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PAPER

Evolution of cooperation in synergistically evolving dynamic interdependent networks: fundamental advantages of coordinated network evolution

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Abstract

Real networks are not only multi-layered yet also dynamic. The role of coordinated network evolution regarding dynamic multi-layer networks where both network and strategy evolution simultaneously show diverse interdependence by layers remains poorly addressed. Here, we propose a general and simple coevolution framework to analyze how coordination of different dynamical processes affects strategy propagation in synergistically evolving interdependent networks. The strategic feedback constitutes the main driving force of network evolution yet the inherent cross-layer self-optimization functions as its compensation. We show that these two ingredients often catalyze a better performance of network evolution in propagating cooperation. Coordinated network evolution may be a double-edged sword to cooperation and the network-adapting rate plays a crucial role in flipping its double-sided effect. It often economizes the cost and time consumption for driving the system to the full cooperation phase. Importantly, strongly coupled slow-tuned networks can outperform weakly coupled fast-regulated networks in solving social dilemmas, highlighting the fundamental advantages of coordinated network evolution and the importance of synergistic effect of dynamical processes in upholding human cooperation in multiplex networks.

1. Introduction

Decoding the origin of cooperation as well as its persistence in humans and other species is the core mission of evolutionary game theory [1]. The heart of this enduring puzzle lies in two aspects. First, altruistic cooperation often benefits others yet incurs a cost to its carriers. In sharp contrast, however, selfish defection most of the time free rides on others' contributions at almost no cost to its carriers. From an evolutionary point of view, as a consequence, cooperation becomes the one that should have been wiped out [2, 3]. This theoretical prediction gives prominence to an increasing inconsistency with ubiquitous cooperation that has laid a solid foundation for the success of human societies. Aside from this point, if both cooperate it commonly maximizes the collective good in this context, thus also establishing a dilemma regarding what is best for an individual and what is best for the group [4]. Myriad mechanisms such as kin selection [5, 6], punishment [7–10] and voluntary participation [11] are proposed to rescue such cooperation tragedy in this spectrum (see review [12, 13]). In particular, by taking into account the following aspects, the recent shift from evolutionary games in well-mixed populations [14–16] and static networks [17–23] to evolutionary games in multiplex networks [24–27] and dynamic networks [28–34] has stimulated mounting efforts in exploring cooperation dynamics in more realistic scenarios (see review [35–38]).

First, the vast majority of literatures to date have suggested that in many circumstances single-layer networks are no longer suitable for capturing the nature of backbones of physical, biological, and social systems due to their inability to encode the universal multiplexity of real systems [39, 40]. More importantly, system dynamics in multiplexes in general differs dramatically from that in simplexes by virtue of diverse problem-oriented multi-domain interdependence. For instance, in the study of interdependent networks, Buldyrev *et al* [41] reported that failure of nodes in one layer is likely to beget failure of dependent nodes in other layers, thus causing a cascade of failure. The analysis around topology interdependence has become a primary means of mitigating cascading failures. In [42], Parshani *et al* also showed analytically that two different types of links are needed for proper characterization of real networks. For a detailed review, refer to [43–46]. In particular, regarding cooperation dynamics, Gómez-Gardeñes *et al* [47] established a prisoner's dilemma game in multiplex networks and evidenced that the increased multiplexity favors cooperation by means of enhancing the so-called cooperation resilience, whereby evolutionary dynamics strikingly profits the survival of cooperation even in an environment overwhelmingly unfavorable to altruistic behaviour. Likewise, Battiston *et al* [48] further investigated key determinants of public cooperation by focusing on the interplay between the structure overlap and inherent game parameters in such networks. In addition, a series of reports on directed and undirected interdependence between two populations by Wang *et al* [37, 49, 50] also revealed that the state-feedback based interdependence as well as its optimal organization can dramatically uphold cooperation in the form of a variety of biased utility functions. To sum up, these efforts not only injected new impetus into the activity of exploring cooperation dynamics yet also broke through the limitation of the traditionally and frequently used single-layer network and therefore broadened the theoretical framework for addressing the cooperation puzzle.

Second, real networks are not static yet continuously evolving dynamic entities. Compared with static networks, dynamic networks have inherent advantages owing to their ability to adapt the structure flexibly and properly in response to the rich variety of real-time feedback. More importantly, there has been an increasing urgency to consider network dynamics and dynamics in networks as a whole rather than separately. Adhering to this thought, by synthesizing strategy dynamics and network dynamics, Pacheco *et al* [51, 52] designed several delicate coevolutionary models. Their novel insights verified analytically an equivalent transformation between different social dilemmas provided that network dynamics proceeds much faster than strategy dynamics, whereby cooperation can be greatly boosted. In particular, they also tested and extended their conclusion to a broader scope using extensive numerical simulations. Yet, the study of dynamic networks has never been confined to the theoretical level. As an excellent experiment work, [53] by Rand *et al* explained clearly with real human interactions that the fast network-regulation is a necessary requirement for cooperation to evolve in a dynamically organized game. Quite recently, Akcay [54] also reported how cooperation collapses and is rescued in evolving dynamic networks that are synergistically driven by social inheritance and random connections. He introduced interaction based selection into their previous novel dynamical process model [55], focused on the coevolution of social network and social traits, and conducted comprehensive analysis on the conditions favorable and unfavorable to cooperation. For the detailed literature review on coevolutionary dynamics, refer to the systematic review by Perc *et al* [35]. In fact, in addition to underpinning cooperation, dynamic networks also retain merits in many other areas such as optimizing disease control [56, 57] and economizing control energy [58].

Yet, despite the abundance of studies regarding above two eye-catching properties of real networks, much less is known about the issue of coordinated network evolution that should be a fundamental problem in the analysis of evolving dynamic interdependent networks where both strategy dynamics and network dynamics exhibit significant interdependence at the layer level. In addition, the universal multi-domain nature of complex networks generally endues agents an inherent compensation channel to self-assess behaviour in pursuit of personal evolutionary success. Taking these regards into consideration, here we design a minimal coevolutionary model, in which network dynamics of two sub-layers coordinates with each other based on an inherent cross-layer self-assessment compensation rule, to focus on how the coordination of different network dynamics transforms the propagation of altruistic behaviour in this context. Unlike previous studies, the fate of each link is determined not only by evaluations from two terminal nodes in the current habitat layer yet also directly or indirectly by feedback from the other interdependent layer. The duration of each link evolves in a more moderate and rational way, then the update and remove of links distinguish apparently from the previous 'either-keep-or-delete' mode that is in general deterministic in the link updating stage. In subsequent sections, we show that the cross-layer self-optimization compensation method, proposed on the basis of multi-domain nature of multiplex networks, most of the time enables dynamical processes of different layers to serve as a mirror to each other, which to some extent allows the possibility of inherent self-correction in multiplex networks and upgrades in some aspects the functionality of dynamic networks.

