December 2016.[19]

To test our approach we construct 22 trailing sub-samples of six years each: five years are used for estimation (July 1, 1990 - June 30, 1995; July 1, 1991 - June 30, 1996 etc.) with the remaining one year for out-of-sample performance (July 1, 1995 - June 30, 1996; July 1, 1996 - June 30, 1997, etc.). Portfolios are rebalanced on July 1 (or the next available trading day) of every year to ensure availability of fundamental data from previous calendar years. In each in-sample sub-period we select 500 stocks with the highest market values on the date of portfolio construction; these are closely related to our main benchmark, the S&P 500.[20] Please see Table 1 for descriptive statistics of the data for stocks that are included at least once in our sample (1095 stocks, for the period of 27 years).[21]

3.2 *Data Preparation*

Since the total return index (RI) reflects both the price of an asset and any dividend disbursements, we obtain daily stock returns as follows:

$$r_{i,t} = \frac{RI_{i,t} - RI_{i,t-1}}{RI_{i,t-1}} \tag{8}$$

Note, that using the simple return formula is essential for accurate aggregation of assets in portfolios, whereas log returns are convenient for time aggregation but result in inaccurate estimates when aggregated across several securities.

Our next section discusses the results of out-of-sample tests on the proposed blended portfolios comparing their performance to common market benchmarks, namely the S&P 500

[17] Available at http://www.econ.yale.edu/~shiller/data.htm.

[18] Available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[19] Available at https://sites.google.com/site/danlingjiang/ data-library.

[20] We find a high degree of concordance between the market values and free float market capitalization resulting in minimal changes in composition of our universe of 500 stocks.

[21] We do not require normality for the distribution of returns, as we use the Ledoit and Wolf [2008] test to calculate heteroskedasticity and autocorrelation-consistent p-values for statistical significance tests of portfolios' Sharpe ratios.

Table 2: OUT-OF-SAMPLE SHARPE RATIO ANALYSIS. This table outlines the results of significance tests for the difference in Sharpe ratios (Sharpe ratios are highlighted in bold) of various portfolios (in rows) against the two benchmarks (in columns 2-3-4-5) for the period from July 1, 1995 to June 30, 2017. We apply the methodology in Ledoit and Wolf [2008] to calculate heteroskedasticity and autocorrelation-consistent (HAC) p-values for the difference in Sharpe ratios of two portfolios. (*) and (**) represent the 5% and 1% significance levels, respectively. The out-of-sample Sharpe ratios of our constructed portfolios are, generally, higher than those of the two benchmarks considered. The bottom two rows contain the best and the worst blends of FI and GMV portfolios under unrealistic perfect foresight scenarios, representing the most liberal and conservative thresholds.

	Portfolios:		S&P 500		Eq.-Weighted S&P 500	
		Sharpe ratio	**0.331**		**0.505**	
			p-values		p-value	
			HAC	HAC (pre-whitened)	HAC	HAC (pre-whitened)
Out-of-sample	Predictive Blend	**0.647**	0.001**	0.000**	0.029*	0.031*
	Tangency	**0.558**	0.058	0.030*	0.543	0.550
	GMV	**0.591**	0.014*	0.008**	0.098	0.099
	25%FI+75%GMV	**0.566**	0.007**	0.002**	0.073	0.075
	50%FI+50%GMV	**0.527**	0.008**	0.001**	0.136	0.140
	75%FI+25%GMV	**0.485**	0.025*	0.002**	0.705	0.710
	FI	**0.446**	0.116	0.019*	0.240	0.247
	Best Blend	**0.791**	0.000**	0.000**	0.001**	0.001**
	Worst Blend	**0.295**	0.853	0.819	0.006**	0.007**

Index, the Equally-Weighted portfolio comprised of the S&P 500 constituents, the GMV and Arnott's FI portfolios.

4 Results

We analyze portfolios when a "no short-sales" constraint is implemented with maximum holding weights of at most 10% of the portfolio at the time of construction.[22] Table 2 shows the following central findings of our paper. The first, and the most important result of this study is that over the period 1995-2017 in out-of-sample tests, the Predictive Blended (PB) portfolio, based on the Buffett Indicator discussed in Section 2.3, outperforms in terms of Sharpe ratio scores the Markowitz Tangency, GMV, Arnott FI portfolios and any fixed blend of the GMV and FI portfolios. In Table 2 refer to the second column and the first row: the Sharpe ratio of the PB portfolio is 0.647; this is the highest in the out-of-sample calculations. The PB portfolio is the only portfolio that has a statistically significant outperformance compared to the Equally-Weighted S&P 500 portfolio. Given that all these methods use the same universe

[22]The choice of 10% for the maximum holding weight is well justified in practice, although for the majority of mutual funds and ETFs this constraint is even more conservative. The liberal threshold of 10% is sufficient to avoid "error maximization" in covariance matrix estimation in our optimization procedures. In the Appendix, we provide additional robustness checks by constructing portfolio frontiers based on sample, linear shrinkage [Ledoit and Wolf, 2004] and non-linear shrinkage [Ledoit and Wolf, 2017] covariance estimators "with" and "without" the constraints on holdings. We confirm large discrepancies in portfolio frontiers and portfolio compositions when no weight constraint is applied, and minimal or no discrepancies when the weight constraint (no short sales, less or equal 10% long positions) is enforced.

of stocks (the S&P 500 constituents lists), the only source of better performance is likely to be a superior methodological approach.

Second, even if the Predictive Blended approach is not applied, blending the GMV and FI portfolios in fixed proportions (for example 25%FI + 75%GMV) produces results stronger (Sharpe ratio is 0.566) than those of the Markowitz's tangency portfolio (0.558), S&P 500 (0.331), Equally-Weighted S&P 500 (0.505), or the FI portfolio (0.446).[23] The difference in Sharpe ratios between the fixed blend (25% FI, 75% GMV) and the Equally-Weighted S&P 500 portfolio is statistically significant at the 10% level[24], a result that is only outmatched by the Predictive Blended portfolio (see the right hand column in Table 2). This confirms the point we made earlier in Section 2 that blending portfolios produce better results than the pure Markowitz MVO (GMV) or Arnott's FI approaches.

Third, even if the Predictive Blended portfolio is based on consistently flawed forecasts the result would not be much different from the capitalization-weighted S&P 500: Table 3 shows that even when we consistently choose the worst blend, the since-inception Sharpe ratio is 0.295, compared to 0.331 for the S&P 500. In contrast, in the equally unrealistic case, when our forecasts are consistently right (the best blend), the Sharpe ratio is 0.791, compared to 0.331 for the S&P 500.

Fourth, the Predictive Blended portfolio produces higher since-inception return (13.22%) than the GMV (10.84%) and FI (12.01%) portfolios taken separately (see Table 8 in the Appendix). The S&P 500 and GMV portfolios have the lowest returns since inception: 10.11% and 10.84% respectively; in contrast, returns on the FI (12.01%), PB (13.22%), Equally-Weighted S&P 500 (13.39%), and Tangency (13.79%) portfolios are similar, but higher. The Predictive Blended portfolio provides marginally lower returns than the Tangency and the Equally-Weighted S&P 500 portfolios, but with the benefit of much lower volatility.

Fifth, the PB portfolio is less volatile (σ =14.33%) than the Tangency (σ =17.62%), FI (σ =18.06%), S&P 500 (σ =18.60%), and Equally-Weighted S&P 500 (σ =18.70%) over the period 1995 - 2017. This property makes the PB portfolio the portfolio of choice for investors with high aversion to volatility, but who still would like to make returns higher than those of the GMV portfolio (with the lowest volatility of σ =11.65%).

Interestingly, the Sharpe ratio in out-of-sample tests of the GMV portfolio (0.591) is superior to the Tangency portfolio (0.558). It may be due to the fact that in out-of-sample-tests the Tangency portfolio moves further inside the Minimum Variance Set than the GMV portfolio. This illustrates the point we made earlier that the GMV portfolio does not suffer as much from estimation errors of its inputs: the covariance structure needed for both of them is more robust than the hard-to-predict expected returns needed for the Tangency portfolio. Table 3 shows that Predictive Blended (PB) is the best strategy over the long-term, even though, there is a possibility that some other strategy might be better in specific years (for the year-by-year

[23]However, the GMV portfolio outperforms fixed blends, having a Sharpe ratio of 0.591.

[24]Interestingly, we noticed that the fixed blend (25% FI, 75% GMV) has higher statistical significance and a lower Sharpe ratio than those of the GMV and Tangency portfolios.

Table 3: Out-of-sample Sharpe ratios for portfolios first built in 1995, and ending in various years, assuming annual rebalancing on July 1 of each year. The bottom row represents the number of years a portfolio had the highest Sharpe ratio among the benchmarks considered. In each row, the highest out-of-sample Sharpe ratios are **emphasized**. Sharpe ratios for perfect foresight strategies are included for reference only.

