Solving Selected Problems on American Option Pricing with the Method of Lines

A Thesis Submitted for the Degree of

Doctor of Philosophy

by

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Certificate

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirement for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Signed Production Note: Signature removed prior to publication.

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Contents

Abstract

The American option pricing problem lies on the inability to obtain closed form representation of the early exercise boundary and thus of the American option price. Numerous approaches have been developed to provide approximate solution or numerical schemes that derive American prices at a sufficient level of pricing accuracy and computational effort.

Jump-diffusion models have been used in literature with stochastic volatility models to better explain shorter maturity smiles[1]. They are also able to capture the skewness and kurtosis features of return distributions often observed in a number of assets in the market. By including jumps in volatility to the model dynamics, a rapidly moving but persistent component driving the conditional volatility of returns is incorporated (Eraker, Johannes & Polson (2003)).

This thesis considers the pricing of American options using the Heston (1993) stochastic volatility model with asset and volatility jumps and the Hull & White (1987) short rate model. Since the early exercise boundary depends on the term structure of interest rates, stochastic interest rates are important. The American option prices are numerically evaluated by using the Method of Lines (MoL) (Meyer (2015)). This method is fast and efficient, with the ability to provide the early exercise boundary and the Greeks[2] at the same computational effort as the option price. The main contributions of the thesis are:

- *Pricing American Options with Jumps in Asset and Volatility - Chapter 2 and Chapter 3.* In the first chapter, a MoL algorithm is developed to numerically evaluate American options under the Heston-Hull-White (HHW) model with asset and volatility jumps. A sensitivity analysis is then conducted to gauge the impact of jumps and stochastic interest rates on prices and their free boundaries. The second chapter assesses the importance of asset and volatility jumps in American option pricing models by calibrating the Heston (1993) stochastic volatility model with asset and volatility jumps to S&P 100 American options data. In general,

[1] A volatility smile is a graph of implied volatility as a function of strike for options with the same maturity.
[2] Greeks measure the sensitivity of the option price to changes in parameter values on which the option price depends.

jumps improve the model's ability to fit market data. Further the inclusion of asset jumps increases the free boundary, whilst volatility jumps marginally drops the free boundary, a result that is more pronounced during periods of high volatility.

- *Evaluation of Equity Linked Pension Schemes With Guarantees. - Chapter 4.* In the third chapter American Asian option are priced with application to two types of equity linked pension schemes, namely, the investment guarantee and the contribution guarantee schemes. This extends work by Nielsen, Sandmann & Schlögl (2011), where the American feature is introduced and allows for early termination of the policy. The investment fraction of the investors contributed amount that is required to achieve a certain level of investment return guarantee is analysed. A sensitivity analysis shows that asset volatility and interest rate volatility have a positive impact on the free boundary.