2. Evolution of cooperation in synergistically evolving dynamic multiplex networks

2.1. Population structure

In this study, we employ two ER-like random graphs, namely ***A*** and ***B***, to characterize the initial interdependent contact networks. Nodes that compose two networks are the same, while links between them are different. One possible exemplification to such scenario is the interdependent multi-domain social network in which identical agents engage in nonidentical social relationships with generally different strategies.

i. M links are utilized to randomly pair N nodes to generate each of two aforementioned nonidentical ER-like networks. Therefore, such an interdependent network totally includes $2M$ links and N nodes.

ii. Once a link in layer A (B) is created between two agents, termed i_A and j_{1A} (i_B and j_{2B}), by initializations or by linking dynamics, i_A and j_{1A} (i_B and j_{2B}) are assigned with an expected duration $\tau_{i_A j_{1A}}$ and $\tau_{j_{1A} i_A}$ ($\tau_{i_B j_{2B}}$ and $\tau_{j_{2B} i_B}$) towards this link, respectively.

iii. In general, $\tau_{i_A j_{1A}} \neq \tau_{j_{1A} i_A}$ and $\tau_{i_B j_{2B}} \neq \tau_{j_{2B} i_B}$, also $\tau_{i_A j_{1A}}$, $\tau_{j_{1A} i_A}$, $\tau_{i_B j_{2B}}$, $\tau_{j_{2B} i_B} \in [0,\ \tau_{\max}]$, and $\tau_{\max} < 1$. The rationale behind these assumptions lies in the following aspects. On the one hand, there are no permanent relationships in reality, thus $\tau_{\max} < 1$. On the other hand, consider that different agents often have different perceptions towards the importance of a given partnership, so in most cases $\tau_{i_A j_{1A}} \neq \tau_{j_{1A} i_A}$ and also $\tau_{i_B j_{2B}} \neq \tau_{j_{2B} i_B}$. Without loss of generality, we use $\tau_{\max} = 0.95$ throughout this research.

In the initialization stage, each agent in each network is designed either as a cooperator (**C**) or as a defector (**D**) with equal probability.

2.2. Two-player prisoner's dilemma game

In the prisoner's dilemma game [17], there exist three possible strategy pairs among which mutual cooperation produces the reward **R**, mutual defection leads to the punishment **P**, and the mixed offers the cooperator the sucker's payoff **S** yet the defector the temptation **T**. Thus, the payoff matrix of this game is formulated as:

$$\begin{array}{cc} & \begin{array}{cc}\mathbf{C} & \mathbf{D}\end{array} \\ \begin{array}{c}\mathbf{C}\\ \mathbf{D}\end{array} & \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{1+u} & \mathbf{u} \end{pmatrix} \end{array}, \tag{1}$$

where $\mathbf{R} = \mathbf{1}, \mathbf{T} = \mathbf{1} + \mathbf{u}, \mathbf{S} = \mathbf{0}, \mathbf{P} = \mathbf{u}$ and **u** is a rescaled parameter quantifying dilemma strength.

In each time step, an agent, namely i, is chosen randomly from the population as the game organizer. Then another two agents, namely j_{1A} and j_{2B}, are randomly drawn from i's neighborhood as i's co-players in layer A and layer B, respectively. i plays the prisoner's dilemma game with j_{1A} and j_{2B} simultaneously and all these three agents collect their payoff in accordance with the matrix (1). If i has no neighbors in both layers, another agent will be randomly chosen as the game organizer.

2.3. Multi-player public goods game

In the public goods game [59], an agent, namely i, plays the game with all adjacent neighbors in layer X ($X = A$ or B) by simultaneously deciding whether or not to contribute to the common pool according to their strategies. Specifically, the cooperator contributes a fixed amount $\mathbf{c} = 1$, however, the defector contributes nothing. Finally, the total contributions are multiplied by a synergy factor g and shared by each player of the focal group in this layer regardless of their contributions. Then each group member collects the payoff in line with equation (2):

$$\pi_{x_X} = \sum \frac{s_{y_X}}{k_{i_X} + 1} * g - s_{x_X}, \qquad x_X,\, y_X \in \Omega_{iX}, \tag{2}$$

where X is one of A or B. π_{x_X} and s_{x_X} are the payoff and strategy of x in X, respectively. We assume that $s_{j_X} = 1$ means j carries a **C** strategy in X and $s_{j_X} = 0$ indicates j adopts a **D** strategy in X. Ω_{i_X} is the neighborhood of i in layer X and k_{i_X} is its size.

2.4. Self-reliant and coordinated network evolution

After game interaction, i, together with other randomly chosen co-players, namely j_{1A} and j_{2B}, simultaneously adjusts their own expected duration towards corresponding links in two layers. Without loss of generality, as for the multi-player public goods game, we let i randomly select one link with her co-players in each layer to update. Here we introduce two classes of network evolution according to the information applied in linking dynamics. One is the self-reliant network evolution based solely on strategic feedback from the current habitat layer. The other is the coordinated network evolution that considers also mutual affections of different network-adapting processes. For comparison, we classify the population into two categories. One type are common agents who

