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WAVELET TRANSFORM-BASED STRATEGY FOR IDENTIFYING IMPACT FORCE ON A COMPOSITE PANEL

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An algorithm based on wavelet analysis for automatically estimating the location and magnitude of impact forces exerted on a rectangular carbon fibre-epoxy honeycomb composite panel is developed. The technique employs a single piezoelectric sensor mounted distant from the impact zone and presumes that an impact is applied at one of several pre-established locations. Furthermore, it is presumed that the recorded vibration response is the superposition of the simultaneous 'assumed' impacts at these locations, with the aim of simultaneously identifying the actual impact location and force magnitude through an under-determined regularisation scheme. The algorithm aims to detect the most probable impact location amongst the spurious locations. Since a normal impact introduces a narrow-band time-localised event with high energy, the wavelet transform is an effective tool to locate this event, with the wavelet coefficient representing how closely correlated the wavelet is with the reconstructed forces. The larger the coefficient is in absolute value, the greater the similarity. As a case study, an under-determined problem with four potential impact locations is considered. The results demonstrate successful localisation and reconstruction of the impact force using both orthogonal and non-orthogonal wavelets.
Keywords: Impact force identification; Inverse problem; Wavelet transform.

1. Introduction

Composite panels are already used widely in the aviation industry but are increasingly deployed in other application domains, primarily due to the high stiffness-to-weight ratio. Due to the nature of the construction, they are in general, however, vulnerable to damage due to various types of impact loadings. Bird strikes, for example, are a major event in aviation and represent a substantial and inevitable safety threat to aircraft [1]. It has been reported that bird strikes can impose costs of more than $1.2 billion on the aviation industry for aircraft repairs and delays or cancellations of flights [2].

Examples of common impact-induced failures in honeycomb sandwich panels include delamination of carbon fibre/epoxy laminate skins, de-bonding of the skins from the core and crushing of core itself. Such failures are not necessarily catastrophic, nor are they detectable by the naked eye through routine visual inspection; the vital need for efficient, low-cost and (semi-) automated structural health monitoring (SHM) systems is therefore demonstrated.

The inverse estimation of determining an impact force is desirable when details as to the nature of the applied force are essential but the impact location is unknown or unreachable for direct measurement. Inverse algorithms make use of the impact responses, such as surface acceleration or strain, which are readily measured by ubiquitous, convenient sensors such as accelerometers or strain gauges attached to the structure distant from the impact location. Complete recognition of an impact force is achieved by determination of both its location and its magnitude (i.e. the force-time history). De-convolving the response signals from the transfer function of the system is the essential component for reconstructing the impact force history [3].

In this study, deconvolution is used to recognise to localise and reconstruct the impact forces applied on a rectangular carbon fibre-epoxy honeycomb composite sandwich panel. A certain number of specific locations on the panel are indicated as potential slots for the occurrence of impact and a single piezoelectric sensor is mounted underneath the panel to record the vibration responses. It is assumed that impact forces are concurrently applied on all potential locations, but the magnitude of all forces except one is zero. The problem aims to identify the actual impact location as well as its magnitude through a minimisation problem. As a result of the minimisation problem, for each potential impact location a reconstructed force is obtained. However, unlike in reality, there may be some non-zero reconstructed forces at spurious locations. Therefore, it is worth designing an automated algorithm capable of detecting the most probable location. As a powerful tool, a wavelet transform using MATLAB is utilised to find the actual force. The technique is based on computing the wavelet coefficients for the reconstructed forces at all potential locations. It is demonstrated that the reconstructed force at the actual location is characterised by the highest wavelet coefficient and energy index. Two experimental case studies with different numbers of unknown impact locations are investigated. The results in both case studies indicate successful identification of the impact location. Moreover, the effect of using different mother wavelets on the performance of the algorithm is studied

