

UNIVERSITY OF TECHNOLOGY SYDNEY  
Faculty of Engineering and Information Technology

**MODELING AND CONTROL OF SMART  
STRUCTURES EMBEDDED WITH  
MAGNETORHEOLOGICAL DEVICES**

by

**Sayed Ahmed Royel**

A THESIS SUBMITTED  
IN PARTIAL FULFILLMENT OF THE  
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## Certificate of Authorship/Originality

I, Sayed Royel declare that this thesis, is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Electrical and Data Engineering at the University of Technology Sydney.

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## ABSTRACT

### **MODELING AND CONTROL OF SMART STRUCTURES EMBEDDED WITH MAGNETORHEOLOGICAL DEVICES**

by Sayed Ahmed Royel

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Dr. Ricardo P. Aguilera

Engineering structures are the essence of supporting the development of society. However, quite often do they suffer from hostile dynamic loadings or external disturbances that may affect structural health or function. Modeling and control techniques can be applied to resiliently preserve structural health and function with a low energy cost, which is the main theme of the thesis. Smart structures embedded with semi-active control devices offer a promising solution to the problem, such as the magnetorheological (MR) damper (MRD), pin joint (MRP), and elastomer base isolator (MRE). This study first aims at exploring the solutions to the present problem in system modeling and controller design of MR based systems to effectively damp out unwanted vibrations as well as control the embodied energy level. Multi-variable hysteresis models for these structural members are developed, capable of effectively working in a wide scale of loading amplitude and frequency. The modeling objective is to illustrate the intrinsic nonlinearity with traceable relationships between model parameters and control signals in order to realize the field-controlled method for MR structure systems. Experimental data are obtained from a long-stroke MRD, a recent prototype of MRP and an MRE under different loading conditions for model identification and performance assessment. To achieve robustness, a second-order sliding mode controller is designed and applied to the MRE to provide a real-time feedback control of structures. The performance of the proposed technique is evaluated in the simulation of a seismically excited three-

storey benchmark building model. To exploit the frequency domain advantage, this study also focuses on the cyclic dissipation of vibration-induced energy in the smart devices under a controlled magnetic field to analyze the energy relationships of the smart devices in the structures. A frequency-shaped second-order sliding mode controller (FS2SMC) is designed along with a low-pass filter to implement the desired dynamic sliding surface. The proposed controller can shape the frequency characteristics of the equivalent dynamics for the MR structure against induced vibrations, and hence, dissipate the mechanical energy flow within the devices to prevent structural damage. The energy spectra of a 10-floor building subject to four benchmark earthquakes are analyzed in terms of kinetic, damping, strain, and input energies to illustrate the capability of an energy-efficient embedded structure. The merits of FS2SMC in engineering structures can also be verified in a half-car model for reducing the roll angle while adjusting the spectrum to prevent natural modes of the structure under external excitations.

## Dedication

To my parents associate professor *Sayed Jahangir Alam* and *Hafiza Khatun*, wife *Farjana Hoq*, and son *Sayed Shoayb Ibraheem*.

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## List of Publications

### *Journal Papers*

- J-1. Q. P. Ha, **S. Royel**, and C. Balaguer (2018), “Low-energy Structures Embedded With Smart Dampers,” *Energy and Buildings*, vol. 177, pp. 375 - 384, doi: 10.1016/j.enbuild.2018.08.016.
- J-2. Q. P. Ha, **S. Royel**, J. Li and Y. Li (2016), “Hysteresis Modeling of Smart Structure MR Devices using Describing Functions,” *IEEE/ASME Transactions on Mechatronics*, vol. 21, no. 1, pp. 44 - 50, doi: 10.1109/TMECH.2015.2506539.
- J-3. Y. Yu, **S. Royel**, J. Li, Y. Li and Q. P. Ha (2016), “Magnetorheological Elastomer Base Isolator for Earthquake Response Mitigation on Building Structures: Modeling and Second-Order Sliding Mode Control,” *Earthquakes and Structures*, vol. 11, no. 6, pp. 943-966, doi: 10.12989/eas.2016.11.6.943.
- J-4. Y. Yu, Y. Li, J. Li, X. Gu, and **S. Royel** (2018), “Nonlinear Characterisation of the MRE Isolator Using Binary-coded Discrete CSO and ELM,” *International Journal of Structural Stability and Dynamics*, vol 18, no 8, doi: 10.1142/S0219455418400072.