Period		Out-of-sample Sharpe ratios						Perfect foresight	
Start	End	PB	Tangency	GMV	FI	S&P 500	Eq.Weighted	Best Blend	Worst Blend
1995	1996	1.899	2.847	**3.123**	1.899	1.718	2.576	3.132	1.893
1995	1997	1.947	2.430	**2.602**	1.947	1.771	2.155	2.600	1.944
1995	1998	1.720	2.096	**2.451**	1.720	1.499	1.725	2.471	1.719
1995	1999	**1.352**	1.281	1.240	**1.352**	1.225	1.222	1.405	1.171
1995	2000	0.949	0.904	0.737	0.949	**0.952**	0.904	0.882	0.833
1995	2001	**0.878**	0.695	0.817	**0.878**	0.589	0.877	0.914	0.767
1995	2002	**0.757**	0.416	0.701	0.619	0.286	0.678	0.824	0.482
1995	2003	**0.703**	0.376	0.614	0.501	0.273	0.573	0.747	0.377
1995	2004	**0.780**	0.461	0.741	0.565	0.312	0.661	0.836	0.460
1995	2005	0.831	0.518	**0.840**	0.564	0.313	0.670	0.908	0.468
1995	2006	0.815	0.505	**0.830**	0.557	0.317	0.669	0.893	0.468
1995	2007	**0.866**	0.522	0.857	0.612	0.371	0.707	0.948	0.499
1995	2008	0.662	0.484	**0.665**	0.465	0.255	0.551	0.780	0.370
1995	2009	**0.495**	0.285	0.423	0.266	0.116	0.332	0.548	0.180
1995	2010	**0.531**	0.336	0.474	0.303	0.152	0.379	0.587	0.225
1995	2011	**0.593**	0.437	0.550	0.370	0.226	0.455	0.651	0.300
1995	2012	**0.572**	0.468	0.563	0.351	0.226	0.421	0.657	0.285
1995	2013	**0.625**	0.510	0.598	0.401	0.264	0.468	0.715	0.315
1995	2014	**0.668**	0.577	0.607	0.440	0.308	0.511	0.761	0.327
1995	2015	**0.653**	0.593	0.572	0.433	0.311	0.502	0.740	0.310
1995	2016	**0.622**	0.566	0.607	0.419	0.305	0.484	0.764	0.300
1995	2017	**0.647**	0.558	0.591	0.446	0.331	0.505	0.793	0.295
No. of superior years		15	0	5	3	1	0		

Table 4: One-year out-of-sample Sharpe ratios for portfolios built in various periods, starting on July 1 of each year. The bottom row represents the number of years a portfolio had the highest Sharpe ratio among the benchmarks considered. In each row, the highest out-of-sample Sharpe ratios are **emphasized**. Sharpe ratios for perfect foresight strategies are included for reference only.

Period		Out-of-sample						Perfect foresight	
Start	End	PB	Tangency	GMV	FI	S&P 500	Eq.Weighted	Best Blend	Worst Blend
1995	1996	1.899	2.847	**3.123**	1.899	1.718	2.576	3.132	1.899
1996	1997	2.019	2.137	**2.175**	2.019	1.848	1.908	2.263	2.019
1997	1998	1.462	1.689	**2.300**	1.462	1.184	1.248	2.300	1.462
1998	1999	0.686	0.161	-0.659	0.686	**0.796**	0.463	0.686	-0.659
1999	2000	-0.298	-0.037	-0.672	-0.298	**0.125**	0.065	-0.298	-0.672
2000	2001	0.514	0.059	**1.221**	0.514	-0.906	0.758	1.221	0.514
2001	2002	-0.343	-1.412	**-0.033**	-0.764	-1.438	-0.326	0.064	-0.764
2002	2003	**0.318**	0.166	**0.318**	0.095	0.223	0.230	0.318	0.095
2003	2004	**1.611**	1.474	**1.611**	1.312	0.860	1.539	1.660	1.312
2004	2005	1.611	1.332	**1.761**	0.602	0.372	0.819	1.761	0.602
2005	2006	0.640	0.354	**0.727**	0.500	0.440	0.685	0.727	0.500
2006	2007	**1.674**	0.744	1.249	**1.674**	1.480	1.362	1.674	1.249
2007	2008	-0.907	**0.265**	-0.864	-0.907	-0.918	-0.822	-0.864	-0.920
2008	2009	**-0.273**	-0.623	**-0.273**	-0.439	-0.530	-0.411	-0.273	-0.439
2009	2010	**1.260**	1.023	**1.260**	0.775	0.708	0.953	1.260	0.775
2010	2011	2.074	**2.391**	2.097	1.720	1.864	1.901	2.097	1.720
2011	2012	0.324	**0.887**	0.750	0.117	0.227	0.014	0.750	0.117
2012	2013	1.763	1.421	1.378	**1.766**	1.404	1.764	1.765	1.378
2013	2014	1.884	2.031	0.851	1.884	1.929	**2.042**	1.883	0.851
2014	2015	0.296	**1.077**	-0.233	0.296	0.451	0.298	0.289	-0.233
2015	2016	0.098	-0.021	**1.246**	0.098	0.163	0.060	1.246	0.098
2016	2017	1.668	0.370	0.161	1.758	**1.766**	1.575	1.752	0.161
No. of superior years		5	4	12	2	3	1		

performance see Table 4). In fact, the PB strategy has the highest Sharpe ratios since inception in 16 out of the 22 years we considered (see Table 3). Even though the year-by-year Table 4 shows that the GMV portfolio outperforms other portfolios in 12 out of 22 years taken separately, it is not a reliable strategy in the long term. For example, during the Asian and the Long Term Capital Management crises in 1998-1999, the GMV portfolio was the only portfolio in our set to show negative returns (from Table 9 on page 29 in the Appendix: the GMV return was -2.11%, while the S&P 500 return was +22.45%, and FI +17.37%) and consequently, Sharpe ratios (for the GMV it was 2.300, while for the S&P 500 Sharpe ratio was 0.796, and for the FI it was 0.686). The following year, in 1999 to 2000, this situation was similar: returns were -1.03% for the GMV vs +8.58% for the S&P 500, and +0.79% for the FI portfolio.[25]

Figure 2 helps visualize the results described above. The top panel traces a one-dollar investment in the Predictive Blended portfolio and two of its constituents, FI and GMV portfolios. We include the S&P 500 index for comparison. The performance is assessed out-of-sample, that is we invest/rebalance on the first trading day of July in each year following the derivation of weights of the corresponding portfolios. The gray shaded areas denote periods when FI constituted 100 percent in the Predictive Blended portfolio. We detail the percentage of FI in the Predictive Blended portfolios in the bottom panel of Figure 2. These percentages are calculated using equation 6 based on the Buffett Indicator, and on Shiller's CAPE (Cyclically Adjusted Price-to-Earnings) ratio. Based on BII, for the first six years of our sample, FI constituted 100 percent in Predictive Blended portfolios, and, as a consequence, the performance of the PB and FI portfolios coincide over the period 1995 to 2001 (the first shaded area in the top panel). The ability of the PB strategy to re-allocate to minimum variance portfolios in times of financial crises explains its superior performance in the long-run. We tested alternative blending strategy, relying on Shiller's CAPE. For the period 1995 to 2017, the PB portfolio based on the Buffett Indicator provided a higher out-of-sample Sharpe ratio compared to the PB portfolio based on Shiller's CAPE ratio, although with a statistically insignificant difference (0.647 vs 0.620).[26]

4.1 *Asset-pricing and robustness tests*

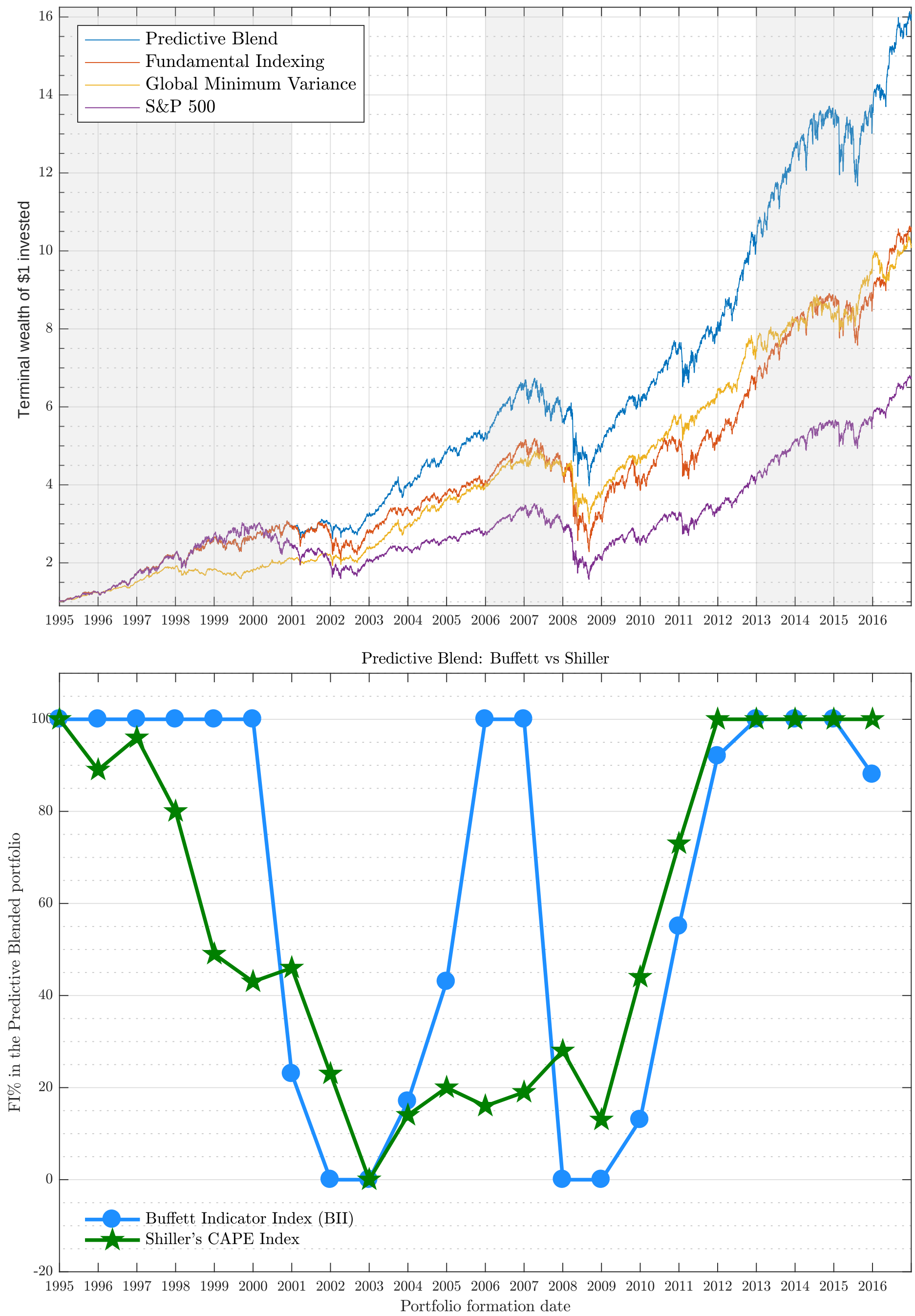
In this section, we examine the risk-adjusted performance of the PB, GMV and FI portfolios using the CAPM as well as the three-, four-, and five-factor models of Fama and French [1993], Fama and French [1996] and Carhart [1997] as well as Walkshäusl [2016] parsimonious two-factor model. The results are summarized in Table 5. The Predictive Blended portfolio performance cannot be explained by any of the standard asset-pricing models we consider below - we find a highly statistically significant alpha, with t-statistic of at least 3.39 and abnormal monthly returns ranging from 0.33% (3.96% annually)[27] for the Fama-French five