2. Inverse algorithm

This While in principle, inverse reconstruction problems may be readily understood, they are not so straightforward to realise with confidence in reality. The challenge stems from the ill-posed nature of the transfer function of the structure and is described as follows. Firstly, let's simply assume that the impact force, $f(t)$, on a structure is mapped to the impact-induced response, $r(t)$, by a linear operator, Ω, where:

$$\Omega[f(t)] = r(t) \tag{1}$$

Through a perturbation investigation, it is shown [4] that:

$$\frac{\|\Delta f(t)\|}{\|f(t)\|} \leq \mathrm{cond}(\Omega) \ \frac{\|\Delta r(t)\|}{\|r(t)\|}, \tag{2}$$

where $\mathrm{cond}(\Omega)$ is the condition number of the linear operator. This number indicates intensification of error in the linear equation. Consequently, any small perturbation in the signal, $\Delta r(t)$, is multiplied by the condition number of Ω, which is usually a very large number. To prevent the reconstructed force $f(t)$ from a large deviation $\Delta f(t)$, the problem must be regularised. It should be noted that Ω is a convolution operator in impact force problems.

Using a convolution operator, Equation (1) can be represented as

$$\int_0^t q(\eta,\alpha,t-\tau) \ f(\eta,\tau) \ d\tau = s(\alpha,t) \tag{3}$$

where $q(\eta,\alpha,t-\tau)$ denotes the transfer function between impact location η and measurement point α and *s* is the dynamic strain response of the system. Discretising Equation (3), using numerical methods, leads to

$$\sum_{i=0}^{n-1} q_{i+1}\ f_{n-i} = s_n \quad , \quad (n=1,...,p), \tag{4}$$

where $s_n\ (n=1,...,p)$ is the response at time $t_n = n.\Delta t$ where Δt is the sampling time and *p* is the number of samples. The matrix form of Equation (4) can be given as $QF = S$ where Q is a lower triangular matrix.

Considering a number of impact forces, $F_i (i=1...M)$ concurrently applied to a panel at certain different locations, the corresponding dynamic strain signal *S* measured at a given point is a superposition of the responses caused by each single impact force.

$$Q^1F_1 + Q^2F_2 + ... + Q^M F_M = \sum_{i=1}^{M} Q^i F_i = S \tag{5}$$

Where Q^i is the transfer function between the force location *i* and the response measurement point. The matrix form of Equation (5) is given by

$$\begin{bmatrix} Q^1 & Q^2 & \cdots & Q^X \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_M \end{bmatrix} = \begin{bmatrix} S \end{bmatrix}, \tag{6}$$

where *M* is the number of impact locations. Since the number of impact locations is greater than the number of sensors (one, in this paper), the problem is under-determined from a mathematical point of view. For simplicity, Eq. (6) is written as $QF = S$. The least-square calculation of the matrix equation is expressed as

$$\min \|QF - S\|_2^2 . \tag{7}$$

Measurement errors and uncertainties in vector *S*, and the large condition number of matrix *Q* create an unstable minimisation problem. As a result, Equation (7) must be regularised. Tikhonov regularisation searches for an appropriate estimate of *F* by swapping Equation (7) with a penalised least-squares problem of the form

$$\min \left\{ \|QF - S\|_2^2 + \varphi \|IF\|_2^2 \right\} \tag{8}$$

where *I* is the identity matrix and $\varphi \geq 0$ is the regularisation parameter, which can be computed through the generalised cross-validation (GCV) method.

3. Wavelet analysis

For a discrete signal *F*(*t*), the continuous wavelet transform (CWT) can be expressed as the convolution of *F*(*t*) and a scaled normalised mother wavelet function as

$$C_\psi^F(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} F(t) \psi^* \left[\frac{(t-b)}{a} \right] dt , \tag{9}$$

where *a* is the scale parameter, *b* is the space parameter (translation), ψ is the mother wavelet and * denotes the complex conjugate.

The CWT is basically a convolution of the input data sequence and a set of functions generated by the mother wavelet. The CWT decomposes a signal at different time and frequency scales, such that each scale may emphasise different signal characteristics. Wavelet coefficients represent how closely correlated the wavelet is with a section of the signal. The larger the absolute value of the coefficient, the greater the similarity.