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- C-2. **S. Royel** and Q. Ha (2017), “Frequency Shaped Sliding Mode Control of Magnetorheological Smart Structure Systems,” *Proc. IEEE International Confer-*

- ence on Mechatronics*, pp. 117–122, Gippsland, Australia, Feb. 13-15, doi: 10.1109/ICMECH.2017.7921090.
- C-3. **S. Royel**, Y. Yu, Y. Li, J. Li and Q. P. Ha (2015), “A Hysteresis Model and Parameter Identification for MR Pin Joints using Immune Particle Swarm Optimization,” *Proc. IEEE International Conference on Automation Science and Engineering*, pp. 1319-1324, Gothenburg, Sweden, Aug. 24 - 28, doi: 10.1109/CoASE.2015.7294281.
- C-4. **S. Royel**, Z. Movassaghi, N. M. Kwok and Q. P. Ha (2012), “Smart Structures Using MR dampers with Second Order Sliding Mode Control,” *Proc. IEEE International Conference on Control, Automation and Information Science*, pp. 170-175, Ho Chi Minh, Vietnam, Nov. 26 - 29, 2012, doi: 10.1109/ICCAIS.2012.6466580.



# Contents

Certificate	ii
Abstract	iii
Dedication	v
Acknowledgments	vi
List of Publications	vii
List of Figures	xiv
List of Tables	xix
Notation	xx
Abbreviation	xxii
<b>1 Introduction</b>	<b>1</b>
1.1 Background . . . . .	1
1.2 Research objectives . . . . .	7
1.3 Thesis organization . . . . .	8
<b>2 Literature Survey</b>	<b>12</b>
2.1 Smart Structures . . . . .	12
2.2 Structural Control Systems . . . . .	13
2.2.1 Passive control systems . . . . .	15
2.2.2 Active control systems . . . . .	17
2.2.3 Semi-active control systems . . . . .	17

2.2.4	Hybrid control systems . . . . .	18
2.2.5	Smart structure control systems . . . . .	19
2.3	Smart Materials and Magnetorheological Devices . . . . .	20
2.3.1	Smart materials . . . . .	20
2.3.2	Magnetorheological devices . . . . .	20
2.3.3	Mathematical models of MR devices . . . . .	24
2.4	Sliding Mode Control . . . . .	25
2.4.1	Background . . . . .	25
2.4.2	First-order sliding mode control . . . . .	37
2.4.3	Higher-order sliding mode control . . . . .	38
2.5	Frequency-Shaped Control . . . . .	40
<b>3</b>	<b>Magnetorheological Devices</b>	<b>45</b>
3.1	Introduction . . . . .	45
3.2	Magnetorheological Damper . . . . .	46
3.2.1	Experimental setup . . . . .	47
3.2.2	Energy cycle of MR dampers . . . . .	49
3.2.3	Dissipation in MR dampers . . . . .	52
3.3	Magnetorheological Pin Joint . . . . .	55
3.3.1	Experimental setup and device characteristics . . . . .	57
3.3.2	Static hysteresis model . . . . .	60
3.3.3	Model identification using immune particle swarm optimization	63
3.3.4	Identification results and analysis . . . . .	65
3.4	Magnetorheological Elastomer Base Isolator . . . . .	69
3.4.1	Experimental setup and device characteristics . . . . .	71

3.4.2	Hyperbolic hysteresis model . . . . .	73
3.4.3	Model identification using fruit fly optimization algorithm . . . . .	74
3.4.4	Identification results and analysis . . . . .	77
3.5	Summary . . . . .	82
<b>4</b>	<b>Frequency-based modeling of MR devices</b>	<b>84</b>
4.1	Introduction . . . . .	84
4.2	Describing Function . . . . .	85
4.3	MRD Describing Function Model . . . . .	86
4.4	MRP Describing Function Model . . . . .	89
4.5	MRE Describing Function Model . . . . .	90
4.6	Fractional-Order DF and Comparison . . . . .	92
4.7	Summary . . . . .	93
<b>5</b>	<b>Second-order Sliding Mode Control of Smart Structures Using MR Devices</b>	<b>95</b>
5.1	Introduction . . . . .	95
5.2	System Model . . . . .	96
5.3	SOSMC Design . . . . .	99
5.4	Simulation Results and Discussion . . . . .	101
5.5	Summary . . . . .	106
<b>6</b>	<b>Frequency-Shaped Structural Control</b>	<b>107</b>
6.1	Introduction . . . . .	107
6.2	Modeling in the Frequency Domain . . . . .	108
6.2.1	Modal decomposition . . . . .	108