[25]Refer to Table 9 in the Appendix.

[26]Detailed results for Shiller's CAPE ratio-based PB performance are presented in Tables 10 and 11 in the Appendix.

[27]It is interesting to see that Walkshäusl [2016] parsimonious two-factor model produces estimates ($\alpha = 0.0034$, $t = 3.39$), that are very close to those of Fama-French five factor model ($\alpha = 0.0033$, $t = 3.51$).

Figure 2: Out-of-Sample Portfolio Performance: Buffett vs Shiller. The top panel presents the out-of-sample performance of Predictive Blended portfolio and two of its constituents, the FI and GMV portfolios. The S&P 500 index is included for comparison. Shaded gray areas denote periods of 100% of FI in the Predictive Blended portfolio. The bottom panel plots the percentage of FI in the Predictive Blended portfolios on the day of portfolio construction (first trading day after July 1st of every year from 1995 to 2016). This percentage is calculated using equation 6 based on the Buffett Indicator, and on Shiller's CAPE (Cyclically Adjusted Price-to-Earnings) ratio. For example, in 1995 both methods recommend allocating 100% of the assets to the FI portfolio; in 2003 both methods recommend allocating 100% of the assets to the GMV portfolio; in 2004 both methods recommend allocating about 15-18% in the FI portfolio and 82-85% to the GMV portfolio. The two methods are not perfectly synchronized. They give similar signals before and after the dot-com bubble, as well as in recent years. However, right before the Great Financial Crisis, the Buffett Indicator Index was more "pessimistic", telling portfolio managers to switch fully to the FI portfolio. On the other hand, Shiller's CAPE ratio-based index was only getting "pessimistic" by increasing the percentage of the FI portfolio, while still being mostly invested in the GMV portfolio.



factor model to 0.51% (6.12% annually) for the CAPM model. Moreover, the PB strategy is the only technique that has a statistically significant alpha in the Fama-French five factor model and Walkshäusl [2016] two-factor model. The FI portfolio is best explained by these asset-pricing models, with adjusted R^2 ranging from 0.8628 to 0.9677, possibly because the fundamentals used for FI portfolio construction correlate with the factors used in asset pricing models. Consequently, the GMV portfolio is explained the least by these asset-pricing models, with adjusted R^2 ranging from 0.4046 to 0.5246. Not surprisingly, the adjusted R^2 of the PB portfolio falls between those of the FI and GMV portfolios, with adjusted R^2 ranging from 0.7007 (in the CAPM case) to 0.7855 (in the Fama-French five-factor model).

In pursuit of the optimal blend between the two distinct portfolio construction techniques, MVO and FI, we find that the ratio of market capitalization-to-GDP, being a leading indicator for an overpriced market, demonstrates remarkably advantageous properties. Our Predictive Blended portfolio based on BII features statistically significant positive alpha even after accounting for the misvaluation factor (UMO) in Walkshäusl's two-factor model (refer to Table 5 Panel E).

In what follows, we provide a detailed account of the performance of each of the components of the Predictive Blended portfolio contrasting it with fixed blends as well as other benchmarks.

Table 8 presents out-of-sample performance statistics beginning in 1995 and ending in various years, while Table 9 presents out-of-sample performance statistics for every year (July 1 - June 30). We list returns, standard deviations and Sharpe ratios for the six portfolios we study: Global Minimum Variance (GMV), Arnott Fundamental Index (FI), Predictive Blended (PB), Tangency based on the restricted MVO frontier, the S&P 500 index, and Equally-Weighted S&P 500. Note, that the S&P 500 and the tangency portfolios are not identical. The S&P 500 is a purely passive portfolio, whereas the Markowitz tangency portfolio is purely active. Table 9 illustrates that when the dot-com bubble burst, the active Tangency portfolio outperformed the passive S&P 500 index: in 2000-2001 the return of the Tangency portfolio was 6.58% vs. - 13.57% for the S&P 500; in 2001-2002 it was -15.36% for the Tangency portfolio vs. -22.38% for the S&P 500. Similarly, during the Great Financial Crisis, in 2007-2008, the return for the Tangency portfolio was 10.51% vs. -14.34% for the S&P 500; in 2008-2009 it was -21.59% for the Tangency portfolio vs. -20.43 for the S&P 500. Even during the recovery, in 2009-2010, the Tangency portfolio earned 23.17% compared to 16.34% for the S&P 500. This seems to indicate, that in these periods the active strategy (Tangency portfolio) preserves wealth better than the passive strategy (S&P 500). The Tangency portfolio selects the subset of stocks from the S&P 500 universe which have higher return, and lower variability - two properties, that seem to carry-over from in-sample analysis to out-of-sample performance. In Tables 8 and 9 we observe that the Tangency portfolio (often referred to as the "Market" portfolio) is substantively different from the S&P 500 index. The Tangency portfolio outperformed the S&P 500 in the period from 1995 to 2017 with higher annualized returns (13.79% vs. 10.11%)

Table 5: This table shows the results from regressing monthly returns of the PB, GMV and FI portfolios on the asset pricing model common factors over the period from July 1995 to June 2017. R^2 is adjusted for degrees of freedom. * and ** denote significance at 1% and 5% levels using Newey and West [1987] adjusted t-statistics for the coefficients. CAPM stands for Capital Asset Pricing Model, and FF stands for Fama-French. The Fama-French data source is Kenneth French's web site at Dartmouth for July 1995 to June 2017. The data on the misvaluation factor (UMO) for Walkshäusl's two-factor model is obtained from Danling Jiang's website (at https://sites.google.com/site/danlingjiang/ data-library) for the period from July 1995 to December 2016 (note: in Panel E the testing period is six months shorter due to availability of UMO data).

Portfolios (i):	PB		GMV		FI	
	Estimates	t-stats	Estimates	t-stats	Estimates	t-stats
Panel A: (CAPM) $r_{it} = a_i + b_i MKT_t + \varepsilon_{it}$						
a_i	0.0051**	4.54	0.0047**	3.22	0.0022	1.75
b_i	0.6285**	14.72	0.4067**	9.59	0.8925**	18.28
Adj.R^2	*0.7007*		*0.4046*		*0.8628*	
Panel B: (Three-factor FF) $r_{it} = a_i + b_i MKT_t + c_i SMB_t + d_i HML_t + \varepsilon_{it}$						
a_i	0.0046**	4.98	0.0039**	3.22	0.0012*	2.32
b_i	0.6728**	14.23	0.4349**	14.21	0.9513**	50.63
c_i	-0.1152*	-2.18	0.0083	0.16	-0.0987**	-3.37
d_i	0.2433**	4.29	0.2712**	4.63	0.4011**	9.53
Adj.R^2	*0.7675*		*0.4911*		*0.9586*	
Panel C: (Four-factor Carhart) $r_{it} = a_i + b_i MKT_t + c_i SMB_t + d_i HML_t + e_i MOM_t + \varepsilon_{it}$						
a_i	0.0045**	4.57	0.0038**	3.03	0.0015**	3.04
b_i	0.6776**	14.56	0.4430**	13.40	0.9342**	53.35
c_i	-0.1168*	-2.25	0.0057	0.10	-0.0932**	-3.98
d_i	0.2483**	4.28	0.2793**	4.52	0.3839**	8.25
e_i	0.0001	0.43	0.0002	0.56	-0.0004	-1.76
Adj.R^2	*0.7670*		*0.4903*		*0.9608*	
Panel D: (Five-factor FF) $r_{it} = a_i + b_i MKT_t + c_i SMB_t + d_i HML_t + e_i RMV_t + f_i CMA_t + \varepsilon_{it}$						
a_i	0.0033**	3.51	0.0022	1.86	0.0000	0.00
b_i	0.7305**	13.97	0.5165**	13.10	1.0113**	74.85
c_i	-0.0386	-0.89	0.0648	1.29	-0.0664*	-2.51
d_i	0.1578*	2.16	0.1023	1.35	0.2685**	8.22
e_i	0.2316**	4.42	0.2120**	3.23	0.1351**	3.46
f_i	0.0342	0.36	0.2455*	2.58	0.2154**	4.43
Adj.R^2	*0.7855*		*0.5246*		*0.9677*	
Panel E: (Two-factor Walkshäusl): $r_{it} = a_i + b_i MKT_t + c_i UMO_t + \varepsilon_{it}$						
a_i	0.0034**	3.39	0.0024	1.82	-0.0001	-0.16
b_i	0.6991**	15.96	0.4935**	15.23	0.9877**	36.24
c_i	0.0017**	4.71	0.0021**	4.98	0.0023**	4.49
Adj.R^2	*0.7275*		*0.4622*		*0.8930*	