Due to its multi-resolution properties, the wavelet transform acts as a microscope with the ability to analyse the details of a signal, enabling temporal localisation of signal features e.g. discontinuities. Since the presence of an impact may introduce a time-localised event with narrow-band high energy, the wavelet transform is deemed a powerful tool to locate this event.

It is expected to observe a local variation with extremum of the wavelet coefficients within this time-localised event throughout the different scales. This property is adopted in the present study to identify the source location of impact.

According to the wavelet transform-based energy, an index δ is introduced as

$$\delta = \max\left\| C(a,b,\psi) \right\| \tag{10}$$

where δ denotes the maximum of absolute wavelet coefficients for the entire range of the scale and translation parameters. It is expected that δ reaches its maximum value at the true source location of impact among all the other possible locations, as

$$\Delta = \arg\max(\delta_i), \quad i = 1 : X \tag{11}$$

where Δ corresponds to the location of the source of impact and X is the number of potential impact locations

4. Experimental set-up and procedure

A carbon fibre composite sandwich panel (600 mm in length and 400 mm in width) encompassing 4 woven plies with a nominal thickness of 0.88 mm and a honeycomb core of 20 mm thickness was used in the experiment. The panel was instrumented with a circular PIC 151 piezoceramic disk (10 mm diameter and 1 mm thickness) which was surface-bonded on the underside of the specimen using Loctite Super Glue. Figure 1 depicts the experimental set-up.

A grid was drawn on the specimen, as shown in Figure 2, to specify 12 possible locations of impact, evenly distributed on upper face of the panel. Impact force was applied using a modal hammer and the signals of the piezoelectric sensor and the modal hammer were collected by an oscilloscope. Signal acquisition was set to begin as soon as the impact force produced by the hammer exceeded 20 N. Signal acquisition was performed at a sampling rate of 50 kHz and captured signals for the duration of 10 ms. A complete procedure for establishing the transfer function can be found in previous studies [6-9].

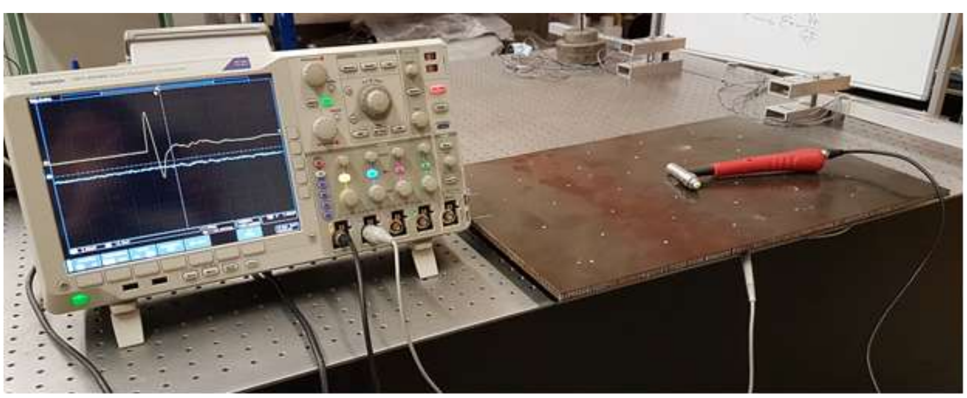

Figure 1. The experimental set-up including the composite panel, impact hammer, oscilloscope, and PZT disk.