6.2.2	Frequency response function . . . . .	113
6.3	FSSMC of MR Smart Structure Systems . . . . .	116
6.3.1	FSSMC design . . . . .	116
6.3.2	Application to smart building structures . . . . .	119
6.3.3	Simulation results and discussion . . . . .	120
6.4	FS2SMC for Smart Suspension Systems . . . . .	127
6.4.1	FS2SMC design . . . . .	128
6.4.2	Vehicle suspension structure control . . . . .	130
6.4.3	Simulation results . . . . .	134
6.4.4	Discussion . . . . .	138
6.5	Summary . . . . .	142
<b>7</b>	<b>Low-Energy Structures Embedded with Smart Dampers</b>	<b>144</b>
7.1	Introduction . . . . .	144
7.2	Energy balance equations of buildings with smart devices . . . . .	145
7.2.1	Relative energy balance equation . . . . .	147
7.2.2	Absolute energy balance equation . . . . .	148
7.3	FS2SMC of low-energy structures . . . . .	148
7.3.1	Energy-aware design . . . . .	149
7.3.2	Smart structural control . . . . .	151
7.4	Application and simulation . . . . .	153
7.4.1	Modal decomposition and control design . . . . .	155
7.4.2	Simulation results . . . . .	160
7.5	Discussion . . . . .	163
7.6	Summary . . . . .	167

<b>8 Thesis Contributions, Conclusions and Future Works</b>	<b>169</b>
8.1 Summary . . . . .	169
8.2 Thesis contributions . . . . .	171
8.3 Future works . . . . .	172
<b>Appendix A Structure Model &amp; MR Device Parameters</b>	<b>175</b>
<b>Appendix B Half-car Test Rig Identification</b>	<b>178</b>
<b>Appendix C Testo 875-2i Report</b>	<b>182</b>
<b>Bibliography</b>	<b>189</b>

## List of Figures

Figure 1.1	The (a) tallest bridge (Millau Viaduct 343 m); (b) longest suspension bridge (Akashi Kaikyō Bridge 1991 m); (c) longest and widest multi-pylon cable-stayed bridge (Jiaying-Shaoxing Sea Bridge 10138 m); (d) tallest building (Burj Khalifa 828 m); (e) tallest tower (Tokyo Skytree 634 m); (f) UTS building 2; (g) longest cable-stayed bridge (Russky Bridge 3100 m) [1]. . . . .	2
Figure 1.2	Schematic diagram of the lumped mass distribution of an $n$ -dof system with single-support excitation; $k = 1, 2, \dots, n$ . . . . .	6
Figure 1.3	The 325 m long London Millennium footbridge over the river Thames and resonance phenomenon [35]. . . . .	7
Figure 2.1	Schematic view of a set of structures. . . . .	14
Figure 2.2	Functional summary of the smart structure. . . . .	14
Figure 2.3	Schematic diagram of semi-active control systems. . . . .	18
Figure 2.4	MR device photos: (a) MRD, (b) MRP and (c) MRE base isolator. . . . .	22
Figure 2.5	Smart MR fluid with (a) no magnetic field and (b) an applied magnetic field [62]. . . . .	23
Figure 2.6	Schematic representation of Filippov's definition. . . . .	28
Figure 2.7	Approximations of $\text{sgn}(\cdot)$ nonlinearity; $\varepsilon = [0.1, 0.3, 0.5, 0.7, 0.9]$ . In the limit $\varepsilon \rightarrow 0$ , saturation, tangent and sigmoid functions approach to signum function. . . . .	30