and lower annualized standard deviation (17.62% vs. 18.60%). Consequently, the Sharpe ratio of the Tangency portfolio is 0.56 against 0.33 for the S&P 500. This confirms, that the S&P 500 is not as mean-variance efficient in out-of-sample tests as the past information-based Tangency portfolio, possibly due to sub-optimal portfolio allocation methods or conservative weight constraints. It highlights the need for improved portfolio allocation techniques that use available information better.

The proposed Predictive Blended portfolio technique uses information from firms' fundamentals, the stocks' variance-covariance structure, and market timing to produce superior portfolio performance. We note in the Table 8 (bottom row 1995-2017) that the PB portfolio outperforms all other portfolios in terms of Sharpe ratios, and the difference in performance is statistically significant at the 1% level compared with the S&P 500, and at the 5% level compared with the Equally-Weighted S&P 500 as our main benchmarks (note that the results of significance tests for the difference in Sharpe ratios are presented in Table 2 on page 11). Even though the cumulative over-performance of the PB over the GMV portfolio is not statistically significant, the cumulative returns of the PB portfolio dominate those of the GMV portfolio in 15 out of 22 years, not a bad property for practitioners.

In Table 10 we follow the performance of various blended portfolios over the period from 1995 to 2017. To evaluate the performance of our actual PB portfolio, we provide a range of possible blending outcomes by computing the performance of the "best" portfolios, assuming unrealistic perfect foresight, and "worst" portfolios, assuming the unrealistic situation of being wrong every time. The Sharpe ratio for the Predictive Blended portfolio (0.65) is closer to the Sharpe ratio of the perfect foresight portfolio (0.79) compared to the Sharpe ratio of the "always wrong" portfolio (0.30). Within those limits we also construct three "fixed-blended" portfolios: (25%FI+75%GMV), (50%FI+50%GMV), (75%FI+25%GMV). While the "fixed-blended" portfolios outperform the S&P 500, they also have lower variance than the Equally-Weighted S&P 500. The (25%FI+75%GMV) fixed-blended portfolio has a statistically significant larger Sharpe ratio than the difficult-to-beat Equally-Weighted S&P 500 (Table 2).

In Table 11 we are particularly concerned with the performance of fixed-blended and "worst" and "best" blended portfolios during two crises periods: from 2000 to 2002, and 2007 to 2009. During the Dot-com crash even the "worst" blended portfolio had returns[28] of -7.65% and 6.35% vs. the S&P 500 returns of -13.57% and -22.38% (Table 9). Over this period our Predictive Blended portfolios performed much better than the market: the Buffett Indicator-based PB (1.79% and 8.20%) and the Shiller's CAPE-based PB (-1.02% and 7.53%). During the Great Financial Crisis, even the "worst" blended portfolio had returns of -10.95% and -17.25% vs. the S&P 500 returns of -14.34% and -20.43%. Over this period our Predictive Blended portfolios performed much better than the market: the Buffett Indicator-based PB (-13.18% and -5.25%) and the Shiller's CAPE-based PB (-8.46% and -8.91%). In both periods of crises the blended strategy enabled investors to win by not losing as much as we could have lost

[28] During crises, when returns are negative Sharpe ratios cannot be interpreted in the conventional way, thus the comparison is better done using average returns.

with a passive strategy of investing in the S&P 500 index.

4.2 *Practitioners' short-cut for implementing the Predictive Blended method.*

In evaluating the practicality of our proposed Blended Portfolio strategy we employ readily available financial products and consider two sets of proxies for the GMV and FI portfolios constructed earlier: ETFs and indices. The performance of equity portfolios optimized to have the lowest possible variance, declining less than the market during market downturns, has attracted investor attention only recently. In a somewhat delayed response to recent financial crises, a series of minimum variance indices and ETFs were launched (refer to Table 6). ETFs provide very low cost implementation of an asset allocation decision but have limited data availability, often insufficient to make reliable long-term inference. The main benefit of using indices over ETFs is data availability: indices backdate their strategies thus providing a much longer data history.[29] We matched the ETFs to their parent indices in Table 6.

Table 6: Indices and ETFs summary. The table provides a summary of candidate proxies for the FI and GMV portfolios constructed in this study. Panel A lists ETFs commonly associated with the FI and GMV portfolio strategies and Panel B lists the associated parent indices. Since the number of securities in ETFs varied over time, figures in the column "No. of stocks" are as of 13 Nov 2018. For the ETFs, first value dates coincide with launch dates, however, indices backdate strategies to earlier dates than their launch dates. The last two columns list portfolio return correlations between the GMV/ FI portfolios constructed in this study and the corresponding readily available financial products. The correlation estimates arefor the maximum period available within our dataset: for example, for SPLV it is from May 5, 2011 to June 30, 2017 (i.e., the final date in our dataset). The period we study ends before the launch of the SPMV (we report *n.a.* in the relevant cell).

	Ticker	Parent	No. of stocks	First value date	Launch Date / Inception	Correlation with GMV	FI
Panel A: ETFs							
Invesco S&P 500 Low Volatility	SPLV	SP5LVI	100	5 May 2011	5 May 2011	0.9097	
Invesco S&P 500 Min Variance	SPMV	SP5MV	121	12 Jul 2017	12 Jul 2017	n.a.	
iShares Edge MSCI Min Vol USA	USMV	MSUSMV	205	18 Oct 2011	18 Oct 2011	0.8734	
Invesco FTSE RAFI US 1000	PRF	FTR100	989	19 Dec 2005	19 Dec 2005		0.9953
Panel B: Indices							
S&P 500 Low Volatility	SP5LVI		100	16 Nov 1990	4 Apr 2011	0.8502	
S&P 500 Min Volatility	SP5MV		120	21 Dec 1990	9 Nov 2012	0.8571	
MSCI USA Min Volatility	MSUSMV		n.a.	31 Dec 1998	30 May 2008	0.8336	
FTSE RAFI US 1000	FTR100		1000	31 Dec 1999	28 Nov 2005		0.9915

Among the available ETFs and indices we identified several matches to our constructed FI portfolio. Returns on the PRF (Invesco FTSE RAFI US 1000) ETF and the FTR100 (FTSE RAFI US 1000) Index correlate nearly perfectly with our FI portfolio, with coefficients of 0.9953 and 0.9915 respectively. Finding a good proxy for our GMV portfolio proved to be more difficult. We were unable to find a good match for our GMV portfolio. On the one hand, low volatility funds (such as SPLV, SP5LVI) have weightings that are conceptually different from the optimization-based compositions. The majority of these indices and funds weigh their constituents "...relative to the inverse of their corresponding volatility, with the least volatile

[29] Index data prior to the index launch date is back-tested data (i.e., calculations of how the index might have performed over that time period had the index existed).

Table 7: Annualized Out-of-Sample Portfolio Performance Statistics beginning in 2000 and ending in various years. The "PB" and "Index-based PB" portfolios are rebalanced every year on July 1. We estimate annualized average portfolio returns, $\bar{r}$, portfolio standard deviations, $\hat{\sigma}$, and Sharpe ratios, S, for various portfolio strategies and benchmarks. In each row, the highest out-of-sample Sharpe ratios are **emphasized**. Sharpe ratios for PB portfolio are included for reference only.