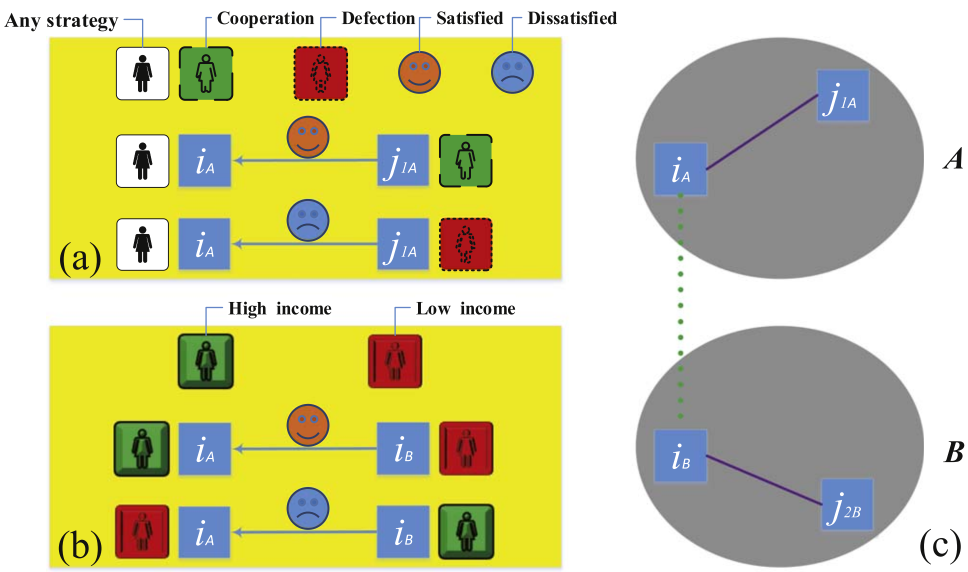

Figure 1. Schematic diagram for link duration adjustment. For simplicity, it only illustrates how the focal agent i_A updates her expected duration $\tau_{i_A j_{1A}}$ regarding the link $l_{i_A j_{1A}}$ in network A. In the present study, the driving force responsible for network evolution generally includes two parts. One is the strategic feedback from the current habitat layer (a) and the other is the cross-layer self-assessment (b). If i_A is a smart agent, she updates her local contact network by integrating feedbacks transferred by both channels in (a) and (b). If i_A is a common agent, she only considers the strategic feedback as shown in (a). Regardless of the specific channel, the driving force tends to push i_A to lengthen (shorten) her expected duration regarding the concerned link when she is satisfied (dissatisfied) with the corresponding feedback. The arrow indicates the source of the feedback. The typical structure slice in a two-layer interdependent network is plotted in (c).

adopt the self-reliant dynamical linking. The others are smart agents who employ the coordinated dynamical linking. And we denote ρ the fraction of smart agents in the population. More details about the evolution of link duration are shown in equation (3) for the common:

$$\begin{bmatrix} \tau^{*}_{i_A j_{1A}} \\ \tau^{*}_{j_{1A} i_A} \\ \tau^{*}_{i_B j_{2B}} \\ \tau^{*}_{j_{2B} i_B} \end{bmatrix} = \begin{bmatrix} \tau_{i_A j_{1A}} \\ \tau_{j_{1A} i_A} \\ \tau_{i_B j_{2B}} \\ \tau_{j_{2B} i_B} \end{bmatrix} + \begin{bmatrix} (-1)^{s_{j_{1A}}+1} & \\ (-1)^{s_{i_A}+1} & \\ & (-1)^{s_{j_{2B}}+1} \\ & (-1)^{s_{i_B}+1} \end{bmatrix} \begin{bmatrix} \Delta_A \\ \Delta_B \end{bmatrix} \tag{3}$$

and in equation (4) for the smart:

$$\begin{bmatrix} \tau^{*}_{i_A j_{1A}} \\ \tau^{*}_{j_{1A} i_A} \\ \tau^{*}_{i_B j_{2B}} \\ \tau^{*}_{j_{2B} i_B} \end{bmatrix} = \begin{bmatrix} \tau_{i_A j_{1A}} \\ \tau_{j_{1A} i_A} \\ \tau_{i_B j_{2B}} \\ \tau_{j_{2B} i_B} \end{bmatrix} + \begin{bmatrix} (-1)^{s_{j_{1A}}+1} & \epsilon_{i_A i_B} \\ (-1)^{s_{i_A}+1} & \epsilon_{j_{1A} j_{1B}} \\ \epsilon_{i_B i_A} & (-1)^{s_{j_{2B}}+1} \\ \epsilon_{j_{2B} j_{2A}} & (-1)^{s_{i_B}+1} \end{bmatrix} \begin{bmatrix} \Delta_A \\ \Delta_B \end{bmatrix}, \tag{4}$$

where $\tau^{*}_{i_A j_{1A}}$ is the updated $\tau_{i_A j_{1A}}$ for the next generation. Δ_A (Δ_B) characterizes A's (B's) network-adapting speed. $\epsilon_{i_A i_B}$ encodes the inter-layer coordination when agents reshape local contact networks and highlights the significance of self-assessment. In this way, unlike previous 'either-keep-or-remove' model, a certain amount of stochasticity is introduced into linking dynamics and accounts for the widespread impermanency of partnership in real-world. Concretely, if i_A's co-player j_{1A} is carrying a **C** (**D**) strategy, then no matter what strategy i_A adopts she will be satisfied (dissatisfied) with this partnership. In this regard, $\tau_{i_A j_{1A}}$ will increase (decrease) by Δ_A, as illustrated by figure 1(a). If $\tau_{i_A j_{1A}}$ reaches its limits, i.e. zero as the minimum or $\tau_{\max}$ as the maximum, in the evolution, then it only shifts in the other direction. Compared with equation (3), the rationale lies behind equation (4) is aside from strategic feedback that comes from current habitat layer, it most of the time incorporates the message captured in the other layer. Such supplementary message often offers an agent a state-dependent compensation choice.

In each time step, without loss of generality, we let the game organizer, i.e. i_A and i_B, to determine either to keep, with the probability $\tau_{i_A j_{1A}}$ and $\tau_{i_B j_{2B}}$, or to remove, with the likelihood $1 - \tau_{i_A j_{1A}}$ and $1 - \tau_{i_B j_{2B}}$, corresponding links (i.e. $l_{i_A j_{1A}}$ and $l_{i_B j_{2B}}$) in both layers. In order to highlight the interdependence between different social or nonsocial domains, we make the following assumptions: In the case that $l_{i_A j_{1A}}$ and $l_{i_B j_{2B}}$ are a pair of overlapping links in two layers, each of them is retained with the probability $\frac{\tau_{i_A j_{1A}} + \tau_{i_B j_{2B}}}{2}$ and cut off with the probability $1 - \frac{\tau_{i_A j_{1A}} + \tau_{i_B j_{2B}}}{2}$. If any one of them, e.g. $l_{i_A j_{1A}}$ ($l_{i_B j_{2B}}$), is removed, then the other $l_{i_B j_{2B}}$ ($l_{i_A j_{1A}}$) is simultaneously rewired. In the case that i breaks a link, i establishes a new connection to another randomly drawn agent in the same layer excluding i's existing neighbors, keeping the population size constant. Self and cross-layer connections are forbidden in this study. Because each agent will be selected once on average to be as the game organizer in a complete round, there are no bias or preferences towards any agents. In addition, if agents are isolated nodes in one layer, we assume that they only adopt the self-reliant dynamical linking because of invalid inter-layer feedback in this case, regardless of their types (i.e. the common or the smart).

