FAST SCENARIO-BASED OPTIMAL CONTROL FOR STOCHASTIC PORTFOLIO OPTIMIZATION with Application to a Large-Scale Portfolio

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Certificate of Original Authorship

I, Marc Weibel declare that this thesis, is submitted in fulfilment of the requirements for the award of Degree of Doctor of Philosophy, in the Finance Discipline Group of the Faculty of Sciences at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise reference or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

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Signature of Student: ____________________________ Date: ________________

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Abstract

This thesis contributes towards the development of a fast optimal control algorithm, relying on the Alternating-Direction of Multipliers (ADMM), for solving large-scale linear convex multi-period optimization problems as well as the design of investment strategies aiming at stabilizing portfolio performance over time.

The first part of the thesis focuses on a statistical risk-budgeting method to improve naive diversification strategies. We extend the so-called minimum-torsion approach and use advanced modern techniques for covariance estimation and shrinkage purposes. We propose a novel factor investing approach, which dynamically identifies statistical risk factors over time. We device dynamic investment strategies aiming at diversifying idiosyncratic risk left unexplained by the factors.

We develop in the second part of this thesis a fast algorithm for solving scenario-based model predictive control (MPC) arising in multi-period portfolio optimization problems efficiently. We derive an alteration of the termination criterion, using the probabilities assigned to the scenarios and provide a convergence analysis. We show that the proposed criterion outperforms the standard approach and highlight our results with a numerical comparison with a state-of-the-art algorithm. We also enhance the standard two-set splitting algorithm of the ADMM method, by including inequality constraints through a so-called embedded splitting, without recourse to an additional (costly) splitting set.

We present a real-world large-scale multi-period portfolio application, where we combine the different concepts derived in this thesis. We propose an approach to generate scenarios relying on a Hidden Markov Model (HMM) and solve the constrained multi-period MPC problem with the ADMM algorithm developed. We also suggest an innovative concept to steer the risk-aversion used in the objective function dynamically, building on the probability assigned to each scenario. We back-test the strategy and show that the results obtained do provide the expected risk-adjusted outperformance and stability, without deviating significantly from the strategic asset allocation.

Key words: Risk-Budgeting, Diversification, Convex Optimization, Model Predictive Control, Alternative-Direction Method of Multipliers, Optimal Control.
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My first contact with financial theory was during my Master Studies in Economics at the University of Neuchatel, Switzerland. I took my first finance class of Prof. Michel Dubois and resolved to take every class Michel has proposed since. Michel was an incredible lecturer and teacher and sparked my interest in financial topics.

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It's not about finding your limits. It's about finding what lies just beyond them.
— Unknown

To my wife Coralie ...
Abbreviations

AADMM ........ Accelerated Alternating Direction Method of Multipliers
ADP ............ Approximate Dynamic Programming
ADMM ........... Alternating Direction Method of Multipliers
CVaR ............ Conditional Value-at-Risk
DP ............... Dynamic Programming
HMM ............. Hidden Markov Model
MPC ............. Model Predictive Control
PCA ............. Principal Component Analysis
RMT ............. Random Matrix Theory
VaR ............. Value-at-Risk