

DOCTORAL THESIS

---

FAST SCENARIO-BASED OPTIMAL CONTROL  
FOR STOCHASTIC PORTFOLIO OPTIMIZATION  
with Application to a Large-Scale Portfolio

---

*Author:*  
Marc WEIBEL

*Supervisor:*  
Associate Professor. Juri HINZ

*Co-supervisor:*  
Professor. Marc WILDI

A thesis submitted to the [Finance Discipline Group](#) of the [University of Technology Sydney](#), in  
fulfilment of the requirements for the degree of Doctor of Philosophy.

June 2019

Finance Discipline Group  
University of Technology Sydney  
PO Box 123, Broadway, NSW 2007,  
Australia

# Certificate of Original Authorship

I, Marc Weibel declare that this thesis, is submitted in fulfilment of the requirements for the award of Degree of Doctor of Philosophy, in the Finance Discipline Group of the Faculty of Sciences at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise reference or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

This research is supported by the Australian Government Research Training Program.

Signature of Student:

Production Note:

Signature removed prior to publication.

Date:

10/26/2018

# Abstract

This thesis contributes towards the development of a fast optimal control algorithm, relying on the Alternating-Direction of Multipliers (ADMM), for solving large-scale linear convex multi-period optimization problems as well as the design of investment strategies aiming at stabilizing portfolio performance over time.

The first part of the thesis focuses on a statistical risk-budgeting method to improve naive diversification strategies. We extend the so-called minimum-torsion approach and use advanced modern techniques for covariance estimation and shrinkage purposes. We propose a novel factor investing approach, which dynamically identifies statistical risk factors over time. We devise dynamic investment strategies aiming at diversifying idiosyncratic risk left unexplained by the factors.

We develop in the second part of this thesis a fast algorithm for solving scenario-based model predictive control (MPC) arising in multi-period portfolio optimization problems efficiently. We derive an alteration of the termination criterion, using the probabilities assigned to the scenarios and provide a convergence analysis. We show that the proposed criterion outperforms the standard approach and highlight our results with a numerical comparison with a state-of-the-art algorithm. We also enhance the standard two-set splitting algorithm of the ADMM method, by including inequality constraints through a so-called embedded splitting, without recourse to an additional (costly) splitting set.

We present a real-world large-scale multi-period portfolio application, where we combine the different concepts derived in this thesis. We propose an approach to generate scenarios relying on a Hidden Markov Model (HMM) and solve the constrained multi-period MPC problem with the ADMM algorithm developed. We also suggest an innovative concept to steer the risk-aversion used in the objective function dynamically, building on the probability assigned to each scenario. We back-test the strategy and show that the results obtained do provide the expected risk-adjusted outperformance and stability, without deviating significantly from the strategic asset allocation.

Key words: Risk-Budgeting, Diversification, Convex Optimization, Model Predictive Control, Alternative-Direction Method of Multipliers, Optimal Control.

# Acknowledgements

During my career in the industry and at the Zurich University of Applied Sciences, I have had the privilege and pleasure of working and interacting with many talented persons. These people have contributed to my education and my evolution and it would be impossible to name each of them.

First of all, I would like to thank my principal advisor, Juri Hinz. He gave me the opportunity to pursue PhD study, his dedication and support have helped me to navigate through my thesis and to conduct a proper research. He undertook the necessary steps for enrolling me into the program at the University of Technology in Sidney and helped me in every aspect of the PhD thesis. I would like to express my deepest gratitude and respect to Juri for his support and encouragement.

I would like to thank my co-adviser, Marc Wildi, who played an important role in the process of my PhD study. Our close interaction brought me further in my research and his funded knowledge in econometrics have helped me in taking the correct direction when I felt uncertain.

I would also like to express my gratitude to my management at the Zurich University of Applied Sciences, in particular Wolfgang Breymann and Jürg Hosang, who encouraged me to accomplish this PhD.

My first contact with financial theory was during my Master Studies in Economics at the University of Neuchatel, Switzerland. I took my first finance class of Prof. Michel Dubois and resolved to take every class Michel has proposed since. Michel was an incredible lecturer and teacher and sparked my interest in financial topics.

I would like to thank the academics at UTS, who offered me the opportunity to pursue my study out of Switzerland and the reviewers who took the time necessary for reading my progress reports and provided me with thoughtful advice.