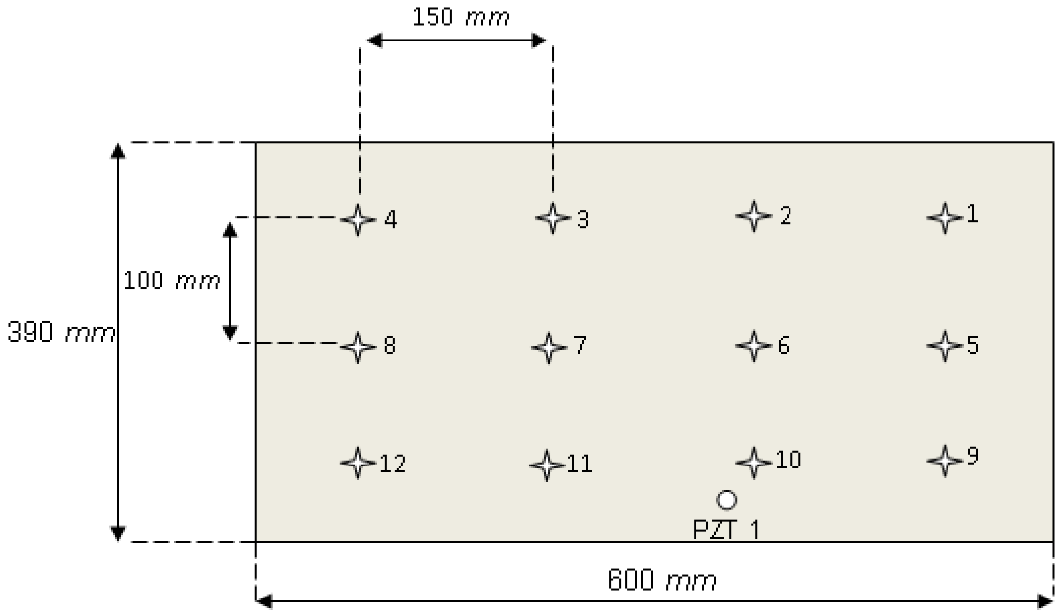

Figure 2. Location of sensors and potential impact forces.

5. Results and discussion

Theoretically, all the reconstructed impact forces but the one at the true impact location must have small magnitudes (around zero). However, this condition did not take place in most of the analyses carried in this study. Qualitative and quantitative evaluation criteria based on the characteristics of the reconstructed forces were already proposed in previous study [7]. Nevertheless, the qualitative analysis necessitated a manual deciding process. As previously mentioned, the wavelet transform provides an automated system for analysing both quantitative and qualitative features of the reconstructed forces.

From a qualitative standpoint, a usual impact force contains localised high-energy components with temple shapes with or without multiple reflections. These features are localised in both time and frequency domains. Based on these characteristics, application of the CWT can be quite advantageous for identifying these time-localised events and scale-localised components.

Additionally, due to the compressive nature of an impact force, there should be no negative portion in the reconstructed impact force. Besides, if there are multiple reflections or local peaks in an impact force due to a usual free strike, the first peak typically carries a higher energy than the next peaks.

In the present study, different mother wavelet functions were adopted. All the mother functions from the Haar, Daubechies, Biorthogonal, Morlet and Symlet wavelet families were tested and successful identification of the impact location was obtained. Because of space constraints, only the results from the 2nd order Daubechies wavelet (db2) are presented.

5.1 Case study: under-determined problem with four potential impact locations

As a case study, the impact is applied at location 3 and four locations, 1, 2, 3 and 4, are considered as potential impact locations.

Figure 3 depicts the reconstructed impact forces at the four possible locations utilising the strain responses captured by the sensor. At first glance, the reconstructed forces at locations 1, 2, and 4 can be disregarded since they are not analogous to a normal temple-shaped impact and have low amplitudes. This finding is discussed in more detail as follows.

The reconstructed impact force at location 1, on one hand, begins with a negative gradient and includes a negative portion which makes it similar to a tensile force. This is contrary to the known fact that an impact results in a compressive force. On the other hand, the amplitude of the produced force is around zero, with a very small maximum peak. In addition, with reference to Figure 3 (b) and (d), the reconstructed forces with low amplitude together with the existence of some negative values of amplitude eliminate locations 2 and 4 from the candidates for actual impact occurrence. The shape of the reconstructed impact force at location 3 resembles a normal impact force, as evident in Figure 3 (c). The maximum peak of the reconstructed impact force at this location (208 N) is remarkably higher than the maximum peaks of the forces at the other locations (all less than 50 N).