Compared with single-layer networks, the multi-domain nature of multiplex networks can inherently offer agents a natural compensation channel to self-correct behaviour by means of allowing them the possibility of self-assessment of evolutionary success between different layers. And a very important point lies in that such convenient and inherent compensation choices incur almost no cost to agents. With this regard, we propose an inherent cross-layer self-optimization compensation approach as shown in equation (5):

$$\epsilon_{x_X x_Y} = \frac{(-1)^{H\left(\frac{1}{1+\exp[\alpha*(\pi_{x_X} - \pi_{x_Y})]} - z\right)}}{1 + \exp[\alpha * (\pi_{x_X} - \pi_{x_Y})]}, \tag{5}$$

where z is a variable randomly and evenly distributed in the interval [0, 1]. $x \in \{i, j_1, j_2\}$ and $X, Y \in \{A, B\}$. α is a tunable parameter. $H(x)$ is the step function and satisfies the following form:

$$H(x) = \begin{cases} 1, & x > 0, \\ 0, & x \leqslant 0. \end{cases} \tag{6}$$

According to equation (5), agents autonomously choose appropriate compensation including positive and negative in exact accordance with the inherent self-assessment of evolutionary outcome in two layers. For instance, regarding $\alpha > 0$, if the income in the other layer surpasses that in the current layer, by a large probability negative compensation is applied and the feedback from the other layer prefers to revise the concerned link, and vice versa, as shown in figure 1(b). The greater the income gap, the stronger the compensation effect. Naturally, the multi-domain nature of multiplex networks allows a convenient realization of inherent self-optimization compensation method. This is very important and cautious in terms of grabbing and making use of information from the perspective of game theory, and is also a point frequently overlooked by previous literatures. In this regard, we claim that such an inherent self-optimization compensation scheme is in large part able to avoid far-fetched spies on evolution information of others and meanwhile advises agents to shift their focus to themselves. By this approach, the input to power the evolution of network is definitely enriched to comprise two elements. One part comes from the outside, i.e. rival's strategy, and one part comes from the inside, i.e. self-assessment. We show that such a simple yet reliable compensation approach yields an optimization of short-term strategy feedback. Importantly, the well-reported optimal partner choice that used to be seen as the best response to the rival's behaviour seems to be not necessarily optimal, from a long-term perspective, for structure adapting in evolving dynamic interdependent networks.

2.5. Strategy evolution

In this section, an agent, namely i_X, updates her strategy by imitating the strategy of another randomly chosen neighbor, namely j_X, in the same layer with the probability given by the Fermi function:

$$\Gamma(s_{i_X} \to s_{j_X}) = \frac{1}{1 + \exp[\beta * (\Pi_i - \Pi_j)]}. \tag{7}$$

In line with the previous study [26], we use the mean payoff that i collects in all layers to quantify i's effective payoff Π_i that ultimately maps into strategy dynamics, i.e. $\Pi_i = \frac{\pi_{i_X} + \pi_{i_Y}}{2}$. β is the selection intensity and introduces a noise into strategy-learning [34]. Cross-layer strategy-learning is forbidden in this work. We calculate the final cooperation level in the equilibrium state by averaging over 100–1000 independent runs after a sufficiently long transient time.

3. Results

It is a primary feature that each sub-system (i.e. each sub-layer) always equilibrates either in the full cooperation state or in the full defection state, making coexistence of different strategies impossible. As a consequence, from the perspective of the entire population there are three equilibrium combinations among which *full-**C*** means two layers converge to all cooperation equilibrium, *full-**D*** indicates they evolve to all defection equilibrium, and *mixture* implies one converges to all cooperation while the other to all defection. In figure 2, we show in detail three representative time evolution converging to different equilibrium states in one complete run. In evolving

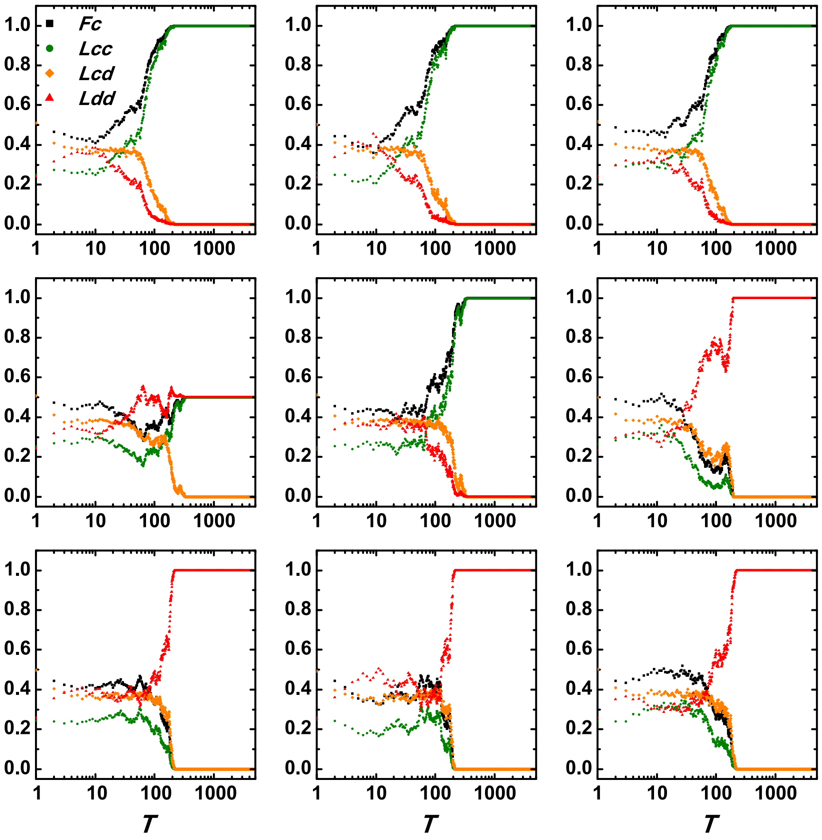

Figure 2. Representative time evolution residing in three different equilibrium states including full-***C*** (first row), *mixture* (second row) and full ***D*** (third row), with the same system configuration in the prisoner's dilemma game. For each row, the first column depicts the statistical quantities of the entire population and the rest for other two sub-populations (sub-layers). F_C is the cooperation level. L_{CC}, L_{CD}, and L_{DD} denote the abundance of links paring two ***C*** strategies, one ***C*** and one ***D***, and two ***D*** strategies, respectively. $M = 2000$, $N = 1000, \alpha = 10, \beta = 10, \rho = 1, \Delta_A = \Delta_B = 0.1, u = 0.23$.

dynamic interdependent networks, coordinated network-adapting is able to make cooperative strategy more competitive even in the parameter region overwhelmingly unfavorable to cooperation provided that agents can adaptively adjust their social ties in accordance with the integrated feedback from sub-layers. Remarkably, it is often the case that the intertwined coevolutionary dynamics reinforces social ties between cooperators even if the abundance of cooperation in the population declines. In most cases, L_{cc} climbs or holds steady in figure 2 when F_c declines in the early stage of evolution, independent of the ultimate evolutionary outcome. It implies that coordinated linking dynamics is likely to increase the probability by which cooperation mutually feeds each other. In other words, it may help enhance the well-reported direct reciprocity [12] that remarkably contributes to the diffusion of cooperation in structured populations.

