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Linearly Polarized Shaped Power Pattern Synthesis With Dynamic Range Ratio Control for Arbitrary Antenna Arrays

ZHIYAO XU[1,2], YANHUI LIU[2,3], (Senior Member, IEEE), MING LI[2,3], AND YINGSONG LI[4], (Senior Member, IEEE)

[1]Institute of Electromagnetics and Acoustics, Xiamen University, Fujian 361005, China
[2]Shenzhen Research Institute, Xiamen University, Shenzhen 518057, China
[3]Global Big Data Technologies Centre, University of Technology Sydney, Ultimo, NSW 2007, Australia
[4]College of Information and Communication Engineering, Harbin Engineering University, Harbin 150001, China

Corresponding author: Yanhui Liu (yanhuiliu@xmu.edu.cn)

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ABSTRACT This paper extends the semidefinite relaxation (SDR) method to be capable of synthesizing linearly polarized shaped patterns with accurate control of sidelobe level (SLL), cross-polarization level (XPL), and dynamic range ratio (DRR) of the excitation distribution for arbitrary antenna arrays. In addition, by using the vectorial active element patterns, mutual coupling and platform effect can be also incorporated into the proposed vectorial shaped pattern synthesis. Three examples for synthesizing linearly polarized patterns with different pattern shape requirements and different antenna array geometries have been conducted to check the effectiveness and robustness of the proposed method. Compared to the original vectorial shaped pattern synthesis without DRR control, the proposed method with the DRR control can significantly reduce the obtained DRR which is very useful in many antenna array applications.

INDEX TERMS Vectorial shaped pattern synthesis, semidefinite relaxation (SDR), cross-polarization level (XPL), dynamic range ratio (DRR), mutual coupling.

I. INTRODUCTION

Antenna arrays capable of yielding shaped power patterns have received intense attention in the antenna community. In the past decades, researchers have explored many synthesis methods concerning the problems of shaped power patterns [1]–[12]. On account of the complexity of practical antenna array and nonlinearity of some synthesis constraints, many of shaped power pattern synthesis methods assume isotropic element patterns and consequently handle only array factors without considering mutual coupling effect. Some techniques such as stochastic algorithms in [13]–[15] and iterative convex optimization techniques in [16], [17], adopt a more general array model by considering element pattern diversity, and they can be potentially used to synthesize shaped patterns for more practical arrays by using active element patterns (AEPs) [18], [19]. Despite the success, most of the existing shaped pattern synthesis methods consider only the shape of the total power distribution over a certain angular region, and they lack controlling the power distributions for the co-polarization (COP) and cross-polarization (XP) components, respectively. Consequently, it sometimes happens that even the synthesized pattern meets the desired shape requirement in the total power, but the truly useful COP pattern is not like what we expect.

As is well known, adjusting element structure and array arrangement is a useful means to suppress the XP level (XPL). However, when it comes to the pattern synthesis problem for a given antenna array with a fixed structure, synthesizing appropriate excitations to feed the antenna elements is also a way to reduce the XPL to a certain degree. Although there are not many papers talking about this issue, few methods can be still found in the literature. They include, for example, polarization-controllable pattern synthesis for vector antenna arrays in [20], polarized focused and shaped pattern synthesis for arbitrary antenna arrays in [21]–[23], frequency-invariant polarized pattern synthesis for wideband conformal arrays [24], and circular-polarization (CP)

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pattern synthesis for spherical arrays [25]. However, all these research assumed a fixed desired polarization state for all angular space of interest. This is usually not the case for practical linearly polarized shaped pattern antenna arrays. As we know, for a linearly polarized antenna array, the realizable COP always changes with the propagation direction and it may be different from a prescribed desired polarization direction.

Another issue to be addressed is the large dynamic range ratio (DRR) of excitation amplitude distribution which usually happens in the shaped pattern synthesis results due to the complicated requirement on the pattern shapes. Reducing the DRR is significant for simplifying the feeding network design and also improving the aperture efficiency. Some techniques have been presented to reduce the obtained DRRs for shaped pattern synthesis problems [26]–[32]. However, most of these techniques consider only scaled radiation pattern synthesis problems without considering the polarization control. Only a few methods such as the synthesis method based on equivalent principle and trust region method in [31] and the iterative synthesis technique by introducing two auxiliary phase patterns to simplify the power pattern synthesis problem in [32] have considered both the XPL control and DRR reduction. However, similar to other vectorial pattern synthesis methods, both of them do not carefully consider the variation of the realizable COP direction with the changing of propagation direction.

Recently, we introduced a definition of the realizable COP direction which is formulated as the projection of the prescribed desired polarization direction onto the wavefront plane, as well as the definition of the realizable XP direction which is given by vector cross-product of the realizable COP and the propagation direction [33]. With these definitions, the realizable COP and XP patterns can be easily expressed. In [33], the semidefinite relaxation (SDR) method was extended to synthesize linearly polarized shaped patterns with XPL suppression for practical antenna arrays including mutual coupling and platform effect. The SDR method has been recognized as an effective method to place both upper and lower bounds for pattern shape control [34]. However, in [33], the DRR control has not been considered, and consequently the obtained DRR is usually considerably large due to complicated vectorial shaped pattern requirements. In this work, we will further extend the SDR method to be capable of synthesizing linearly polarized shaped patterns for arbitrary antenna arrays integrated with the ability of SLL, XPL and DRR control. Three examples including sectorial, cylindrical and conical conformal arrays are conducted to validate the effectiveness and advantages of the proposed method. Synthesis results show that the proposed further extended SDR method can effectively control the DRR of obtained excitation amplitude distribution while maintaining almost the same vectorial shaped pattern synthesis performance as the previous SDR method without the DRR control.