On the private side, I would like to thank my family and especially my lovely wife Coralie for her unconditional and constant support throughout the years. She let me work sometimes very late in the night to finish a chapter and put a smile back on my face, when I felt demoralized. This thesis is dedicated to you Coralie.

# Contents

<b>Declaration</b>	<b>i</b>
<b>Abstract</b>	<b>ii</b>
<b>Acknowledgements</b>	<b>iii</b>
<b>List of Figures</b>	<b>ix</b>
<b>List of Tables</b>	<b>x</b>
<b>Abbreviations</b>	<b>xii</b>
<b>Introduction</b>	<b>1</b>
<b>1 Alternative Diversification Strategies</b>	<b>3</b>
1.1 Introduction . . . . .	3
1.2 Naive Diversification . . . . .	4
1.2.1 Mean-Variance and Naive Diversification . . . . .	5
1.3 Random Matrix Theory . . . . .	5
1.3.1 Overview . . . . .	6
1.3.2 Theory . . . . .	6
1.3.3 Random correlation matrices . . . . .	7
1.4 Minimum-Torsion . . . . .	11

1.4.1 Corrected-Benchmark Portfolio . . . . .	14
1.5 Case Study . . . . .	15
1.6 Conclusion . . . . .	18
<b>2 Statistical Risk Budgeting</b>	<b>19</b>
2.1 Background . . . . .	19
2.2 Factor Investing . . . . .	20
2.2.1 Smart Beta . . . . .	20
2.2.2 Risk Drivers . . . . .	20
2.3 Statistical Factors . . . . .	21
2.3.1 Minimum-Torsion Approach . . . . .	22
2.3.2 Uncorrelated Factors . . . . .	22
2.3.3 Effective Rank . . . . .	23
2.3.4 Factor Risk Budgeting . . . . .	24
2.4 Diversification . . . . .	25
2.4.1 Idiosyncratic Risk . . . . .	25
2.4.2 Measuring Diversification . . . . .	26
2.5 Investment Strategies . . . . .	27
2.6 Application . . . . .	29
2.6.1 Goal . . . . .	29
2.6.2 Data . . . . .	29
2.6.3 Benchmark Strategies . . . . .	30
2.6.4 Risk Parity Strategies . . . . .	31
2.6.5 Risk Budgeting Strategies . . . . .	31
2.6.6 Diversification Analysis . . . . .	35
2.7 Conclusion . . . . .	36

<b>3 Portfolio Optimization</b>	<b>37</b>
3.1 Single-Period Optimization . . . . .	37
3.1.1 Modern Portfolio Theory . . . . .	38
3.1.2 Mean-Variance Framework . . . . .	38
3.2 Multi-Period Optimization . . . . .	39
3.3 Dynamic Programming Techniques . . . . .	43
 <b>4 Model Predictive Control</b>	 <b>45</b>
4.1 Introduction . . . . .	45
4.2 Background . . . . .	46
4.3 Scenario-Based MPC . . . . .	46
4.4 Portfolio, Benchmark and Trading . . . . .	47
4.5 Constraints . . . . .	48
4.5.1 Minimum and Maximum Weights . . . . .	48
4.5.2 Brokerage Costs and Bid-Ask Spread . . . . .	49
4.5.3 Price impact . . . . .	49
4.6 Problem Description . . . . .	49
4.6.1 Portfolio Restrictions . . . . .	50
4.6.2 Objective function . . . . .	51
4.7 Decomposition Quadratic / Non-Quadratic . . . . .	51
4.7.1 Quadratic Component . . . . .	52
4.7.2 Non-Quadratic Component . . . . .	53
4.8 Conclusion . . . . .	54
 <b>5 Fast Scenario-Based Optimal Control</b>	 <b>55</b>
5.1 Definitions . . . . .	56
5.1.1 Convex Functions . . . . .	56