Figure 2.8	Closed-loop block diagram of the SMC system with LPF. . . .	31
Figure 3.1	MR damper structure. . . . .	46
Figure 3.2	A photograph of the MRD RD-8041-1 test system. . . . .	48
Figure 3.3	Measured RD-8041-1 hysteresis. . . . .	50
Figure 3.4	Graphical representation of dissipated-energy per cycle at resonance. . . . .	51
Figure 3.5	Heat generation in the field-controllable hysteretic MRD. . . .	52
Figure 3.6	Experimental temperature rise history within MRD due to harmonic wave motion $x = E \sin(2\pi ft)$ . . . . .	54
Figure 3.7	MRD (a) dissipated energy and (b)-(d) energy rate during a vibration cycle operated at a constant magnetic field: $i \in [0 \ 2]$ A, $E = [4, 8, 12, 15]$ mm, $f = [0.5, 1, 2, 3]$ Hz, $R = 5 \ \Omega$ and $\theta \in [25.6 \ 43.9]^\circ\text{C}$ . . . . .	55
Figure 3.8	MR pin joint schematic. . . . .	56
Figure 3.9	An example of the MRP-embedded structure. . . . .	57
Figure 3.10	Test setup schematic for dynamic testing of MRP. . . . .	58
Figure 3.11	Characteristics of MRP at $i \in [0, \ 2]$ A. . . . .	59
Figure 3.12	(a)-(c) MRP hysteresis cycle, mean and thickness curve at $i = 1.5$ A, (d) a family of hysteresis by using the proposed model at $f = 2$ Hz, $E = 35.20$ mm. . . . .	61
Figure 3.13	A family of hysteresis by using the proposed model at $i \in [0, \ 2]$ A, $f = 3$ Hz, $E = 28.20$ mm. . . . .	62
Figure 3.14	Schematic of model identification. . . . .	63
Figure 3.15	$x = E \sin(2\pi ft)$ ; $i \in [0 \ 3]$ A, $f \in [1 \ 4]$ Hz, $E \in [2 \ 8]$ mm. . . .	67
Figure 3.16	RMSE comparison of the proposed and hyperbolic hysteresis models. . . . .	68

Figure 3.17	Convergence comparison of two types of PSO. . . . .	69
Figure 3.18	MRE base isolator schematic. . . . .	70
Figure 3.19	Test setup for dynamic testing of MRE base isolator at UTS. . . . .	72
Figure 3.20	MRE's testing results under $x = 8 \sin(2 \times 2\pi t)$ mm. . . . .	73
Figure 3.21	Flow chart of FFOA to identify the hyperbolic hysteresis model. . . . .	76
Figure 3.22	Iteration process of parameter identification using FFOA. . . . .	77
Figure 3.23	$x = E \sin(2\pi ft)$ ; $i \in [0 \ 3]$ A, $f \in [1 \ 4]$ Hz, $E \in [2 \ 8]$ mm. . . . .	80
Figure 3.24	RMSE and running time of model identification using FFOA. . . . .	82
Figure 3.25	Under random excitation and 1 A current level. . . . .	83
Figure 4.1	Nonlinear feedback system for DF analysis. . . . .	85
Figure 4.2	DF gain for MRD hysteresis. . . . .	87
Figure 4.3	DF phase for MRD hysteresis. . . . .	88
Figure 4.4	$N_D$ with different currents. . . . .	89
Figure 4.5	Hysteresis DFs for smart pin joint. . . . .	90
Figure 4.6	Hysteresis DFs for MRE base isolator. . . . .	91
Figure 4.7	Comparison of gain DFs for MRP hysteresis. . . . .	94
Figure 5.1	Smart building integrated with MRE base isolators. . . . .	97
Figure 5.2	Applied current signals under four earthquake excitations. . . . .	103
Figure 5.3	Response comparisons between passive and controlled structures under four earthquake excitations. . . . .	104
Figure 5.4	Comparisons of inter-storey drift and peak acceleration at each floor between passive and control structures. . . . .	105