How the cooperation level varies in dependence on Δ_A, Δ_B and u in the two-player prisoner's dilemma game (on Δ_A, Δ_B and g in the multi-player public goods game) in figure 3 stresses the familiar dominating effect of network-adapting speed on the evolution of cooperation in evolving dynamic networks [28, 29, 34, 52, 53]. Consider that there are usually critical parameter thresholds marking the emergence, extinction of cooperation and *full-**C*** phase in the aforementioned social dilemmas. Hence, the evolutionary performance of concerned mechanisms can be examined by analyzing specific shifts of these critical points. In figure 3, a higher Δ_A (Δ_B) always results in a larger threshold value of u beyond which cooperation vanishes in the prisoner's dilemma game and meanwhile a smaller threshold value of g around which cooperation emerges in the public goods game. Simultaneously, a higher Δ_A (Δ_B) usually corresponds to a larger value of u beyond which *full-**C*** phase fades away in the prisoner's dilemma game and also a smaller value of g around which *full-**C*** phase emerges in the public goods game. It thus demonstrates, no matter in which game, coevolution dynamics commonly seats cooperation at a more competitive position provided that the rapid adjustment to network structure in response to immediate game feedback is possible. In general, the more agile the network adapting, the more competitive the cooperation.

To demonstrate the role of coordinated network-adapting in the evolution of cooperation in dynamic interdependent networks, we show in figure 4 the $\Delta - u$ phase diagram for the prisoner's dilemma game, as obtained for $\rho = 0$ (figures 4(a) and (d)), $\rho = 0.5$ (figures 4(b) and (e)), $\rho = 1$ (figures 4(c) and (f)), for

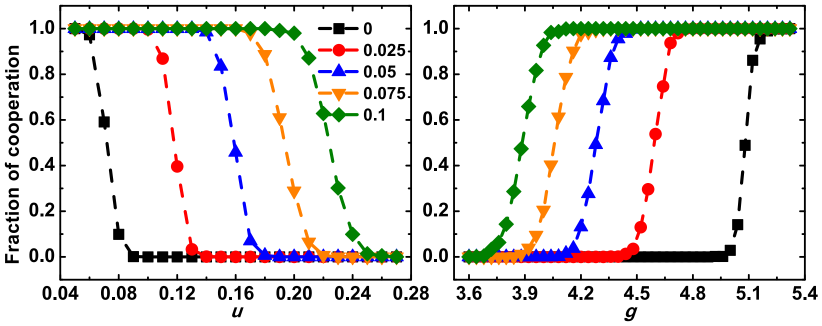

Figure 3. The dominating role of network-adapting rate in promotion of cooperation, as obtained for the prisoner's dilemma game (left panel) and multi-player public goods game (right panel), respectively. $M = 2000, N = 1000, \alpha = 10, \beta = 10, \rho = 1$. For simplicity, here we first focus on situations where $\Delta_A = \Delta_B$ and the rest will be further considered in later sections.

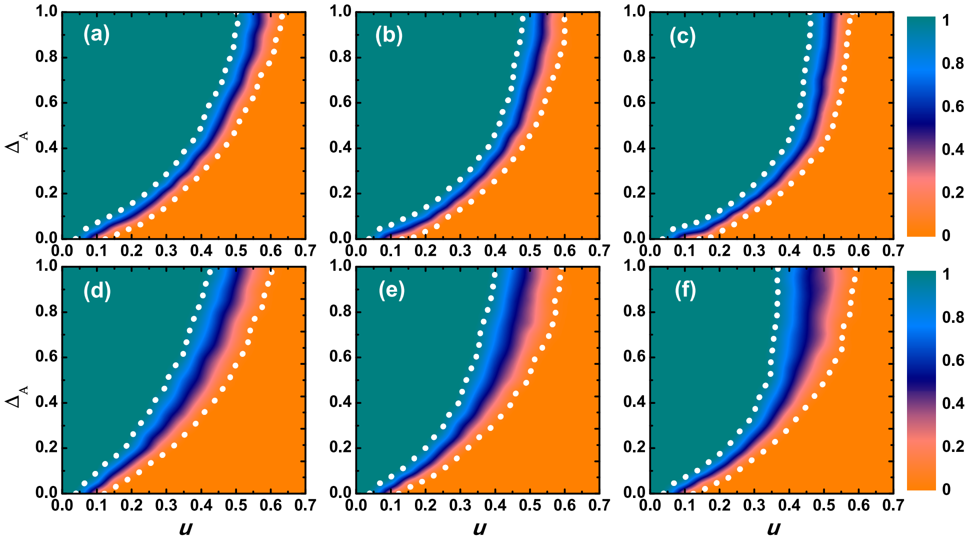

Figure 4. $\Delta_A - u$ phase diagram for the two-player prisoner's dilemma game subject to the coordinated linking dynamics, as obtained for $\rho = 0$ ((a), (d)), $\rho = 0.5$ ((b), (e)), $\rho = 1$ ((c), (f)), for $\Delta_A = \Delta_B$ ((a)–(c)), and for $\Delta_A = 2\Delta_B$, i.e. $\Delta_A \neq \Delta_B$ ((d)–(f)). Cyan denotes the *full-**C*** phase and Orange the *full-**D*** phase. The dashed lines are boundaries separating different phases and the area between two dashed lines is the *mixture* phase. The plot indicates a double-edged sword effect of coordinated network-adapting on cooperation and its potential association with network-adapting rate. $M = 2000, N = 1000, \alpha = 10, \beta = 10$.

$\Delta_A = \Delta_B$ (figures 4(a)–(c)), and for $\Delta_A \neq \Delta_B$ (figures 4(d)–(f)), respectively. The figure manifests a strong and uncommon impact of coordinated network evolution on cooperation. First, the growth of u and network-adapting rate for $\Delta_A = \Delta_B$ ($\Delta_A \neq \Delta_B$) leads to the slight (remarkable) territory expansion of *mixture* phase (areas enclosed by dashed separating lines in figure 4). This difference can be explained as follows. On the one hand, the collaborative network evolution has the homogenization effect for apparently unequal sub-layer adapting rates as a result of self-optimization compensation as implied in equation (5). On the other hand, phase separating lines for $\Delta_A \neq \Delta_B$ are expected to be non-coincident considering the dominating role of network-adapting rate in this study. Accordingly, the homogenization of non-coincident coexistence areas in two sub-layers leads to the notable territorial extension of *mixture* phase. Second, regarding relatively low network-regulation rates, cooperation most of the time thrives as ρ gradually intensifies. On the contrary, for relatively high network-regulation rates, decreasing ρ fertilizes the flourish of cooperation. In this regard, consider ρ encodes the timescale between the self-reliant and coordinated network evolution. Hence, the change in curvature of dashed phase separating lines in figure 4 (follow the order from figures 4(a) to 4(c) as well as from figures 4(d) to 4(f)) probably suggests a double-edged sword effect of coordinated network evolution on cooperation and also its intimate association with the network-adapting rate. In order to verify and make it easy to comprehend this point, we clarify in figure 5 a sharp contrast between the positive effect of collaborative network evolution on cooperation in the slow-regulated network and its negative effect on cooperation in the fast-regulated network and this further validates the above analysis.