II. LINEARLY POLARIZED SHAPED POWER PATTERN SYNTHESIS WITH DRR CONTROL

A. VECTORIAL ARRAY PATTERN EXPRESSION

Consider an arbitrary antenna array consisting of N elements that locate at positions $\vec{r}_n = (x_n, y_n, z_n)$ $(n = 1, 2, \cdots, N)$ of the Cartesian system. The far-field radiation pattern can be formulated as

$$\vec{E}_{tol}(\theta,\varphi) = \sum_{n=1}^{N} w_n \left[\vec{e}_\theta E_{n\theta}(\theta,\varphi) + \vec{e}_\varphi E_{n\varphi}(\theta,\varphi)\right] \times e^{j\beta\vec{e}_\beta(\theta,\varphi)\cdot\vec{r}_n} \tag{1}$$

where w_n denotes complex excitation of the nth element, $\beta = 2\pi/\lambda$ is wavenumber in the free space, and

$$\vec{e}_\theta = \cos\theta\cos\varphi\vec{e}_x + \cos\theta\sin\varphi\vec{e}_y - \sin\theta\vec{e}_z \tag{2}$$

$$\vec{e}_\varphi = -\sin\varphi\vec{e}_x + \cos\varphi\vec{e}_y \tag{3}$$

$$\vec{e}_\beta(\theta,\varphi) = \sin\theta\cos\varphi\vec{e}_x + \sin\theta\sin\varphi\vec{e}_y + \cos\theta\vec{e}_z \tag{4}$$

In the above, $E_{n\theta}(\theta,\varphi)$ and $E_{n\varphi}(\theta,\varphi)$ denote the $\vec{e}_\theta$- and $\vec{e}_\varphi$-polarization components of vectorial AEP (VAEP) radiated by the nth element, respectively. The VAEP is adopted to consider element mutual coupling and platform effect [19]. For convenience, (1) can be concisely expressed in the matrix form given by

$$\vec{E}_{tol}(\theta,\varphi) = \vec{e}_\theta s_\theta^H \mathbf{w} + \vec{e}_\varphi s_\varphi^H \mathbf{w} \tag{5}$$

where

$$\mathbf{s}_\theta = \left[E_{1\theta}e^{j\beta\vec{e}_\beta(\theta,\varphi)\cdot\vec{r}_1}, \cdots, E_{N\theta}e^{j\beta\vec{e}_\beta(\theta,\varphi)\cdot\vec{r}_N}\right]^T \tag{6}$$

$$\mathbf{s}_\varphi = \left[E_{1\varphi}e^{j\beta\vec{e}_\beta(\theta,\varphi)\cdot\vec{r}_1}, \cdots, E_{N\varphi}e^{j\beta\vec{e}_\beta(\theta,\varphi)\cdot\vec{r}_N}\right]^T \tag{7}$$

$$\mathbf{w} = [w_1, w_2, \cdots, w_N]^T \tag{8}$$

and the superscript 'H' denotes Hermitian transposition of a matrix.

To formulate the linearly polarized shaped pattern synthesis problem, the realizable COP and XP directions should be defined at first. Such definitions have been thoroughly discussed in [33]. Assume that the user-desired polarization direction $\vec{p}_d$ is prescribed and it is usually fixed in applications. However, since the realizable COP direction of an antenna array is always perpendicular to the propagation direction $\vec{e}_\beta(\theta,\varphi)$, it is generally different from the fixed user-defined desired polarization direction if it is observed in a wide angular space. In [33], the realizable COP direction is defined as the projection of the $\vec{p}_d$ on the wavefront plane shown in Fig 1. It is given by [33]

$$\vec{p}_{co} = \frac{\vec{p}_d - [\vec{p}_d\cdot\vec{e}_\beta(\theta,\varphi)]\,\vec{e}_\beta(\theta,\varphi)}{|\vec{p}_d - [\vec{p}_d\cdot\vec{e}_\beta(\theta,\varphi)]\,\vec{e}_\beta(\theta,\varphi)|} \tag{9}$$

Accordingly, the realizable XP direction $\vec{p}_X$ which is perpendicular to both $\vec{p}_{co}$ and $\vec{e}_\beta(\theta,\varphi)$, can be expressed by [33]

$$\vec{p}_X = \vec{p}_{co} \times \vec{e}_\beta(\theta,\varphi) \tag{10}$$

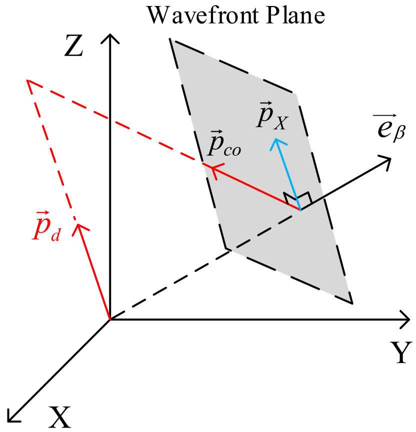

FIGURE 1. Illustration of the definitions of realizable COP and XP directions.

Then the COP and XP components of the radiation pattern can be respectively expressed by

$$\vec{E}_{co}(\theta,\varphi) = \left[\vec{E}_{tol}(\theta,\varphi)\cdot\vec{p}_{co}\right]\vec{p}_{co} = \vec{p}_{co}\,\mathbf{s}_{co}^H\,\mathbf{w} \quad (11)$$

$$\vec{E}_X(\theta,\varphi) = [\vec{E}_{tol}(\theta,\varphi)\cdot\vec{p}_X]\,\vec{p}_X = \vec{p}_X\,\mathbf{s}_X^H\,\mathbf{w} \quad (12)$$

where

$$\mathbf{s_{co}} = (\vec{e}_\theta\cdot\vec{p}_{co})\,\mathbf{s}_\theta + (\vec{e}_\varphi\cdot\vec{p}_{co})\,\mathbf{s}_\varphi \quad (13)$$

$$\mathbf{s_X} = (\vec{e}_\theta\cdot\vec{p}_X)\,\mathbf{s}_\theta + (\vec{e}_\varphi\cdot\vec{p}_X)\,\mathbf{s}_\varphi \quad (14)$$