Figure 6.1	Schematic diagram of the smart structure integrated with auxiliary robust damping devices. . . . .	110
Figure 6.2	Hankel singular values of the resulting system. . . . .	122
Figure 6.3	Open-loop singular value plot for $H(j\omega)$ . . . . .	122
Figure 6.4	TF from input to the floors displacements. . . . .	124
Figure 6.5	Illustration of the quake-prone frequency range. . . . .	125
Figure 6.6	Damper current corresponding to average peak force. . . . .	125
Figure 6.7	Time responses of $x_1(t)$ : first floor (top), $x_2(t)$ : second floor (middle) and $x_3(t)$ : third floor (bottom) with FSSM control (red-solid) and without (green-dot) control. . . . .	126
Figure 6.8	The forces produced by the active HIS, two road disturbance inputs, centrifugal acceleration, body bounce and roll angle are $f_1, f_2, f_{d1}, f_{d2}, a_c, z_s = \frac{z_3+z_4}{2}$ and $\theta = \frac{z_4-z_3}{2l}$ , respectively. Input variable $u = v_a$ is the hydraulic fluid volumetric displacement produced by the actuator. . . . .	132
Figure 6.9	Front view photograph of the combined system. . . . .	134
Figure 6.10	Under lateral acceleration $3 \sin(1.392 \times 2\pi t)$ m/s <sup>2</sup> . . . . .	136
Figure 6.11	Roll angle spectrums of the integrated system. . . . .	137
Figure 6.12	Feedback system defined by FS2SM; $L(s) = \frac{1}{s^2+1.4142s+1}$ but extensions to more orders with an equal number of poles and zeros (or less zeros) can also be made. . . . .	139
Figure 6.13	Actuator velocity due to a (a) thin boundary and (b) thick boundary under lateral acceleration $3 \sin(0.5 \times 2\pi t)$ m/s <sup>2</sup> ; $\eta = 0.25$ . . . . .	139
Figure 6.14	Actuator force due to a (a) thin boundary and (b) thick boundary under lateral acceleration $3 \sin(0.5 \times 2\pi t)$ m/s <sup>2</sup> ; $\eta = 0.25$ . . . . .	140
Figure 6.15	Phase portrait. . . . .	141
Figure 6.16	LFT representation. . . . .	142

Figure 7.1	Smart building integrated with energy-dissipative devices; $x_{dk} = x_k, k = 1, 2, \dots, n.$	146
Figure 7.2	Singular value plot of $H(j\omega).$	157
Figure 7.3	Closed-loop block diagram of the modal FS2SMC of the smart structure.	159
Figure 7.4	Closed-loop and open-loop phase portrait.	161
Figure 7.5	The spectrum of the first, fourth, sixth, eight and top floor modes for the controlled case.	161
Figure 7.6	MRD current.	162
Figure 7.7	MRD force-displacement trajectory	162
Figure 7.8	Uncontrolled relative energy signals under seismic disturbance, $E_i \rightarrow E_k, E_\zeta, E_s.$	164
Figure 7.9	Controlled relative energy signals under seismic disturbance for: $E_k, E_\zeta, E_s \rightarrow E_i.$	165
Figure 7.10	Energy flow in the low-energy smart structure system under external excitation.	166
Figure 8.1	Semi-active suspension system.	174
Figure A.1	MRP model parameter identification results (solid) and polynomial fitted results (dash).	177

## List of Tables

Table 2.1	Classification of structural vibration control. . . . .	16
Table 2.2	Summary of smart materials. . . . .	21
Table 2.3	Summary of frequency-shaped control methods. . . . .	43
Table 3.1	Parameters of the LORD RD-8041-1 MRD. . . . .	47
Table 3.2	$x = E \sin(2\pi t)$ ; $E = 4, 8, 12, 15$ mm, $f = 1$ Hz. . . . .	49
Table 3.3	Identification results for all the excitation conditions. . . . .	79
Table 4.1	Goodness-of-fit statistics. . . . .	92
Table 7.1	Peak relative energy responses (J) under various seismic disturbances. . . . .	167