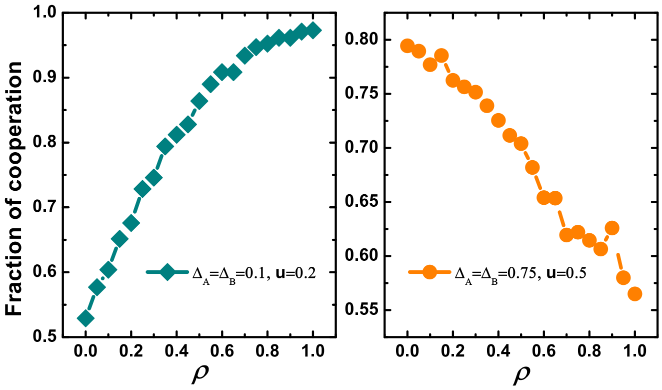

Figure 5. Positive (Negative) effect of coordinated network-adapting on the evolution of cooperation in the slow-regulated (fast-modulated) network for the two-player prisoner's dilemma game. $M = 2000, N = 1000, \alpha = 10, \beta = 10$.

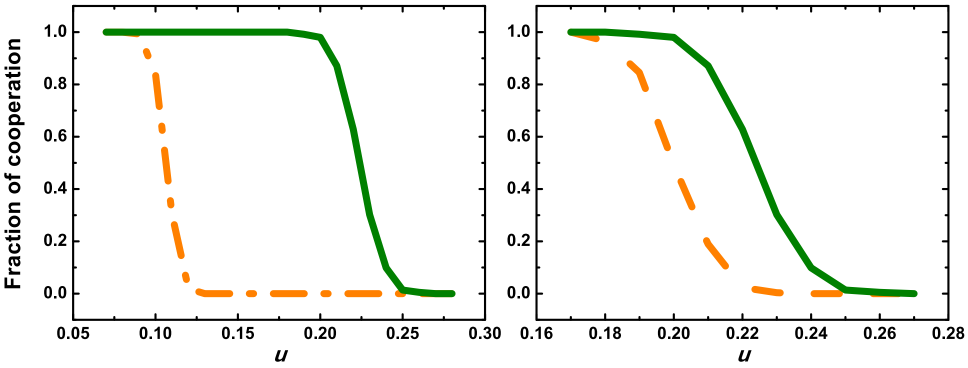

Figure 6. Synergy effect of strategic feedback and self-optimizing compensation on improvement of cooperation. Dashed line denotes the result for strategy based linking dynamics.Dashed–dotted line represents the result for self-optimizing compensation based linking dynamics. Solid line is the result for linking dynamics based on their combination. Note that different horizontal scales are utilized for comparison purposes. $M = 2000, N = 1000, \alpha = 10, \beta = 10, \rho = 1, \Delta_A = \Delta_B = 0.1$.

For a further probe of the answer to this nontrivial double-sided effect, figure 6 makes a comparison of the performance of linking dynamics that is based either on the elementary feedback (i.e. regarding solely either strategy or self-assessment) or on their combination. Intriguingly, it reflects that these two components normally bring about better results when they are used in combination than individually, which emphasizes the synergy effect of strategy feedback and self-optimization compensation on reinforcement of cooperation. First, the $\Delta_A - u$ phase diagram in figure 7 evidences that in the absence of strategy feedback the cooperation level decays quickly to zero once the value of u exceeds 0.2 with all values of Δ_A. That is to say, such inherent compensation is supposed to become invalid and fail to sustain cooperation as soon as the value of u oversteps this critical point. In this regard, further trust in the message transferred by this channel may cause the confusing interference to strategy feedback in the context of coordinated network-adapting. And a foreseeable result is the greater the network-adapting rate, the more serious the interference. Second, remember that a relatively large network-adapting rate generally corresponds to a large threshold of u that labels the extinction of cooperation in the population as shown in figure 3. Consequently, a double-edged sword effect of coordinated network-adapting on cooperation in respect to the relatively large values of Δ_A and u is established in figure 4, as a result of asynchronous failure of these two feedback. In a nutshell, our analysis suggests not only a double-sided effect of coordinated network evolution on cooperation, and more importantly, also that the network adapting rate is a key factor responsible for transforming such a complicate impact. The result highlights the complexity of a multi-layer network as a complete functional entity rather than a simple superposition of monolayer counterparts.

As the most direct and accessible feedback from the game interaction, opponents' strategies have almost been regarded as the only driving force of network evolution in the study of dynamic single-layer networks [28–31]. Rewiring links with unsatisfactory co-players based solely on their strategies has often been referred to as the optimal transient network-adapting for cooperation and has long been seen as a reliable pathway for raising cooperation in these models. In what follows, we evidence that, in the presence of aforementioned

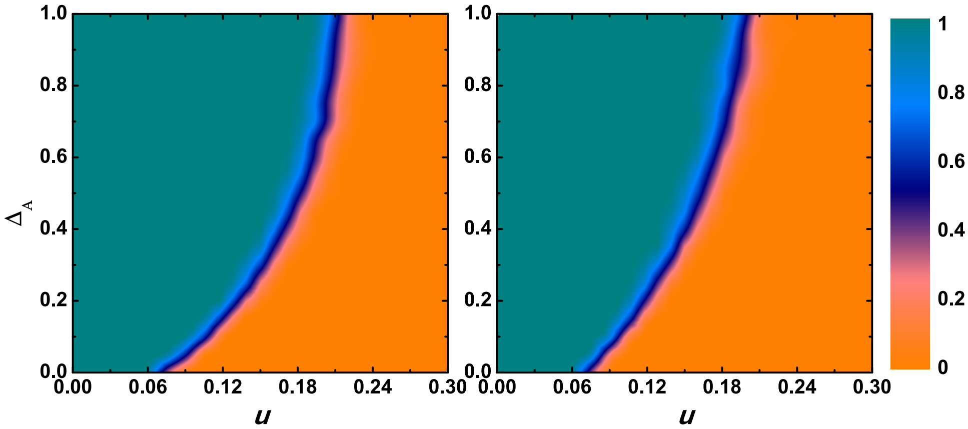

Figure 7. $\Delta_A - u$ phase diagram subject to the self-optimizing compensation based linking dynamics for $\Delta_A = \Delta_B$ (left panel) and $\Delta_A \neq \Delta_B$, i.e. $\Delta_A = 2\Delta_B$ (right panel). It implies that the self-assessment compensation scheme is going to become invalid once the dilemma strength exceeds a certain value. $M = 2000, N = 1000, \alpha = 10, \beta = 10, \rho = 1$.