Consequently, (1) can be reformulated by

$$\vec{E}_{tol}(\theta,\varphi) = \vec{p}_{co}\,\mathbf{s}_{co}^H\,\mathbf{w} + \vec{p}_X\mathbf{s}_X^H\,\mathbf{w} \quad (15)$$

B. DEFINITION OF DYNAMIC RANGE RATIO

Dynamic range ratio is defined as the ratio of the maximum of excitation amplitude to the minimum one. Thus, the DRR of the excitation vector $\mathbf{w}$ is given by

$$DRR(\mathbf{w}) = \frac{\max\limits_{1\le n\le N}\{|w_n|\}}{\min\limits_{1\le n\le N}\{|w_n|\}} \quad (16)$$

For the shaped pattern synthesis without DRR control, the excitation distribution with large DRR is usually obtained due to complicated pattern shape requirement. Large DRR is usually not preferable which increase the difficulty of the feeding network design. Thus, incorporating the DRR control into the shaped pattern synthesis is very significant for practical applications.

C. SHAPED PATTERN SYNTHESIS WITH SLL, XPL AND DRR CONTROL

The proposed synthesis is to find appropriate array excitations which yield a desired linearly polarized shaped power pattern by satisfying multiple constraints given as follows: 1) the COP component of the pattern is constrained by the upper bound $U_{ML}(\theta,\varphi)$ and the lower bound $L_{ML}(\theta,\varphi)$ in the mainlobe region Ω_{ML}; 2) the XP component of the pattern is controlled to be less than Γ_X in the region of interest Ω_X; 3) the total power pattern should be less than Γ_{SL} in the sidelobe region Ω_{SL}; and 4) the DRR of the excitation distribution is not larger than a preset DRR threshold $\rho_{\max}$. All these constraints can be formulated as follows:

$$\begin{cases} L_{ML}(\theta,\varphi) \le |\vec{E}_{co}(\theta,\varphi)|^2 \le U_{ML}(\theta,\varphi), & (\theta,\varphi)\in\Omega_{ML} \\ |\vec{E}_X(\theta,\varphi)|^2 \le \Gamma_X, & (\theta,\varphi)\in\Omega_X \\ |\vec{E}_{tol}(\theta,\varphi)|^2 \le \Gamma_{SL}, & (\theta,\varphi)\in\Omega_{SL} \\ |DRR(\mathbf{w})| \le \rho_{\max}. & \end{cases} \quad (17)$$

To be consistent with other constraint forms, we rewrite the DRR constraint as

$$1 \le \frac{\max\limits_{1\le n\le N}\{|w_n|^2\}}{\min\limits_{1\le n\le N}\{|w_n|^2\}} \le \rho_{\max^2} \quad (18)$$

where $\rho_{\max}$ is a preset DRR threshold. Here we introduce an auxiliary parameter ξ which is used to transform the DRR constraint to be lower and upper bound constraints on each excitation energy. That is,

$$\xi^2 \le |w_n|^2 \le \rho_{\max}^2\cdot\xi^2, \quad n = 1, 2\cdots, N. \quad (19)$$

To express notations simply, the arguement (θ,φ) is dropped in the following. Then, by defining

$$\mathbf{S}_{co} = \begin{bmatrix} \mathrm{Re}(\mathbf{s}_{co}{}^T) & -\mathrm{Im}(\mathbf{s}_{co}{}^T) & 0 \\ \mathrm{Im}(\mathbf{s}_{co}{}^T) & \mathrm{Re}(\mathbf{s}_{co}{}^T) & 0 \end{bmatrix} \quad (20)$$

$$\widetilde{\mathbf{w}} = \begin{bmatrix} \mathrm{Re}(\mathbf{w}) \\ \mathrm{Im}(\mathbf{w}) \\ \xi \end{bmatrix} \quad (21)$$

we obtain that

$$|\vec{E}_{co}|^2 = \widetilde{\mathbf{w}}^T\mathbf{Q}_{co}\,\widetilde{\mathbf{w}}, \text{ with } \mathbf{Q}_{co} = \mathbf{S}_{co}^T\,\mathbf{S}_{co}. \quad (22)$$

Similarly, we have

$$|\vec{E}_X|^2 = \widetilde{\mathbf{w}}^T\mathbf{Q}_X\widetilde{\mathbf{w}}, \text{ with } \mathbf{Q}_X = \mathbf{S}_X^T\,\mathbf{S}_X. \quad (23)$$

$$|\vec{E}_{tol}|^2 = \widetilde{\mathbf{w}}^T\mathbf{Q}_{tol}\widetilde{\mathbf{w}}, \text{ with } \mathbf{Q}_{tol} = \mathbf{S}_{tol}^T\,\mathbf{S}_{tol}. \quad (24)$$

where

$$\mathbf{S}_X = \begin{bmatrix} \mathrm{Re}(\mathbf{s}_X{}^T) & -\mathrm{Im}(\mathbf{s}_X{}^T) & 0 \\ \mathrm{Im}(\mathbf{s}_X) & \mathrm{Re}(\mathbf{s}_X{}^T) & 0 \end{bmatrix}$$

$$\mathbf{S}_{tol} = \begin{bmatrix} \mathrm{Re}(\mathbf{s}_\theta{}^T) & -\mathrm{Im}(\mathbf{s}_\theta{}^T) & 0 \\ \mathrm{Im}(\mathbf{s}_\theta{}^T) & \mathrm{Re}(\mathbf{s}_\theta{}^T) & 0 \\ \mathrm{Re}(\mathbf{s}_\varphi{}^T) & -\mathrm{Im}(\mathbf{s}_\varphi{}^T) & 0 \\ \mathrm{Im}(\mathbf{s}_\varphi{}^T) & \mathrm{Re}(\mathbf{s}_\varphi{}^T) & 0 \end{bmatrix}$$