## Notation

$\mathbb{N}, \mathbb{Z}, \mathbb{R}, \mathbb{C}$	the field of natural, integers, real and complex numbers
$\mathbb{R}^+$	the set of strictly positive real numbers
$\mathbb{C}^-$	the open left half of the complex plane, i.e. $\{z \in \mathbb{C} : \text{Re}[z] < 0\}$
$\text{Re}[z](\text{Im}[z])$	the real (imaginary) part of a complex variable $z$
$\mathbb{R}^n$	the $n$ -dimensional Euclidean space
$\mathbb{R}^{n \times m}$	the set of real matrices with $n$ rows and $m$ columns
$\mathbf{I}^{n \times n}(\mathbf{0}^{n \times n})$	the $n \times n$ identity (zero) matrix
$A^T(x^T)$	the transpose of a matrix $A$ (a vector $x$ )
$A^{-1}$	the inverse of a nonsingular square matrix $A$
$\lambda(A)$	the spectrum of the non-defective matrix $A$ , i.e. the set of eigenvalues
$\text{diag}(a_1, \dots, a_n)$	a diagonal matrix with diagonal elements $a_1$ to $a_n$
$\text{dist}(p, M)$	the distance from a point $p \in \mathbb{R}^n$ to a set $M \subseteq \mathbb{R}^n$
$B_p$	the closed ball of radius $p$ with center at the origin is defined by $\{x \in \mathbb{R}^n \mid \ x\  \leq p\}$ ; $p$ is a nonnegative real number
$f : S_1 \rightarrow S_2$	the function $f$ mapping a set $S_1$ into a set $S_2$
$f^{-1}(\cdot)$	the inverse of a function $f(\cdot)$
$f'(\cdot)$	the first derivative of a real-valued function $f(\cdot)$
$\text{sgn}(\cdot)$	the signum function
$\text{sat}(\cdot)$	the saturation function
$\tanh(\cdot)$	the hyperbolic tangent function

$\dot{x}$	the first derivative of $x$ with respect to time
$\ddot{x}$ ( $\overset{\cdot\cdot}{x}$ )	the second (third) derivative of $x$ with respect to time
$x^{(r)}$	the $r$ th derivative of $x$ with respect to time
$j = \sqrt{-1}$	the imaginary unit
$ p $	the absolute value of a scalar $p$
$p_m$	the minimal value of a positive scalar $p$
$p_M$	the maximal value of a positive scalar $p$
$\ x\ $	the Euclidean norm (or 2-norm) of a vector $x$
$\ \cdot\ _1$	the 1-norm
max (min)	maximum (minimum)
sup	supremum, the least upper bound
inf	infimum, the greatest lower bound
$\forall$	for all
$\in$	belongs to
$\exists$	there exists
$\subset$	subset of
$\rightarrow$	tends to
$\Rightarrow$	implies that
$\equiv$	identically equal
$\approx$	approximately equal
$\Leftrightarrow$	equivalent to, if and only if
$:=$	equal to by definition
(xx)	equation number xx in the thesis
[xx]	reference number xx in the bibliography

## Abbreviation

1(2) SMC	First (Second)-order sliding mode control
1(3)D	One (Three) dimensional
DF	Describing function
CE	Control energy
DE	Damping energy
DFG	Describing function gain
DFP	Describing function phase
dof	Degree-of-freedom
DI	Differential inclusion
FDF	Fractional-order describing function
FFOA	Fruit fly optimization algorithm
FRF	Frequency response function
FSSMC	Frequency-shaped (FS) sliding mode control
FS2SMC	Frequency-shaped second-order sliding mode control
FTSMC	Fast terminal sliding mode control
GSMC	Global sliding mode control
HCM	Half car model
HE	Hysteretic energy
HIS	Hydraulically interconnected system
HOSMC	Higher-order sliding mode control
IE	Input energy

IMPM	Inverse mode participation matrix
IMTM	Inverse modal transformation matrix
IPSO	Immune particle swarm optimization
KE	Kinetic energy
LC	Lyapunov-based control
LPF	Low-pass filter
LQR	Linear quadratic regulator
LUT	Look up table
MR	Magnetorheological
MRF	Magnetorheological fluid
MRD	Magnetorheological damper
MRP	Magnetorheological pin joint
MRE	Magnetorheological elastomer base isolator
RMSE	Root means square error
NTSMC	Nonsingular terminal sliding mode control
SAC	Semi-active (SA) control
SE	Strain energy
SISO	Single input single output
SMC	Sliding mode control
TF	Transfer function
TSMC	Terminal sliding mode control
UC	Uncontrolled case