5.1.2 Proximal Operators . . . . .	56
5.1.3 Proximal minimization . . . . .	57
5.2 ADMM . . . . .	58
5.2.1 Accelerated ADMM . . . . .	59
5.3 Splitting the MPC Problem . . . . .	60
5.3.1 Overview . . . . .	60
5.3.2 Notation . . . . .	61
5.3.3 ADMM Formulation . . . . .	62
5.3.4 Splitting Operator . . . . .	63
5.4 Solving the Convex Quadratic Control Problem . . . . .	65
5.5 Solving the Convex Non-Quadratic Problem . . . . .	68
5.6 Extending the State-of-the-Art . . . . .	69
5.6.1 Improving Convergence . . . . .	69
5.6.2 Extended Two-Set Splitting . . . . .	72
5.6.3 Numerical Results . . . . .	75
5.7 Conclusion . . . . .	76
<b>6 Application</b>	<b>77</b>
6.1 Background . . . . .	77
6.2 Data . . . . .	78
6.3 Scenario Generator . . . . .	80
6.3.1 Hidden Markov Model . . . . .	80
6.3.2 Methodology . . . . .	82
6.4 Large-Scale Dynamic Portfolio Strategy . . . . .	84
6.4.1 Benchmark . . . . .	84
6.4.2 Scenarios and Rebalancing . . . . .	85



6.4.3 Risk Aversion . . . . .	85
6.4.4 Costs and Restrictions . . . . .	85
6.4.5 Results . . . . .	85
6.5 Conclusion . . . . .	87
<b>7 Conclusion and Outlook</b>	<b>88</b>
7.1 Conclusion . . . . .	88
7.2 Further Research . . . . .	89
7.2.1 Risk-Budgeting . . . . .	89
7.2.2 Multi-Period Optimization via ADMM . . . . .	89
7.2.3 Investment Strategies . . . . .	90
7.2.4 Scenario Generator . . . . .	90
<b>A Appendix</b>	<b>92</b>
A.1 The Jacobi and Gauss-Seidel Iterative Methods . . . . .	92
A.1.1 Jacobi Method . . . . .	92
A.1.2 Gauss-Siedel Method . . . . .	92
<b>Bibliography</b>	<b>99</b>

# List of Figures

1.1	Eigenvalues distribution of 200 random assets . . . . .	9
1.2	Eigenvalues spectrum of 75 stocks in the S&P500 . . . . .	9
1.3	S&P500: Marchenko-Pastur density (best fit) . . . . .	10
1.4	S&P500: Reshuffled assets) . . . . .	10
1.5	Eigenvalues spectrum of the S&P500 stocks . . . . .	15
1.6	Cumulative performance . . . . .	17
1.7	Outperformance vs. buy-and-hold portfolio . . . . .	17
2.1	Asset Classes – Correlations. . . . .	30
2.2	Risk Parity Strategies – Performance . . . . .	32
2.3	Risk Budgeting Strategies – Performance . . . . .	33
2.4	Risk Parity vs. Risk Budgeting – Performance . . . . .	34
2.5	Diversification along assets and factors . . . . .	35
5.1	Convergence properties: weighted vs. unweighted scheme . . . . .	71
6.1	Asset Classes – Correlations. . . . .	79
6.2	Dynamic Portfolio – Performance . . . . .	86

# List of Tables

1.1	Minimum-Torsion algorithm . . . . .	14
2.1	Asset Classes – Statistics . . . . .	30
2.2	Risk Parity Strategies – Key figures . . . . .	31
2.3	Risk Budgeting Strategies – Key figures . . . . .	32
2.4	Risk Parity vs. Risk Budgeting – Key figures . . . . .	34
5.1	Computational Time Results for Stochastic MPC Problems . . . . .	75
6.1	Dow Jones Stocks – Statistics . . . . .	78
6.2	Dynamic Portfolio vs. Benchmark – Key figures . . . . .	86

It's not about finding your limits. It's about finding what lies just beyond them.  
— Unknown

To my wife Coralie ...

# Abbreviations

<b>AADMM</b> .....	Accelerated Alternating Direction Method of Multipliers
<b>ADP</b> .....	Approximate Dynamic Programming
<b>ADMM</b> .....	Alternating Direction Method of Multipliers
<b>CVaR</b> .....	Conditional Value-at-Risk
<b>DP</b> .....	Dynamic Programming
<b>HMM</b> .....	Hidden Markov Model
<b>MPC</b> .....	Model Predictive Control
<b>PCA</b> .....	Principal Component Analysis
<b>RMT</b> .....	Random Matrix Theory
<b>VaR</b> .....	Value-at-Risk