For the DRR constraint, it is

$$|w_n|^2 = \widetilde{\mathbf{w}}^T\mathbf{Q}_n(i,i)\,\widetilde{\mathbf{w}}, \quad n = 1,\cdots,N. \quad (25)$$

$$\xi^2 = \widetilde{\mathbf{w}}^T\mathbf{Q}_{\xi 1}(i,i)\,\widetilde{\mathbf{w}} \quad (26)$$

$$\rho_{\max}^2\cdot\xi^2 = \widetilde{\mathbf{w}}^T\mathbf{Q}_{\xi 2}(i,i)\,\widetilde{\mathbf{w}} \quad (27)$$

where

$$\mathbf{Q}_n(i,i) = \begin{cases} 1, & i = n \\ 1, & i = n+N \\ 0, & \text{otherwise} \end{cases} \quad (28)$$

$$\mathbf{Q}_{\xi 1}\ (i,i) = \begin{cases} 1, & i = 2N+1 \\ 0, & \text{otherwise} \end{cases} \tag{29}$$

$$\mathbf{Q}_{\xi 2}\ (i,i) = \begin{cases} \rho_{\max}^2, & i = 2N+1 \\ 0, & \text{otherwise} \end{cases} \tag{30}$$

It is well-known that for any real symmetric matrix $\mathbf{Q}$ and real vector $\widetilde{\mathbf{w}}$, we have

$$\widetilde{\mathbf{w}}^T \mathbf{Q} \widetilde{\mathbf{w}} = \mathrm{Tr}\,(\widetilde{\mathbf{w}}^T \mathbf{Q} \widetilde{\mathbf{w}}) = \mathrm{Tr}\,(\mathbf{Q}\mathbf{W}) \tag{31}$$

where $\mathbf{W} = \widetilde{\mathbf{w}}\widetilde{\mathbf{w}}^T$ is a rank-one and symmetric positive semidefinite matrix. It means that $\mathbf{W} \succeq 0$ and rank $(\mathbf{W}) = 1$. Now, the synthesis problem can reformulated as finding $\mathbf{W}$ such that

$$\begin{cases} ll\mathrm{Tr}\,(\mathbf{Q}_{co}\mathbf{W}) \geq L_{ML}, & (\theta,\varphi) \in \Omega_{ML} \\ \mathrm{Tr}\,(\mathbf{Q}_{co}\mathbf{W}) \leq U_{ML}, & (\theta,\varphi) \in \Omega_{ML} \\ \mathrm{Tr}\,(\mathbf{Q}_X\mathbf{W}) \leq \Gamma_X, & (\theta,\varphi) \in \Omega_X \\ \mathrm{Tr}\,(\mathbf{Q}_{SL}\mathbf{W}) \leq \Gamma_{SL}, & (\theta,\varphi) \in \Omega_{SL} \\ \mathrm{Tr}\ (\mathbf{Q}_n\mathbf{W}) \geq \mathrm{Tr}\,(\mathbf{Q}_{\xi 1}\mathbf{W}), & n = 1,\cdots,N \\ \mathrm{Tr}\ (\mathbf{Q}_n\mathbf{W}) \leq \mathrm{Tr}\,(\mathbf{Q}_{\xi 2}\mathbf{W}), & n = 1,\cdots,N \\ \mathbf{W} \succeq 0, & \mathrm{rank}\,(\mathbf{W}) = 1. \end{cases} \tag{32}$$

In the above, rank$(\mathbf{W}) = 1$ is a typical non-convex constraint. Through dropping this rank-one constraint, the reweighted minimization method is adopted to iteratively minimize the trace of $\mathbf{W}$ that is equivalent to the sum of the eigenvalues of $\mathbf{W}$ until a low-rank $\mathbf{W}$ can be obtained [35]. Such a method is called the semi-definite relaxation (SDR) which is given by

$$\mathbf{W}^{(k)} = \underset{\mathbf{W} \in C}{\mathrm{argmin}}\ \mathrm{Tr}\,(\mathbf{W}^{(k-1)} + \delta\mathbf{I})^{-1}\mathbf{W}^{(k)}\mathrm{Const.}$$
$$\times \begin{cases} ll\mathrm{Tr}\,(\mathbf{Q}_{co}\mathbf{W}^{(k)}) \geq L_{ML}, & (\theta,\varphi) \in \Omega_{ML} \\ \mathrm{Tr}\,(\mathbf{Q}_{co}\mathbf{W}^{(k)}) \leq U_{ML}, & (\theta,\varphi) \in \Omega_{ML} \\ \mathrm{Tr}\,(\mathbf{Q}_X\mathbf{W}^{(k)}) \leq \Gamma_X, & (\theta,\varphi) \in \Omega_X \\ \mathrm{Tr}\,(\mathbf{Q}_{SL}\mathbf{W}^{(k)}) \leq \Gamma_{SL}, & (\theta,\varphi) \in \Omega_{SL} \\ \mathrm{Tr}\left(\mathbf{Q}_n\mathbf{W}^{(k)}\right) \geq \mathrm{Tr}\,(\mathbf{Q}_{\xi 1}\mathbf{W}^{(k)}), & n = 1,\cdots,N \\ \mathrm{Tr}\left(\mathbf{Q}_n\mathbf{W}^{(k)}\right) \leq \mathrm{Tr}\,(\mathbf{Q}_{\xi 2}\mathbf{W}^{(k)}), & n = 1,\cdots,N \\ \mathbf{W}^{(k)} \succeq 0. \end{cases} \tag{33}$$

where $\delta > 0$ is a small regularization parameter. Now, (33) is a convex optimization problem and can be efficiently solved by convex optimization toolboxes, such as the CVX [36].