July 1, *yyyy*		PB			Index-based PB : (blending SP5MV and FTR100)			SP5MV (proxy for GMV)			FTR100 (proxy for FI)			S&P 500		
Start	End	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S
2000	2001	12.73	14.43	0.514	16.48	15.23	**0.734**	8.90	14.26	0.252	16.48	15.23	**0.734**	-13.57	20.84	-0.906
2000	2002	7.26	12.00	0.182	5.94	14.60	**0.059**	3.11	13.76	-0.143	2.77	16.16	-0.143	-17.97	19.91	-1.158
2000	2003	7.57	12.58	0.231	5.37	17.14	**0.041**	3.49	16.65	-0.071	3.60	19.81	-0.054	-8.72	22.24	-0.602
2000	2004	11.02	12.13	0.534	9.14	15.64	**0.294**	7.73	15.24	0.209	7.62	18.15	0.169	-2.93	20.14	-0.371
2000	2005	12.47	11.54	0.694	9.98	14.66	**0.376**	8.90	14.32	0.310	8.56	16.86	0.243	-0.74	18.62	-0.279
2000	2006	12.08	11.12	0.684	9.97	13.92	**0.396**	8.99	13.59	0.333	8.91	15.94	0.279	0.89	17.51	-0.204
2000	2007	13.26	10.90	0.805	11.50	13.40	**0.523**	10.20	13.01	0.439	10.59	15.20	0.401	3.51	16.64	-0.059
2000	2008	9.96	12.22	0.452	7.58	14.43	**0.218**	7.03	13.61	0.190	6.78	15.92	0.147	1.28	17.11	-0.185
2000	2009	8.26	15.48	0.256	4.69	18.66	0.021	4.19	18.08	-0.006	4.80	22.36	**0.023**	-1.14	21.95	-0.248
2000	2010	9.17	15.09	0.328	5.21	18.21	0.055	4.75	17.67	0.030	6.61	22.44	**0.106**	0.61	21.60	-0.167
2000	2011	10.43	14.68	0.431	7.20	17.68	0.175	6.75	17.15	0.154	8.68	21.84	**0.209**	3.19	21.03	-0.043
2000	2012	10.24	15.05	0.419	7.13	17.99	0.178	7.01	17.24	0.178	8.25	22.01	**0.196**	3.53	21.20	-0.019
2000	2013	11.22	14.84	0.502	8.40	17.61	0.263	7.80	16.79	0.240	9.47	21.42	**0.266**	4.68	20.63	0.044
2000	2014	11.94	14.55	0.567	9.38	17.18	**0.331**	8.47	16.36	0.292	10.38	20.82	0.321	5.92	20.06	0.111
2000	2015	11.52	14.36	0.552	9.11	16.87	**0.327**	8.51	16.04	0.307	10.04	20.34	0.317	6.03	19.62	0.124
2000	2016	11.03	14.53	0.519	8.76	16.89	0.312	8.93	15.93	**0.341**	9.63	20.16	0.305	5.94	19.46	0.126
2000	2017	11.32	14.24	0.555	9.11	16.53	**0.345**	8.72	15.57	0.341	10.00	19.68	0.335	6.55	18.98	0.166
No. of superior years				n.a.			11			1			6			0

stocks receiving the highest weights"[30], ignoring the covariance structure of returns. The two candidates, SPLV and SP5LVI, resulted in lower-than-desired correlations of 0.9097 and 0.8502 respectively. On the other hand, minimum variance indices and funds (such as SPMV, MSUSMV, USMV)[31] are designed to provide the lowest return variance for a given covariance matrix of stock returns. However, these only approximate our GMV portfolio due to more conservative restrictions on constituent weights.[32]

We retain the annual rebalancing date (i.e., first trading day in July) and, given the availability of data for the two indices, we evaluate the index-based PB portfolio from Jul 3, 2000 to Jun 30, 2017. In Figure 2 (bottom panel) we observe a variety of mixing proportions between the GMV and FI proxies.[33]

The results in Table 7 (last row) show that although the Sharpe ratio of the index-based PB portfolio is greater than Sharpe ratios of its index-based constituents (i.e., proxies for GMV and FI), the difference is small.

The GMV proxies we identify are too conservative in their weight allocations. In contrast, our GMV portfolio is less constrained, allowing more flexibility with individual weights of up to 10%. As a result, we observe lower out-of-sample variances compared to minimum variance indices in most years, despite more concentrated compositions (100-120 holdings in a typical minimum variance index vs 20-30 holdings in the GMV portfolios constructed using optimization problem in equation (1)). The surprising result of lower variance in our GMV despite a smaller number of holdings in it is explained by Clarke et al. [2011, p.43]: "...small number of securities in [the GMV] solution does not necessarily come from some complex set of exposure constraints, the interaction of variables in expected return forecasts and the risk model...". Instead the authors show that "... the variance minimization component in general mean-variance objective functions is sufficient to disqualify a large majority of investable securities" [Clarke et al., 2011, p.43].

The less stellar performance of the index-based PB portfolio is the result of mismatch between our constructed GMV portfolio and minimum variance index. There is a need for financial products that are better positioned to capture the minimum volatility property of the theoretical GMV portfolio, perhaps by allowing for a higher upper-weight constraint than what is currently used in the construction of indices (and ETFs).

[30] From the S&P 500 Low Volatility Index methodology document at https://spindices.com/indices/strategy/sp-500-low-volatility-index dated October 31, 2018.

[31] Other minimum variance indices are also available (e.g., FTSE USA Min Variance Index) but have a much shorter data span.

[32] For example, the maximum weight of a stock in the index is "...the lower of 20 times its weight in the S&P 500 or 2%" [SP5MV methodology]; "...the lower of 1.5% or 20x the cap-weight" [MSUSMV methodology].

[33] Our ETF-based PB portfolio had a limited evaluation period given the availability of ETF data: from Jul 1, 2011 to Jun 30, 2017, providing us with only 6 years of data to evaluate the performance of our strategy. In Figure 12 (bottom panel), the performance of the ETF-based PB portfolio is closely linked to Invesco FTSE RAFI US 1000 ETF since there is only one value of the BII lower than 85% (in year 2011) and three values of 100% during 2013-2015. For brevity, we omit the results for the ETF-based PB portfolio, focusing on its index-based counterpart instead. Detailed results are available upon request.

5 Conclusion

In this paper, we propose a new portfolio construction technique that combines the benefits of Mean-Variance Optimization (MVO) and Fundamental Indexing (FI). Given that the FI approach is relatively new, and is profoundly different from the MVO, these two approaches have not yet been combined, even though each method offers distinctive benefits for portfolio choice problems. Our paper fills this gap in the literature. Our results attest to the superior performance of the proposed Predictive Blended (PB) portfolio compared to two hard-to-beat benchmarks, the S&P 500 and the Equally-Weighted S&P 500 between 1995 and 2017.

Applying the MVO method proposed by Markowitz (1952), we find the portfolio that contains the most information about the variance-covariance structure of stock returns - the Global Minimum Variance portfolio (GMV). Applying the FI method proposed by Arnott et al. [2005], we construct a portfolio from stocks that are in sound financial health. Blending these two portfolios generates a portfolio that has better diversification than the FI portfolio and better risk-adjusted return characteristics than the GMV portfolio. Although, ad-hoc static fixed-proportion blends provide promising results compared to the benchmarks, we find that the dynamic Predictive Blended portfolio is remarkably superior.

We test the out-of-sample performance of the predictive and fixed blends (for example 25% FI and 75% GMV) using 29 years worth of data from S&P 500 companies. The suggested PB approach is the only portfolio that provides statistically significant superior (over both the S&P 500 and Equally-Weighted S&P 500 benchmarks) Sharpe ratios in out-of-sample tests. The FI, GMV or classic Markowitz Tangency portfolios taken separately do not have statistically significant Sharpe ratios over the hard-to-beat Equally-Weighted S&P 500 benchmark. The performance of the PB portfolio cannot be explained by any of the standard asset-pricing models we considered.

The second major result of our paper is that almost any fixed blend between the GMV and FI portfolios performs better than the S&P 500 (but not necessarily better than the Equally-Weighted benchmark).

Our future research will focus on finding improved FI techniques that would enhance our predictive blended portfolios even further. In particular, within-industry analysis of the FI portfolios could enable portfolio managers to fine-tune prediction metrics and optimal blends during industry-specific crises vs market-wide turmoils. In addition, given the limited number of studies on FI strategies for non-US markets, a comparative study assessing predictive blended portfolios in global markets is worth pursuing.

References

Robert D. Arnott, Jason Hsu, and Philip Moore. Fundamental indexation. *Financial Analysts Journal*, 61(2):83–99, 2005.

Clifford Asness, Andrea Frazzini, Ronen Israel, and Tobias Moskowitz. Fact, fiction, and value investing. *The Journal of Portfolio Management*, 42(1):34–52, 2015.

Anup K Basu and Brigette Forbes. Does fundamental indexation lead to better risk-adjusted returns? New evidence from Australian securities exchange. *Accounting & Finance*, 54(3): 699–728, 2014.

John M Bates and Clive WJ Granger. The combination of forecasts. *Journal of the Operational Research Society*, 20(4):451–468, 1969.

Dave Berger, Kuntara Pukthuanthong, and J Jimmy Yang. Is the diversification benefit of frontier markets realizable by mean-variance investors? The evidence of investable funds. *Journal of Portfolio Management*, 39(4):36, 2013.

Stephen J Brown, William Goetzmann, Roger G Ibbotson, and Stephen A Ross. Survivorship bias in performance studies. *Review of Financial Studies*, 5(4):553–580, 1992.

Warren Buffett and Carol Loomis. Warren Buffett on the stock market. *Fortune Investor's Guide*, pages 80–94, 2001.

John Y Campbell and Robert J Shiller. Stock prices, earnings, and expected dividends. *The Journal of Finance*, 43(3):661–676, 1988.