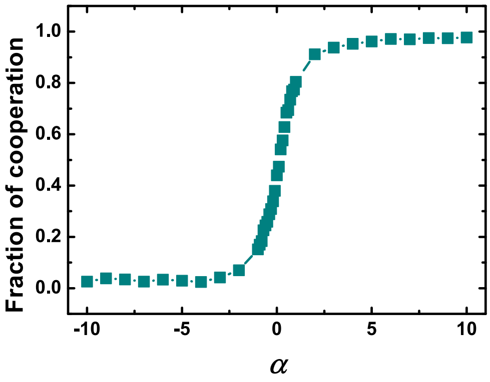

Figure 8. S-shaped curve shows that the preference in compensation choice plays an important role in alternating the function of intertwined linking dynamics. $M = 2000, N = 1000, \beta = 10, \rho = 1, \Delta_A = \Delta_B = 0.1, u = 0.1$.

inherent cross-layer self-optimization compensation, coordinated network-adapting tends to evoke much better promotion of cooperation, issuing a remarkable challenge to the self-reliant network-adapting. Because there is no cross-layer strategy-learning and the same agent often adopts different strategies in two layers, we thus choose income instead of strategy as the potential cross-layer compensation channel bridging nonidentical dynamical processes taking place in different layers. In figure 8, the S-shaped curve discloses a positive correlation between the cooperation level and α. Refer to the introduction in section 2, it elucidates that taking the positive compensation inspires cooperation when an agent has an advantage over her duplicate in the other layer in terms of income. Otherwise, cooperation is to varying degree suppressed. The rationality behind this phenomenon lies in that, for positive α in figure 8, when the positive compensation occurs it probably presages the agent is living in a game environment conducive to reproduction, at least, not the worst in comparison to her duplicate in the other layer. In this regard, if one plans to optimize the network structure in pursuit of personal evolutionary success, the cross-layer feedback should suggest her to continuously preserve the existing social ties, i.e. just consistent with a positive compensation. Especially, the secondary enhancement of cooperation resulted from the addition of inherent cross-layer self-assessment compensation exemplifies that the optimal transient network-adapting relying solely on strategy feedback in the context of single-layer dynamic networks [35] might be, from a long-term perspective, no longer optimal for the evolution of cooperation in the background of multiplex networks. As we have shown, the multi-layer nature of multiplex networks could inherently offer agents a mirror to self-assess their behaviour. Accordingly, the expansion of feedback channel fostered by the inherent multi-domain nature of multiplex networks is inclined to provide more reliable and efficient solutions to the further improvement and optimization of network dynamics in accomplishing some desired and difficult tasks.

The adapting speed quantifies the ability to reorganize network structure dynamically in response to the dynamic state of nodes. Previous studies report that high-speed network-adapting is almost a necessary

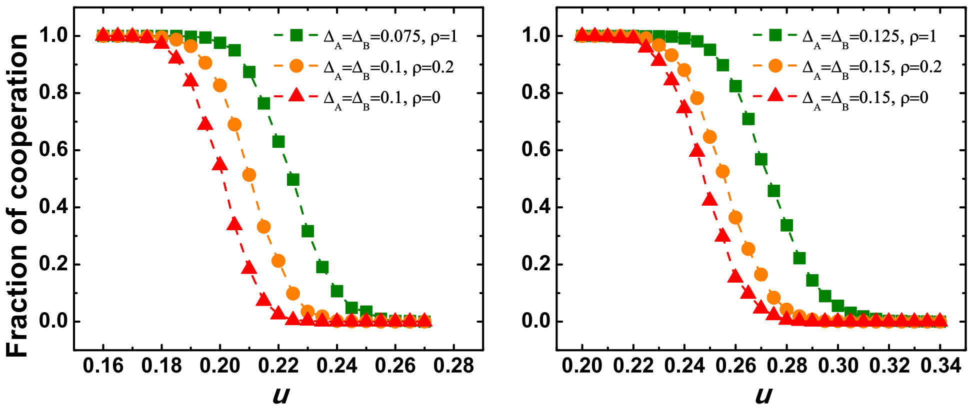

Figure 9. Fundamental advantages of coordinated network evolution over its self-reliant counterpart in boosting cooperation. The plot underlines that with the application of coordinated dynamical linking it is possible for slow-tuned networks to outperform fast-regulated networks in terms of alleviating social dilemmas, destabilizing the dominant role of network-adapting speed in this context. $M = 2000, N = 1000, \alpha = 10, \beta = 10$.

condition for coevolution dynamics to encourage cooperation [28, 29, 34, 52, 53]. In dynamic interdependent networks, however, we claim this issue can be, to some degree, addressed by seeking solutions from the perspective of coordination of different dynamical processes that simultaneously proceed in different layers. In social network analysis, besides, the importance of a node is normally associated with its topological parameters, especially with its degree that is typically interpreted as the mathematical measure of social influence (importance) and social diversity [59, 60]. Next, we advance the coordinated linking dynamics by mapping the so-called social influence into the cross-layer self-optimization compensation. To this end, we rewrite the cross-layer self-optimization compensation modus in equation (5) as:

$$\begin{bmatrix} \epsilon^{*}_{i_A i_B} \\ \epsilon^{*}_{j_{1A} j_{1B}} \\ \epsilon^{*}_{i_B i_A} \\ \epsilon^{*}_{j_{2B} j_{2A}} \end{bmatrix} = \begin{bmatrix} k_{i_B} & & & \\ & k_{j_{1B}} & & \\ & & k_{i_A} & \\ & & & k_{j_{2A}} \end{bmatrix} \begin{bmatrix} \epsilon_{i_A i_B} \\ \epsilon_{j_{1A} j_{1B}} \\ \epsilon_{i_B i_A} \\ \epsilon_{j_{2B} j_{2A}} \end{bmatrix}, \tag{8}$$

where $\epsilon^{*}_{i_A i_B}$ is the rescaled coordination factor. In analogy with the manipulation in [61], the inter-layer feedback arising from the other dependent layer is amplified to a certain extent depending on its influence and importance in the corresponding layer. With this improved compensation formula, the cooperation level is again pulled up. Nontrivially, figure 9 demonstrates the fundamental advantages of collaborative network-adapting in advocating cooperation. It confirms that heavily intertwined slow-tuned networks most of the time have the ability to outperform slightly coupled fast-regulated networks in regard to inspiring the propagation of altruistic cooperation. Meanwhile, it also informs us that in the study of dynamic interdependent networks, coordinating different linking dynamics based on some feasible feedback across different layers can probably provide a new channel to make up for the shortcomings that most of the time are caused by the limited network-adapting speed. In view of such extraordinary observations, we claim that coordinated network-adapting is able to weaken network-adapting rate's dominant effect that is reported to frequently emerge in a coevolution context. Therefore, our findings may further evoke some follow-up research in this direction.