In each iteration, we perform an eigenvalue decomposition of $\mathbf{W}$ as follows

$$\mathbf{W} = \mathbf{V}^T \Sigma \mathbf{V} \tag{34}$$

where Σ is a diagonal eigenvalue matrix, and $\mathbf{V}$ is the corresponding eigenvector matrix. The eigenvalues is sorted in descending order, i.e., $\sigma_1 \geq \sigma_2 \geq \ldots\sigma_n$. If the rank of $\mathbf{W}$ is close to 1, the corresponding largest eigenvalue will be several orders of magnitude larger than the second eigenvalue. In this situation, we have

$$\mathbf{W} \approx \sigma_1 \mathbf{v}\mathbf{v}^T \tag{35}$$

Hence, if $\sigma_2 \leq \eta\sigma_1$ where η is a very small positive number, e.g, $\eta = 10^{-3}$, we can obtain the excitation vector as $\widetilde{\mathbf{w}} = \sqrt{\sigma_1}\mathbf{v}$, and then the iterative procedure can be terminated. Otherwise, the iterative procedure should continue, and the parameter δ can be updated as $\sigma_1/10$ which was obtained in the previous iteration.

D. THE PROPOSED SYNTHESIS PROCEDURE

The overall procedure for synthesizing linearly polarized shaped pattern with SLL, XPL and DRR control is described in Algorithm 1.

Algorithm 1 The SDR Optimization Procedure

1: Set upper bound U_{ML} and lower bound L_{ML} of COP component in shaped region Ω_{ML}, upper bound Γ_X of XP component in certain region Ω_X, and upper bound Γ_{SL} of total power pattern in sidelobe region Ω_{SL}.
2: Set $\rho_{\max}$ as the DRR constraint of array excitation amplitudes.
3: Set user-desired polarization direction $\vec{p}_d$.
4: Obtain the normalized realizable COP direction $\vec{p}_{co}$ through (9) and XP direction $\vec{p}_X$ through (10), respectively.
5: Obtain steering vector $\mathbf{s}_\theta$ along $\vec{e}_\theta$ through (6) and steering vector $\mathbf{s}_\varphi$ along $\vec{e}_\varphi$ through (7), respectively, where active element patterns of $E_{n\theta}(\theta,\varphi)$ and $E_{n\varphi}(\theta,\varphi)$ ($n = 1, 2, \cdots, N$) are exported from the simulation of array antenna model.
6: Compute the COP steering vector $\mathbf{s}_{co}$ through (13) and the XP steering vector $\mathbf{s}_X$ through (14), respectively.
7: Set initial regularization parameter $\delta^{(0)} = 0.02$ and $\mathbf{W}^{(0)} = \mathbf{I}$ where $\mathbf{I}$ is the indentity matrix.
8: Set the maximum iteration number P.
9: **for** $k = 1 : 1 : P$ **do**
 (i) Solve the convex optimization problem (33) and get $\mathbf{W}^{(k)}$.
 (ii) Get the largest eigenvalue $\sigma_{\max}^{(k)}$ of $\mathbf{W}^{(k)}$ through (34) and corresponding eigenvector $\mathbf{v}^{(k)}$.
 (iii) Update the regularization parameter $\delta^{(k)} = \sigma_{\max}^{(k)}/10$.
 (iv) If the largest eigenvalue $\sigma_{\max}^{(k)}$ of $\mathbf{W}^{(k)}$ is several orders than the second large eigenvalue, jump out of the loop.
 end for
10: Output the excitation $\widetilde{\mathbf{w}} = \sqrt{\sigma_{\max}}\mathbf{v}$.

III. NUMERICAL EXAMPLES

In this section, three numerical examples of synthesizing linearly polarized shaped patterns with constrained XPL, SLL and DRR for sectorial array, cylindrical array and conical array are introduced. The mutual coupling and platform effects are taken into account in all the three examples by exploiting the AEPs which are acquired by using High Frequency Structure Simulator (HFSS) software.

A. FLAP-TOP PATTERN WITH XPL CONTROL AND DRR CONTROL FOR A SECTORIAL ARRAY

The first example is synthesizing a linearly polarized flap-top power pattern for a sectorial conformal array which is composed of 16 elements with a uniform spacing of 31.78 mm on an arc with a radius of 242.75 mm. The array element is designed as a curved E-type microstrip antenna working at central frequency of 5 GHz, which is inspired by the planar E-type microstrip antenna designed in [37]. This antenna element consists of an E-type radiating patch, a dielectric layer with thickness of 0.254 mm and relative permittivity of 2.2 as well as an air gap layer with thickness of 4.746 mm. At 5 GHz, the element spacing is 0.53λ. The antenna array geometry is shown in Fig. 2. Our synthesis goal is to generate a linearly polarized flap-top power pattern in XOZ plane. Assume that the user-defined desired polarization $\vec{p}_d = \vec{e}_y$. It can be observed that the realizable COP and XP directions coincide with $\vec{e}_\varphi$ and $\vec{e}_\theta$ on the XOZ plane, respectively. The desired shaped pattern is set as a flat-top shape with a ripple of ±0.5 dB in the region of $|\theta| \leq 25°$. The SLL of the total power pattern is set to be less than −22 dB in the region of $|\theta| \geq 40°$, and the XPL is set to be less than −20 dB in the whole region.

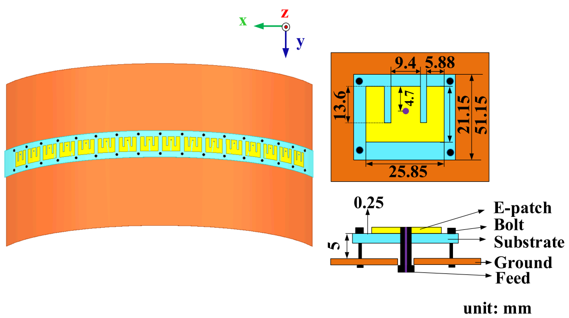

FIGURE 2. Geometry of a 16-element sectorial array (with view of the E-type antenna element on planar surface).