M. Carhart. On persistence in mutual fund performance. *Journal of Finance*, 52:57–82, 1997.

Roger Clarke, Harindra De Silva, and Steven Thorley. Minimum-variance portfolio composition. *Journal of Portfolio Management*, 37(2):31, 2011.

Keith Cuthbertson and Dirk Nitzsche. *Quantitative financial economics: stocks, bonds and foreign exchange*. John Wiley & Sons, 2005.

Ian Domowitz and Ameya Moghe. Donuts: A picture of optimization applied to fundamental portfolios. *The Journal of Portfolio Management*, 44(3):103–113, 2018.

Frederick E Dopfel. The arithmetic of fundamental indexing. *The Journal of Investing*, 17(2): 114–118, 2008.

Jana Eklund and Sune Karlsson. Forecast combination and model averaging using predictive measures. *Econometric Reviews*, 26(2-4):329–363, 2007.

Neil R Ericsson. Economic forecasting in theory and practice: An interview with David F. Hendry. *International Journal of Forecasting*, 2017.

Eugene F Fama and Kenneth R French. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1):3–56, 1993.

Eugene F Fama and Kenneth R French. Multifactor explanations of asset pricing anomalies. *Journal of Finance*, 51:55–84, 1996.

James Douglas Hamilton. *Time Series Analysis*. Princeton University Press, 1994.

KiHoon Hong and Eliza Wu. The roles of past returns and firm fundamentals in driving US stock price movements. *International Review of Financial Analysis*, 43(C):62–75, 2016.

Ravi Jagannathan and Tongshu Ma. Risk reduction in large portfolios: Why imposing the wrong constraints helps. *The Journal of Finance*, 58(4):1651–1684, 2003.

Philippe Jorion. Bayes - Stein estimation for portfolio analysis. *Journal of Financial and Quantitative Analysis*, 21(3):279–292, 1986.

Oliver Ledoit and Michael Wolf. Honey, I shrunk the sample covariance matrix. *The Journal of Portfolio Management*, 30(4):110–119, 2004.

Oliver Ledoit and Michael Wolf. Robust performance hypothesis testing with the Sharpe ratio. *Journal of Empirical Finance*, 15(5):850–859, 2008.

Olivier Ledoit and Michael Wolf. Nonlinear shrinkage of the covariance matrix for portfolio selection: Markowitz meets goldilocks. *The Review of Financial Studies*, 30(12):4349–4388, 2017.

Harry M Markowitz. Portfolio selection. *The Journal of Finance*, 7(1):77–91, 1952.

Harry M Markowitz, G Peter Todd, and William F Sharpe. *Mean-variance analysis in portfolio choice and capital markets*, volume 66. John Wiley & Sons, 2000.

Whitney K. Newey and Kenneth D. West. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55(3):703, 1987.

Eben Otuteye and Mohammad Siddiquee. Overcoming cognitive biases: A heuristic for making value investing decisions. *Journal of Behavioral Finance*, 16(2):140–149, 2015.

James M Poterba and Lawrence H Summers. Mean reversion in stock prices: Evidence and implications. *Journal of Financial Economics*, 22(1):27–59, 1988.

Mark Rubinstein. Markowitz's 'portfolio selection': A fifty-year retrospective. *The Journal of Finance*, 57(3):1041–1045, 2002.

Amartya Sen. Human development index: Methodology and measurement. 1994.

Jeremy Smith and Kenneth F Wallis. A simple explanation of the forecast combination puzzle. *Oxford Bulletin of Economics and Statistics*, 71(3):331–355, 2009.

Jack Treynor. Why market-valuation-indifferent indexing works. *Financial Analysts Journal*, 61 (5):65–69, 2005.

Christian Walkshäusl. The high returns to low volatility stocks are actually a premium on high quality firms. *Review of Financial Economics*, 22(4):180–186, 2013.

Christian Walkshäusl. Mispricing and the five-factor model. *Economics Letters*, 147:99–102, 2016.

Christian Walkshäusl and Sebastian Lobe. Fundamental indexing around the world. *Review of Financial Economics*, 19(3):117–127, 2010.

Appendix

A. *Notation*

Variable	Description
i and t	subscripts denoting stock i and period t
RI	Total Return Index (includes change in price and dividends)
r_{it}	Simple return (based on RI)
FI	Fundamental Index
w^{FI}	Vector of weights of the FI portfolio
w^{GMV}	Vector of weights of the GMV portfolio
w^{PB}	Vector of weights of the Predictive Blended portfolio
Ω	Expected variance-covariance matrix of stocks
BI	Buffett Indicator = Wilshire 5000 / nominal GDP
BII	Buffett Indicator Index = [BI - min(BI)]/[max(BI) - min (BI)]
BV	Book Value
Div	Dividends for the last year
FCF	Free Cash Flows
MV	Market Value, capitalization
Rev	Revenue for the last year

B. *Additional tables*

Table 8: Annualized Out-of-Sample Portfolio Performance Statistics beginning in 1995 and ending in various years. All portfolios are based on S&P 500 stocks. Portfolios are rebalanced every year on July 1. GMV is the Global Minimum Variance portfolio, FI is Arnott's FI, PB is Predictive Blended portfolio (which blends the GMV and FI), Tangency portfolio is a portfolio tangent to the MVO frontier, S&P 500 is market capitalization-weighted portfolio, while Equally-Weighted is a portfolio, where each stock has equal presence. We estimate annualized average portfolio returns, $\bar{r}$, portfolio standard deviations, $\hat{\sigma}$, and Sharpe ratios, S, for various portfolio strategies and benchmarks. Figures in **bold** represent the lowest portfolio risk, the highest portfolio return and Sharpe ratio among the portfolio strategies and benchmarks for a given period under consideration. It is not surprising that the GMV portfolio consistently results in the lowest risk, even in out-of-sample. However, the average portfolio return and Sharpe ratio suggest that the PB portfolio exhibits superior out-of-sample performance.

July 1, *yyyy*		GMV			FI			PB			Tangency			S&P 500			Equally-Weighted		
Start	End	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S
1995	1996	21.28	**4.88**	**3.12**	22.79	8.83	1.90	22.79	8.83	1.90	27.18	7.43	2.85	**22.94**	9.85	1.72	24.45	7.15	2.58
1995	1997	20.05	**5.32**	**2.60**	26.57	10.46	1.95	26.57	10.46	1.95	26.93	8.53	2.43	**27.12**	11.81	1.77	24.99	8.71	2.16
1995	1998	20.83	**6.04**	**2.45**	**26.82**	12.09	1.72	**26.82**	12.09	1.72	26.56	9.80	2.10	26.73	13.81	1.50	24.07	10.47	1.72
1995	1999	15.09	**7.53**	1.24	24.45	13.83	**1.35**	24.45	13.83	**1.35**	21.91	12.61	1.28	**25.66**	16.24	1.23	21.38	12.79	1.22
1995	2000	11.85	**8.21**	0.74	19.70	14.64	**0.95**	19.70	14.64	**0.95**	18.57	14.12	0.90	**22.23**	17.25	**0.95**	18.53	14.08	0.90
1995	2001	12.43	**8.21**	0.82	**18.54**	14.61	**0.88**	**18.54**	14.61	**0.88**	16.58	15.63	0.69	16.28	17.92	0.59	18.30	14.35	**0.88**
1995	2002	11.32	**8.16**	0.70	14.81	14.88	0.62	**16.16**	13.94	**0.76**	12.03	15.46	0.42	10.77	18.08	0.29	15.60	14.74	0.68
1995	2003	10.93	**9.03**	**0.61**	13.75	16.71	0.50	**15.16**	13.91	0.70	11.41	16.07	0.38	10.64	19.29	0.27	14.88	16.57	0.57
1995	2004	12.09	**9.23**	0.74	14.42	16.24	0.57	15.85	13.58	**0.78**	12.42	15.59	0.46	11.05	18.61	0.31	**15.99**	16.25	0.66
1995	2005	12.86	**9.20**	**0.84**	14.01	15.75	0.56	**16.09**	13.18	0.83	12.99	15.15	0.52	10.75	17.96	0.31	15.75	15.85	0.67
1995	2006	12.66	**9.13**	**0.83**	13.60	15.31	0.56	**15.54**	12.84	0.82	12.60	14.90	0.51	10.59	17.41	0.32	15.46	15.52	0.67
1995	2007	12.75	**8.99**	0.86	14.17	14.91	0.61	**15.95**	12.59	**0.87**	12.72	14.71	0.52	11.31	16.91	0.37	15.77	15.18	0.71
1995	2008	11.21	**9.39**	**0.67**	12.06	15.27	0.46	**13.71**	13.21	0.66	12.55	15.68	0.48	9.34	17.17	0.25	13.58	15.63	0.55
1995	2009	10.03	**12.27**	0.42	9.97	19.29	0.27	**12.35**	15.19	**0.49**	10.11	18.49	0.28	7.21	20.40	0.12	11.43	19.84	0.33
1995	2010	10.52	**12.19**	0.47	10.62	19.40	0.30	**12.68**	14.94	**0.53**	10.99	18.55	0.34	7.82	20.26	0.15	12.35	20.05	0.38
1995	2011	11.26	**12.02**	0.55	11.72	19.14	0.37	13.33	14.67	**0.59**	12.65	18.33	0.44	9.14	19.93	0.23	**13.64**	19.81	0.45
1995	2012	11.32	**12.13**	0.56	11.31	19.44	0.35	13.02	14.93	**0.57**	**13.17**	18.56	0.47	9.03	20.12	0.23	12.98	20.16	0.42
1995	2013	11.52	**12.00**	0.60	12.00	19.12	0.40	13.58	14.79	**0.63**	**13.71**	18.38	0.51	9.56	19.76	0.26	13.62	19.83	0.47
1995	2014	11.45	**11.85**	0.61	12.49	18.75	0.44	13.99	14.57	**0.67**	**14.76**	18.23	0.58	10.21	19.37	0.31	14.19	19.46	0.51
1995	2015	10.87	**11.75**	0.57	12.15	18.46	0.43	13.57	14.44	**0.65**	**14.81**	17.99	0.59	10.08	19.06	0.31	13.76	19.14	0.50
1995	2016	11.20	**11.80**	0.61	11.74	18.38	0.42	13.09	14.56	**0.62**	**14.18**	17.93	0.57	9.82	18.96	0.30	13.25	19.03	0.48
1995	2017	10.84	**11.65**	0.59	12.01	18.06	0.45	13.22	14.33	**0.65**	**13.79**	17.62	0.56	10.11	18.60	0.33	13.39	18.70	0.50