Despite the fundamentality of link formation in evolving dynamic networks, hitherto, the effect of link formation cost has often been ignored in the study of cooperation dynamics. The large majority of studies mainly concern the cost and benefit of a link at the level of interaction rather than the inherent cost of establishing and maintaining a social tie. This will not be true for many real situations. In fact, creating and preserving a social tie may be inherently costly [54, 62], for instance, involving long-distance travel or spending tremendous amounts of time. For generality's sake, we let the game organizer to bear the cost of forming a new link in the present work. As the benefit each link produces for an agent only varies between 0 and 2, so we confine this cost to a closed interval [0, 2]. On the one hand, it shows in figure 10 that the addition of link formation cost often has a positive effect on cooperation just as reported in [54]. For a given value of u, if link formation cost grows, then the long-term cooperation level tends to increase too. Of great significance here is that the further study in figure 11 demonstrates not only can coordinated network-adapting impel the population to the full-$\boldsymbol{C}$ state, it also requires much less link formation cost and time consumption than its self-reliant counterparts. It therefore to some extent underlines the universal efficiency and merit that coordinated network evolution may

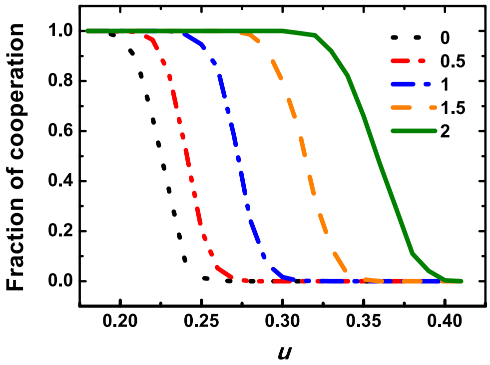

Figure 10. Costly link formation nourishes the evolution of cooperation. The cooperation level in dependence on various link formation cost for $l_{\mathrm{cost}} = 0, 0.5, 1, 1.5$, and 2, respectively. $M = 2000, N = 1000, \alpha = 10, \beta = 10, \rho = 1, \Delta_A = \Delta_B = 0.1$.

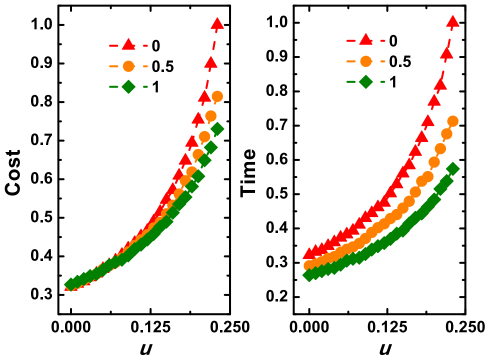

Figure 11. Normalized link formation cost and time resource consumption for driving the system to the full-C phase with respect to $\rho = 0, 0.5$, and 1, respectively. It indicates coordinated network evolution helps to significantly reduce link formation cost and time consumption compared with its counterparts. $M = 2000, N = 1000, \alpha = 10, \beta = 10, \Delta_A = \Delta_B = 0.1, l_{\mathrm{cost}} = 1.5$.

enjoy, in comparison to self-reliant network evolution, in terms of economizing resources reservation when it is used to alternate the system dynamics in order to achieve a specific expected goal.

4. Discussion

In network science, dynamic networks have proven to be capable of innovating better descriptions of real systems whose characteristics constantly adapt to changing environments. Especially in the context of evolutionary game theory, for this sake, dynamical linking, also known as linking dynamics or partner choice [51, 52], has been explored extensively as a feasible avenue for the evolution of cooperation in spatial games [28–30, 32, 34, 52, 54]. Likewise, multiplex networks inherently enjoy the super capability to model coupled structures and coupled dynamical processes [43–46]. On account of their more comprehensive and appropriate portrayals of real contact networks, these two classes of networks have been recognized as highly effective tools for analyzing cooperation dynamics regarding structured populations [35, 37]. However, real networks most of the time are not only multi-layered yet also probably evolving, and moreover, sub-layers could evolve at nonidentical rates. And this key point is clearly often overlooked by existing literatures. In this regard, it is of great urgency and significance to survey the coordination of different dynamical processes in nonidentical sub-layers, which should have been a fundamental problem in the multiplex network context. For this purpose, we propose an intertwined coevolutionary model in which both the self-reliant and coordinated network adapting are incorporated. And we showcase that such a simple yet effective coevolutionary rule enriches the system dynamics and has a profound impact on the evolution of cooperation.

In comparison to previous models by pacheco *et al* [51, 52], Santos *et al* [28] and Rand *et al* [53] where link reconnection as well as its rate largely depends on the type of partners, and by Fu *et al* [29, 31] in which indirect reciprocity is embedded via reputation, here the fate of each link is influenced both by assessments of two terminal nodes in the current residential layer and also by feedback from the other interdependent layer. As a result, it most of the time makes the update and remove of links distinguish remarkably from the most-used 'either-keep-or-remove' rule. The driving force responsible for the evolution of network mainly consists of two parts. One is strategic feedback extracted from game interactions of current habitat layer, which is the main thruster of network evolution. The other pregnant with the self-assessment consciousness that is premised upon the multi-domain nature of multiplex networks encodes cross-layer compensation and provokes the synergistic network evolution.

We show that coordinated network-adapting might be a double-edged sword to cooperation. For instance, it often has a positive (negative) effect on cooperation with respect to slow-tuned (fast-regulated) dynamic networks. And we attribute this to the asynchronous failure of concerned feedback from different layers. Importantly, due in large to the expansion of feedback channel, coordinated network evolution is likely to help improve the dispersal of cooperation in the following ways, such as making up for the deficiency caused by the insufficient network-adapting rate, and cutting down link formation cost as well as time consumption needed for propelling the population to the full-C phase. These results hence emphasize the inherent superiorities of collaboratively evolving dynamic interdependent networks over single-layer dynamic networks in publicizing altruistic behaviour.

In summary, by this work we find that the well-reported multi-domain nature of multiplex networks could naturally offer dynamical processes proceeding in sub-layers some feasible compensation choices by means of inherent self-assessment so that these dynamical processes may act as a mirror to each other for self-optimization in the long-term coevolution, therefore yielding a series of remarkable improvements in functionality of multiplex networks. Given its universality, the observation suggests that the proper coordination of dynamical processes on the basis of reliable feedback channeling sub-layers may be an effective optimizer of network functionality and allow the possibility of further rescue of cooperation, so that much of the previous research on coevolution dynamics [35] might be recast in terms of coordinated network evolution in the multiplex network context.

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