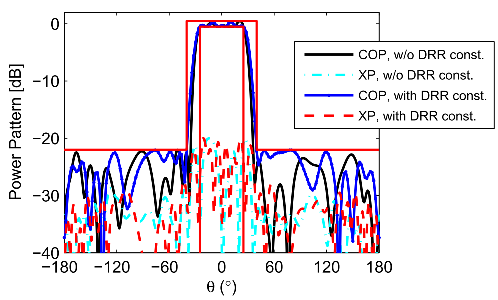

FIGURE 3. The COP and XP patterns synthesized with/without DRR control for the sectorial array.

To show the effectiveness of the proposed method, we perform the synthesis procedure twice: one is without the DRR control and the other has the DRR control with $DRR = 2$. Fig. 3 shows the synthesized COP and XP patterns with/without DRR control. It can be seen that although the synthesized two patterns with/without the DRR control are different in the sidelobe region and XP distribution, both of them meet all the pattern requirements including the COP mainlobe ripple, SLL and XPL. Fig. 4 shows the normalized excitation amplitude distributions obtained in the synthesis with/without DRR constraint. As can be seen, the synthesized excitation distribution without DRR control has a large DRR of 15.22 while the synthesis with DRR control obtains a much smaller DRR which is only equal to 2. This shows that control of DRR is very effective for the proposed vectorial shaped pattern synthesis to reduce the obtained DRR of the synthesized excitations.

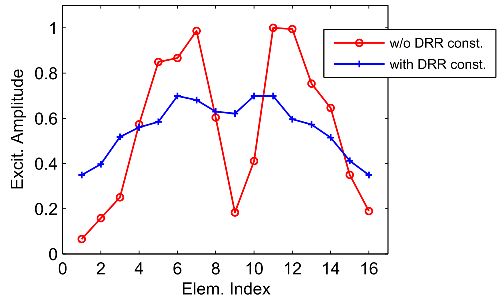

FIGURE 4. The excitation amplitude distributions synthesized with/without DRR control.

B. CIRCULAR FLAP-TOP PATTERN WITH XPL AND DRR CONTROL FOR A CYLINDRICAL ARRAY

To further explore the feasibility of the presented method, synthesizing linearly polarized circular flat-top pattern for a 12× 6 cylindrical conformal array is considered. The element antenna and array arrangement are shown in Fig. 5. The array element is used as a planar E-type patch antenna working at 2.4 GHz [37], and all the elements are uniformly distributed on a multifaceted cylinder with the radius of 380 mm. The element spacings along the circumference and the cylinder height directions are 66.25 mm (0.53λ at 2.4 GHz) and 62.5 mm, respectively. Assume the user-defined desired polarization direction is still $\vec{e}_y$. However, different from the sectorial array case where $\vec{p}_{co} = \vec{e}_\varphi$ and $\vec{p}_X = \vec{e}_\theta$, the realizable COP and XP directions in this case change as the

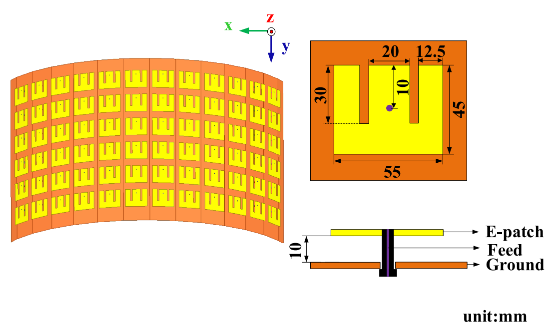

FIGURE 5. Geometry of a cylindrical array with 12 × 6 E-type patch antennas.

propagation direction $\vec{e}_\beta$ varies according to the definitions of (9) and (10). Pattern constraints are set as follows: the mainlobe shape is set as -2 dB $\leq |\vec{E}_{co}(\theta,\varphi)|^2 \leq 0$ dB in a circular-shape region of $\{|\theta| \leq 16^\circ$ and $\varphi \in (0^\circ, 360^\circ)\}$, the SLL is set as $|\vec{E}_{tol}(\theta,\varphi)|^2 \leq -20$ dB in the region of $\{|\theta| \geq 32^\circ$ and $\varphi \in (0^\circ, 360^\circ)\}$, and the XPL is set as $|\vec{E}_X(\theta,\varphi)|^2 \leq -20$ dB in the whole space.

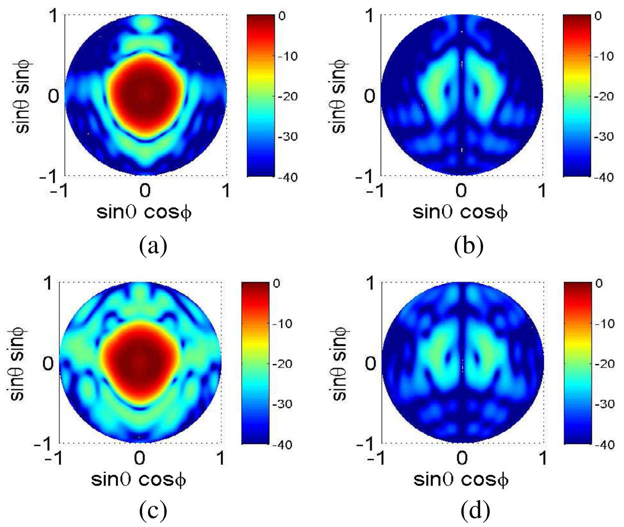

FIGURE 6. Top views of the synthesized linearly polarized circular flat-top patterns for the 12 × 6-element cylindrical array by the proposed method with/without DRR control. (a) COP pattern without DRR control, (b) XP pattern without DRR control, (c) COP pattern with DRR control, and (d) XP pattern with DRR control.