Table 9: Annualized Out-of-Sample Portfolio Performance Statistics of year-by-year portfolios. GMV is the Global Minimum Variance portfolio, FI is Arnott's FI, PB is Predictive Blended portfolio (which blends the GMV and FI), Tangency portfolio is a portfolio tangent to the MVO frontier, S&P 500 is market capitalization-weighted portfolio, while Equally-Weighted is a portfolio, where each stock has equal presence. We estimate annualized average portfolio returns, $\bar{r}$, portfolio standard deviations, $\hat{\sigma}$, and Sharpe ratios, S, for various portfolio strategies and benchmarks. Figures in bold represent the lowest portfolio risk, the highest portfolio return and Sharpe ratio among the portfolio strategies and benchmarks for a given period under consideration. It is not surprising that GMV portfolio consistently results in the lowest risk, even in out-of-sample. The evidence of out-of-sample superior performance of PB portfolios is less apparent from year-by-year results compared to results in Tables 8, suggesting that this strategy is more appropriate for long-term investments. Figures in **bold** represent the lowest portfolio risk, the highest portfolio return and Sharpe ratio among the portfolio strategies and benchmarks for a given period under consideration.

July 1, *yyyy*		GMV			FI			PB			Tangency			S&P 500			Equally-Weighted		
Start	End	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S
1995	1996	21.28	**4.88**	**3.12**	22.79	8.83	1.90	22.79	8.83	1.90	**27.18**	7.43	2.85	22.94	9.85	1.72	24.45	7.15	2.58
1996	1997	18.83	**5.72**	**2.17**	30.33	11.86	2.02	30.33	11.86	2.02	26.68	9.50	2.14	**31.29**	13.48	1.85	25.52	10.03	1.91
1997	1998	22.38	**7.27**	**2.30**	**27.31**	14.81	1.46	**27.31**	14.81	1.46	25.82	11.94	1.69	25.94	17.12	1.18	22.25	13.29	1.25
1998	1999	-2.11	**10.75**	-0.66	17.37	18.06	0.69	17.37	18.06	0.69	7.97	18.62	0.16	**22.45**	21.97	**0.80**	13.32	18.02	0.46
1999	2000	-1.03	**10.45**	-0.67	0.79	17.45	-0.30	0.79	17.45	-0.30	5.28	18.95	-0.04	**8.58**	20.76	**0.12**	7.19	18.32	0.07
2000	2001	15.36	**8.24**	1.22	12.73	14.43	0.51	12.73	14.43	0.51	6.58	21.65	0.06	-13.57	20.84	-0.91	**17.14**	15.62	**0.76**
2001	2002	**4.60**	**7.79**	**-0.03**	-7.65	16.37	-0.76	1.79	8.91	-0.34	-15.36	14.31	-1.41	-22.38	18.93	-1.44	-0.67	16.91	-0.33
2002	2003	8.20	**13.68**	**0.32**	6.35	26.12	0.10	8.20	**13.68**	**0.32**	7.13	19.78	0.17	9.72	26.23	0.22	**9.83**	25.98	0.23
2003	2004	21.30	**10.63**	1.61	19.77	11.88	1.31	21.30	**10.63**	1.61	20.46	11.04	1.47	14.32	11.81	**0.86**	**24.83**	13.42	1.54
2004	2005	**19.87**	8.93	**1.76**	10.31	10.26	0.60	18.28	**8.78**	1.61	18.08	10.47	1.33	8.04	10.49	0.37	13.64	11.60	0.82
2005	2006	10.60	**8.41**	**0.73**	9.50	10.03	0.50	10.09	8.76	0.64	8.74	12.03	0.35	9.03	10.33	0.44	**12.51**	11.71	0.69
2006	2007	13.80	**7.33**	1.25	20.41	9.41	**1.67**	**20.41**	9.41	**1.67**	13.97	12.54	0.74	19.27	9.88	1.48	19.20	10.68	1.36
2007	2008	-7.32	**13.15**	-0.86	-13.18	18.99	-0.91	-13.18	18.99	-0.91	**10.51**	24.45	**0.26**	-14.34	20.03	-0.92	-12.62	20.26	-0.82
2008	2009	**-5.25**	**30.98**	**-0.27**	-17.25	46.59	-0.44	**-5.25**	**30.98**	**-0.27**	-21.59	39.82	-0.62	-20.43	44.61	-0.53	-16.59	48.24	-0.41
2009	2010	17.29	**10.98**	**1.26**	19.72	20.98	0.77	17.29	**10.98**	**1.26**	23.27	19.36	1.02	16.34	18.19	0.71	**25.19**	22.79	0.95
2010	2011	22.36	**9.23**	2.10	28.17	14.62	1.72	23.09	9.68	2.07	**37.57**	14.46	**2.39**	28.98	13.93	1.86	33.13	15.84	1.90
2011	2012	12.33	**13.73**	**0.75**	4.81	23.74	0.12	8.06	18.60	0.32	**21.50**	21.94	0.89	7.24	22.98	0.23	2.39	25.10	0.01
2012	2013	14.87	**9.50**	1.38	23.78	12.46	1.77	23.08	12.08	**1.76**	22.90	14.86	1.42	18.50	11.90	1.40	**24.48**	12.86	**1.76**
2013	2014	10.19	**8.85**	0.85	21.31	9.90	1.88	21.31	9.90	1.88	**33.69**	15.27	2.03	21.98	10.01	1.93	24.44	10.67	**2.04**
2014	2015	-0.04	**9.61**	-0.23	5.61	11.53	0.30	5.61	11.53	0.30	15.83	12.65	**1.08**	**7.55**	11.87	0.45	5.65	11.60	0.30
2015	2016	**17.72**	**12.63**	**1.25**	3.64	16.80	0.10	3.64	16.80	0.10	1.64	16.66	-0.02	4.73	16.83	0.16	3.00	16.75	0.06
2016	2017	3.36	**7.92**	0.16	**17.62**	8.84	1.76	15.98	8.33	1.67	5.45	9.10	0.37	16.25	8.02	**1.77**	16.33	9.05	1.57

Table 10: Annualized Out-of-Sample Portfolio Performance Statistics beginning in 1995 - ending in different years to 2017. First three portfolios are blended portfolios constructed in fixed proportions; the Best and the Worst Blends are constructed given unrealistic perfect foresight scenarios, and CAPE-based PB is an alternative modification of the Predictive Blended portfolio (where Shiller's CAPE is used instead of the Buffett Indicator). We estimate annualized average portfolio returns, $\bar{r}$, portfolio standard deviations, $\hat{\sigma}$, and Sharpe ratios, S, for various portfolio strategies and benchmarks.