First, the DRR constraint is not taken into consideration. Fig. 6(a) and (b) show the top views of the synthesized COP and XP patterns, respectively, by the proposed method without DRR control. As can be seen, the synthesized COP and XP patterns meet their requirements. The normalized excitation amplitude distribution is shown in Fig. 7(a), and the corresponding DRR is 19.6 (i.e., -25.85 dB). Then, we add a constraint of $DRR = 6$ (i.e., -15.56 dB) into the pattern synthesis. The synthesized COP and XP patterns are now shown in Fig. 6(c) and (d), respectively. As can be seen, the obtained two component patterns are slightly different from the corresponding ones, but they still satisfy the requirements. Fig. 7(b) shows the current synthesized excitation amplitude distribution. The obtained DRR is now reduced to 6 which is much smaller than the DRR of 19.6 obtained without the DRR control. This validates the effectiveness of the proposed method for synthesizing two-dimensional vectorial shaped pattern with DRR control for an arbitrary conformal array.

C. TRIANGLE-SHAPED FLAP-TOP PATTERN WITH XPL AND DRR CONTROL FOR A CONICAL ARRAY

In the last example, we consider a more complicated case of synthesizing a linearly polarized triangle-shaped flat-top pattern for a conical conformal array. This array consists of 12 arc arrays with a total of 142 conformal circular patch

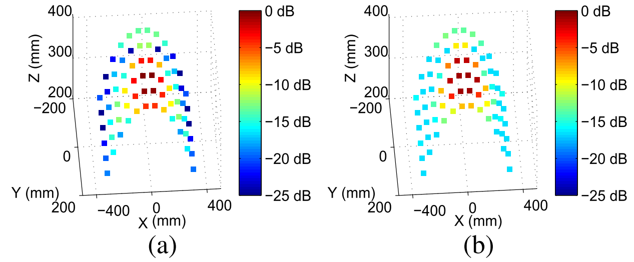

FIGURE 7. The excitation amplitude distributions for the 12 × 6-element cylindrical array synthesized by the proposed method. (a) without DRR control and (b) with DRR control.

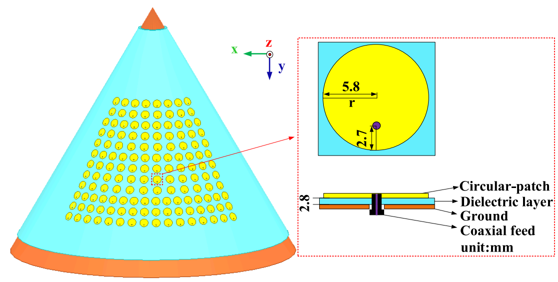

FIGURE 8. Geometry of a conical array with 142 circular patch antennas.

TABLE 1. Some parameters for the conical array configuration.

Ring index	Elem. No	Radius (mm)	Arc Spacing
1	8	76	12°
2	8	84	12°
3	10	92	10°
4	10	100	10°
5	10	108	10°
6	12	116	8°
7	12	124	8°
8	12	132	8°
9	12	140	8°
10	16	148	6°
11	16	156	6°
12	16	164	6°

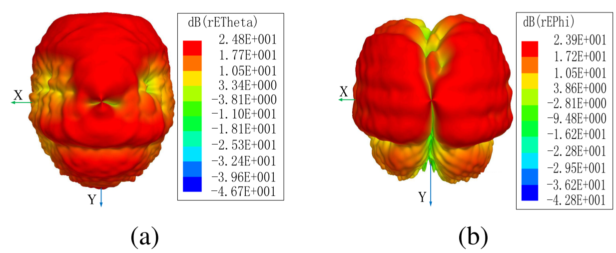

FIGURE 9. The vectorial active element pattern radiated by the element antenna in Fig. (8). (a) $\vec{e}_\theta$- polarization component and (b) $\vec{e}_\phi$- polarization component.

elements working at 10 GHz. In this conical array, adjacent rings have an equal distance of 15 mm (0.5λ at 10 GHz) along the generatrix as shown in Fig. 8. Some parameters for the array configuration are summarized in Table 1. The $\vec{e}_\theta$- and $\vec{e}_\varphi$-polarization components of VAEP radiated by the element in Fig. 8 are shown in Fig. 11, respectively. The user-defined desired direction $\vec{p}_d$ is set as $\vec{e}_y$.

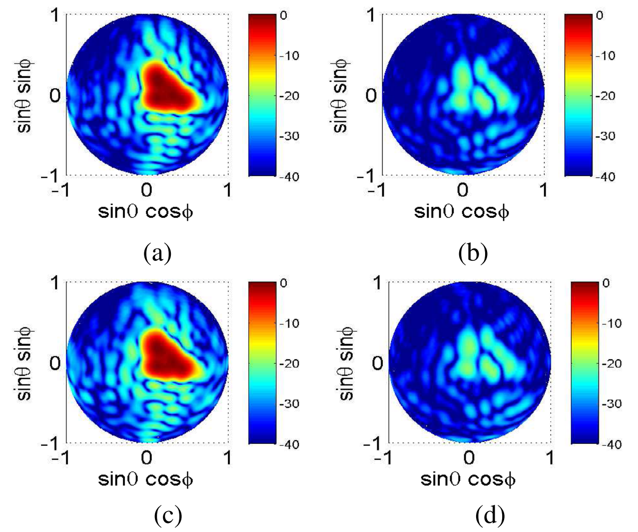

FIGURE 10. Top views of the synthesized linearly polarized triangle-shaped flat-top patterns for the 142-element conical array by the proposed method with/without DRR control. (a) COP pattern without DRR control, (b) XP pattern without DRR control, (c) COP pattern with DRR control, and (d) XP pattern with DRR control.

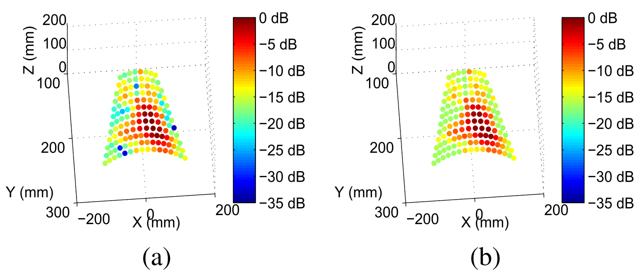

FIGURE 11. The excitation amplitude distributions for the 142-element conical array synthesized by the proposed method. (a) without DRR control and (b) with DRR control.

The constraints are listed as follows: 1) the mainlobe shape is set as -2 dB $\leq |\vec{E}_{co}(\theta,\varphi)|^2 \leq 0$ dB in a triangular region of $\{(u, v); (u+v) \leq 0.4\ \&\ 0.1 \leq u \leq 0.5\ \&\ -0.1 \leq v \leq 0.3\}$ where $u = \sin\theta\cos\phi$ and $v = \sin\theta\sin\phi$; 2) the SLL of the total power pattern is set to be less than -20 dB in the region of $\{(u, v); (u+v) \geq 0.6 \text{ or } u \leq -0.1 \text{ or } v \leq -0.3\}$; and 3) the XP component is set to be less than -20 dB in the whole space. Fig. 10(a) and (b) show the obtained COP and XP patterns without the DRR constraint, respectively. It can be seen that both of the COP and XP patterns meet the requirements. However, the obtained DRR in this case is up to 45.2 (i.e., -33.10 dB). Fig. 10(a) shows the excitation amplitude distribution. Implementing the excitation distribution with so large DRR is somehow difficult. This problem can be mitigated by adding a DRR constraint into the synthesis procedure. Fig. 10(c) and (d) show the obtained COP and XP pattern with a constraint of $DRR = 6$ (i.e., -15.56 dB), respectively. It can be seen that the obtained pattern results still satisfy the prescribed COP and XP constraints. The corresponding excitation amplitude distribution is shown in Fig. 10(b). In contrast to the result in Fig. 10(a), the excitation distribution in Fig. 10(b) appears more evenly.

IV. CONCLUSION

In this paper, the semidefinite relaxation method has been further extended to synthesize linearly polarized shaped patterns with SLL, XPL and DRR control for arbitrary antenna arrays. In addition, by using the vectorial active element patterns, the proposed method can include both mutual coupling and platform effect of practical antenna array structure into the vectorial pattern synthesis. Three synthesis examples for synthesizing sectorial, cylindrical and conical conformal arrays with different pattern shape requirements have been conducted. Synthesis results show that the proposed vectorial shaped pattern synthesis method with the DRR control is very effective and robustness. Compared to the vectorial pattern synthesis without DRR control, the proposed method with the DRR control can significantly reduce the DRR of the obtained excitation amplitude distribution, which is very useful for simplifying the feeding network design as well as improving the array aperture efficiency.

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ZHIYAO XU was born in Henan, China, in 1994. He received the B.S. degree from the University of Electronic Science and Technology of China (UESTC), in 2016. He is currently pursuing the M.S. degree in electromagnetic field and microwave technology from Xiamen University, Xiamen, China.

YANHUI LIU (S'09–M'14–SM'18) received the B.S. and Ph.D. degrees in electrical engineering from the University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 2004 and 2009, respectively. From 2007 to 2009, he was a Visiting Scholar with the Department of Electrical Engineering, Duke University, Durham, NC, USA. In 2011, he joined the Department of Electronic Science, Xiamen University, Xiamen, China, where he is currently a Full Professor. In 2017, he was a Visiting Professor with the State Key Laboratory of Millimeter Waves, City University of Hong Kong, Hong Kong. Since 2017, he has been a Visiting Professor with the Global Big Data Technologies Center, University of Technology Sydney, Ultimo, NSW, Australia. He has authored and coauthored over 120 peer-reviewed journal and conference papers. He holds several granted Chinese invention patents. His current research interests include antenna array design, reconfigurable antennas, and array signal processing.

Dr. Liu has served as a TPC member or reviewer in the IEEE APS, PIERS, APCAP, and NCANT, for many times, and served as a Session Chair in NCANT2015, PIERS2016, ACES2017-China, NCANT2017, and APCAP2017. He was a recipient of the UESTC Outstanding Graduate Award, in 2004, and the Excellent Doctoral Dissertation Award of Sichuan Province of China, in 2012. He is serving as a reviewer for 12 SCI-indexed journals, including the IEEE Transactions on Antennas and Propagation. Since 2018, he has been serving as an Associate Editor for the IEEE Access.

MING LI was born in Anhui, China, in 1991. He received the B.S. and M.S. degrees from the Xiamen University, in 2015 and 2018, respectively.

YINGSONG LI (S'09–M'14–SM'18) received the B.S. degree in electrical and information engineering and the M.S. degree in electromagnetic field and microwave technology from Harbin Engineering University, in 2006 and 2011, respectively, and the Ph.D. degree from both Kochi University of Technology (KUT), Japan, and Harbin Engineering University, China, in 2014.

He was a Visiting Scholar of the University of California, Davis, from 2016 to 2017 and the University of York, U.K. from 2018 to 2018. He has been a Full Professor with Harbin Engineering University, since 2014. He is also a Visiting Professor with Far Eastern Federal University (FEFU) and KUT. His current research interests include remote sensing, underwater communications, signal processing, radar, SAR imaging, compressed sensing, and antennas.

Dr. Li is a senior member of the Chinese Institute of Electronics (CIE). He is an Associate Editor of the IEEE Access and *Applied Computational Electromagnetics Society Journal*, and he is also an Area Editor of *AEÜ-International Journal of Electronics and Communications*. He also serves as a reviewer for more than 20 journals.