July 1, *yyyy*		25%FI+75%GMV			50%FI+50%GMV			75%FI+25%GMV			Worst Blend			Best Blend			CAPE-based PB		
Start	End	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S
1995	1996	21.61	5.29	2.95	21.98	6.21	2.57	22.37	7.44	2.20	22.79	8.83	1.90	21.29	4.89	3.12	22.79	8.83	1.90
1995	1997	21.68	6.09	2.54	23.31	7.34	2.33	24.94	8.84	2.12	26.57	10.46	1.95	21.35	5.83	2.60	25.95	10.01	1.97
1995	1998	22.30	7.06	2.31	23.79	8.53	2.08	25.29	10.24	1.88	26.82	12.09	1.72	21.69	6.35	2.47	26.33	11.68	1.74
1995	1999	17.46	8.47	1.38	19.80	9.99	1.41	22.13	11.82	1.39	19.58	11.79	1.17	20.61	10.57	1.40	23.15	12.93	1.35
1995	2000	13.80	9.06	0.88	15.76	10.58	0.94	17.72	12.50	0.95	15.44	11.55	0.83	16.63	12.28	0.88	18.43	12.87	0.98
1995	2001	13.93	9.06	0.91	15.45	10.56	0.92	16.99	12.47	0.90	14.99	12.07	0.77	16.42	11.70	0.91	17.71	12.44	0.96
1995	2002	12.17	9.06	0.73	13.03	10.64	0.70	13.91	12.64	0.66	11.76	12.78	0.48	14.74	11.23	0.81	15.04	12.21	0.77
1995	2003	11.58	10.19	0.61	12.27	12.00	0.57	12.99	14.23	0.54	11.09	15.11	0.38	13.92	11.57	0.74	14.10	12.71	0.69
1995	2004	12.61	10.17	0.72	13.18	11.80	0.67	13.78	13.88	0.61	12.05	14.79	0.46	14.69	11.41	0.83	14.90	12.50	0.77
1995	2005	13.10	10.04	0.79	13.37	11.54	0.71	13.68	13.50	0.63	11.88	14.40	0.47	15.21	11.18	0.90	15.27	12.18	0.83
1995	2006	12.85	9.91	0.78	13.07	11.33	0.71	13.32	13.18	0.63	11.66	14.06	0.47	14.79	10.96	0.89	14.82	11.89	0.82
1995	2007	13.07	9.74	0.82	13.41	11.09	0.75	13.77	12.86	0.68	11.84	13.63	0.50	15.25	10.84	0.94	14.83	11.59	0.84
1995	2008	11.38	10.17	0.63	11.58	11.51	0.57	11.81	13.26	0.52	10.09	13.85	0.37	13.52	11.04	0.77	13.03	11.80	0.68
1995	2009	9.96	13.34	0.38	9.92	14.94	0.34	9.92	16.95	0.30	8.13	18.26	0.18	12.18	13.49	0.54	11.47	14.60	0.45
1995	2010	10.48	13.32	0.43	10.48	14.98	0.38	10.52	17.04	0.34	8.91	18.46	0.23	12.52	13.34	0.58	11.87	14.44	0.49
1995	2011	11.31	13.15	0.51	11.40	14.79	0.46	11.54	16.82	0.41	10.11	18.24	0.30	13.13	13.12	0.65	12.68	14.26	0.56
1995	2012	11.25	13.32	0.51	11.23	15.01	0.45	11.24	17.08	0.40	9.80	18.61	0.29	13.09	13.15	0.65	12.33	14.71	0.53
1995	2013	11.58	13.15	0.55	11.68	14.79	0.50	11.82	16.81	0.44	10.08	18.23	0.32	13.68	13.12	0.71	12.97	14.59	0.59
1995	2014	11.66	12.95	0.57	11.90	14.54	0.53	12.17	16.50	0.48	10.09	17.86	0.33	14.08	12.97	0.76	13.41	14.38	0.64
1995	2015	11.14	12.81	0.55	11.44	14.35	0.51	11.77	16.26	0.47	9.58	17.54	0.31	13.66	12.90	0.74	13.02	14.25	0.62
1995	2016	11.29	12.82	0.57	11.41	14.33	0.51	11.56	16.21	0.46	9.30	17.50	0.30	13.85	12.89	0.76	12.57	14.39	0.59
1995	2017	11.10	12.62	0.57	11.38	14.09	0.53	11.68	15.93	0.48	9.03	17.18	0.30	14.02	12.73	0.79	12.80	14.18	0.62

Table 11: Annualized Out-of-Sample Portfolio Performance Statistics of year-by-year portfolios. The first three portfolios are blended portfolios constructed in fixed proportions; the Best and the Worst Blends are constructed given unrealistic perfect foresight scenarios, and CAPE-based PB is an alternative modification of the Predictive Blended portfolio (where Shiller's CAPE is used instead of the Buffett Indicator). We estimate annualized average portfolio returns, $\bar{r}$, portfolio standard deviations, $\hat{\sigma}$, and Sharpe ratios, S, for various portfolio strategies and benchmarks.

July 1, *yyyy*		25%FI+75%GMV			50%FI+50%GMV			75%FI+25%GMV			Worst Blend			Best Blend			CAPE-based PB		
Start	End	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S
1995	1996	21.61	5.29	2.95	21.98	6.21	2.57	22.37	7.44	2.20	22.79	8.83	1.90	21.29	4.89	3.12	22.79	8.83	1.90
1996	1997	21.75	6.79	2.26	24.64	8.31	2.20	27.50	10.04	2.10	30.33	11.86	2.02	21.40	6.63	2.26	29.09	11.05	2.05
1997	1998	23.54	8.68	2.06	24.74	10.50	1.82	26.00	12.58	1.62	27.31	14.81	1.46	22.38	7.27	2.30	27.10	14.44	1.48
1998	1999	2.94	11.68	-0.17	7.86	13.42	0.22	12.67	15.62	0.49	-2.11	10.75	-0.66	17.37	18.06	0.69	13.62	16.09	0.54
1999	2000	-0.76	11.06	-0.61	-0.36	12.63	-0.50	0.16	14.85	-0.39	-1.03	10.45	-0.67	0.79	17.45	-0.30	-0.37	12.55	-0.51
2000	2001	14.60	9.02	1.03	13.91	10.44	0.82	13.29	12.30	0.65	12.73	14.43	0.51	15.36	8.24	1.22	14.10	9.99	0.88
2001	2002	1.55	9.04	-0.37	-1.51	11.07	-0.57	-4.58	13.57	-0.69	-7.65	16.37	-0.76	4.60	7.79	-0.03	-1.02	10.71	-0.55
2002	2003	7.48	15.99	0.23	6.92	18.95	0.16	6.54	22.36	0.12	6.35	26.12	0.10	8.20	13.68	0.32	7.53	15.78	0.23
2003	2004	20.82	10.03	1.66	20.41	10.05	1.61	20.06	10.70	1.48	19.77	11.88	1.31	20.80	10.02	1.66	21.30	10.63	1.61
2004	2005	17.52	8.75	1.53	15.15	8.92	1.23	12.74	9.43	0.91	10.31	10.26	0.60	19.87	8.93	1.76	18.56	8.79	1.64
2005	2006	10.30	8.54	0.68	10.02	8.87	0.62	9.75	9.38	0.56	9.50	10.03	0.50	10.60	8.41	0.73	10.36	8.50	0.69
2006	2007	15.47	7.60	1.42	17.13	8.06	1.55	18.77	8.68	1.63	13.80	7.33	1.25	20.41	9.41	1.67	14.87	7.48	1.37
2007	2008	-8.82	14.24	-0.90	-10.30	15.61	-0.92	-11.76	17.20	-0.92	-10.95	16.28	-0.92	-7.32	13.15	-0.86	-8.46	13.95	-0.90
2008	2009	-8.52	33.87	-0.35	-11.66	37.42	-0.40	-14.59	41.66	-0.43	-17.25	46.59	-0.44	-5.25	30.98	-0.27	-8.91	34.26	-0.35
2009	2010	17.75	13.06	1.09	18.32	15.56	0.95	18.98	18.24	0.85	19.72	20.98	0.77	17.29	10.98	1.26	17.52	11.99	1.17
2010	2011	23.77	10.20	2.04	25.21	11.50	1.93	26.68	13.00	1.82	28.17	14.62	1.72	22.36	9.23	2.10	24.87	11.17	1.96
2011	2012	10.36	15.71	0.53	8.44	18.09	0.35	6.58	20.78	0.22	4.81	23.74	0.12	12.33	13.73	0.75	6.73	20.55	0.23
2012	2013	17.12	9.75	1.57	19.35	10.39	1.69	21.57	11.32	1.75	14.87	9.50	1.38	23.78	12.46	1.77	23.78	12.46	1.77
2013	2014	13.05	8.66	1.20	15.86	8.81	1.50	18.61	9.24	1.73	10.19	8.85	0.85	21.31	9.90	1.88	21.31	9.90	1.88
2014	2015	1.36	9.75	-0.09	2.77	10.13	0.06	4.19	10.74	0.19	-0.04	9.61	-0.23	5.61	11.53	0.30	5.61	11.53	0.30
2015	2016	14.32	13.14	0.94	10.84	14.01	0.63	7.28	15.25	0.35	3.64	16.80	0.10	17.72	12.63	1.25	3.64	16.80	0.10
2016	2017	7.05	7.27	0.68	10.66	7.29	1.18	14.18	7.87	1.54	3.36	7.92	0.16	17.62	8.84	1.76	17.62	8.84	1.76

C. *Covariance matrix estimation*

Our portfolios are constructed using the Ledoit and Wolf [2004] shrinkage estimator. We performed a thorough analysis of estimated covariance matrices based on sample covariance, linear [i.e., Ledoit and Wolf (2014)] and non-linear [i.e., Ledoit and Wolf [2017]] shrinkage estimators. The presence of a weight constraint eliminates the need for more sophisticated methods. In Figure 3 we show that in the absence of weight constraints, portfolio frontiers based on the three covariance estimation methods differ substantially. However, when the weight constraint is introduced, the choice of the covariance matrix estimator becomes less relevant.

Figure 3: PORTFOLIO FRONTIERS "WITH" VS "WITHOUT" THE WEIGHT CONSTRAINT. We constructed three portfolio frontiers (based on sample, linear shrinkage and non-linear shrinkage covariance estimators). In the left panel, the optimization procedure did not include asset weights. In the right panel, we enforced a weight constraint, limiting individual assets to be between 0 and 10% of a portfolio. For illustrative purposes, we used data from a five-year sub-period, namely from July 1, 2011 to June 30, 2016. We performed similar analysis on the remaining 22 sub-periods and can confirm that the pattern remains the same: large discrepancies in portfolio frontiers and portfolio compositions with no weight constraint, and minimal or no discrepancies when the weight constraint (no short sales, less or equal 10% long positions) is enforced.