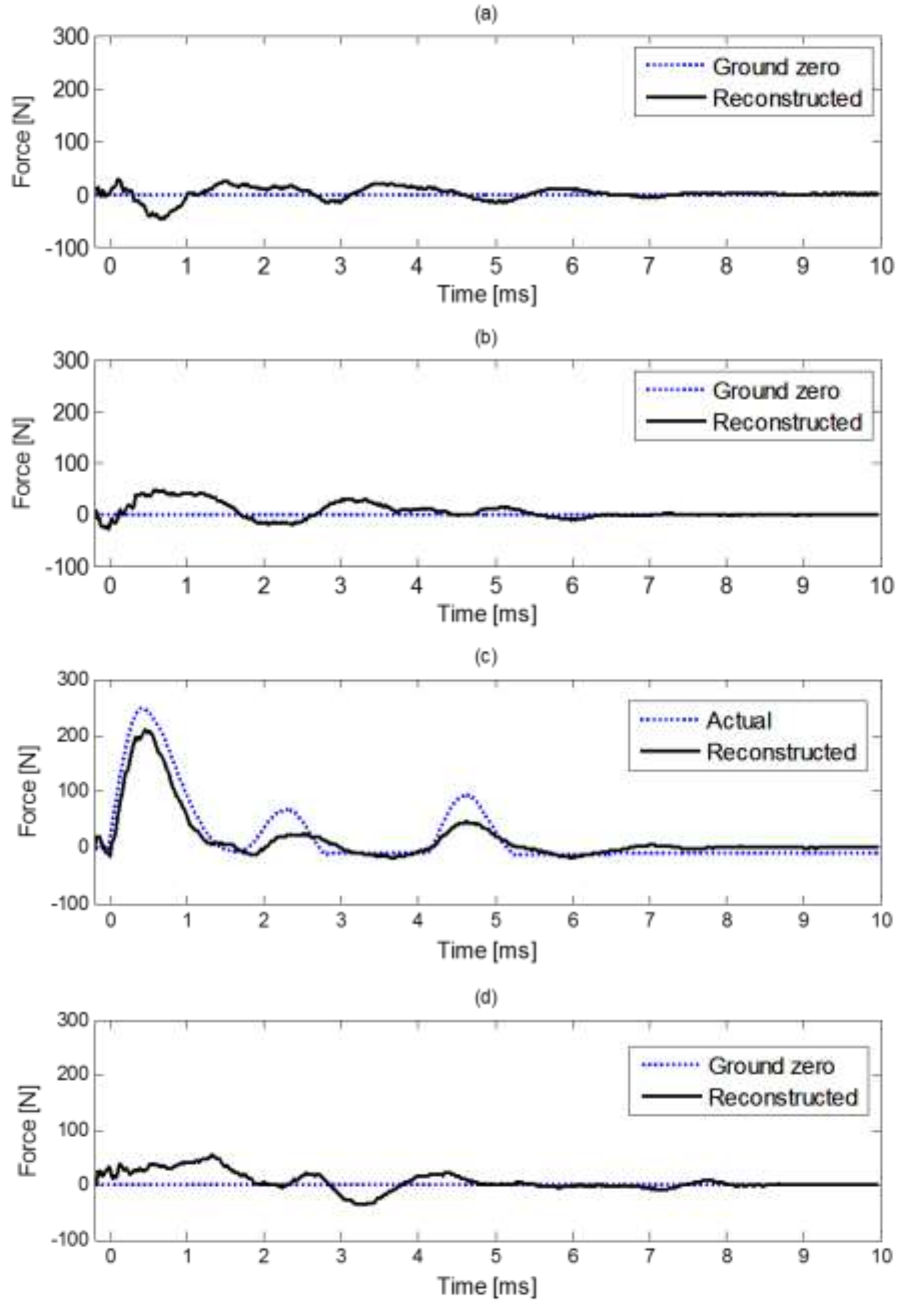

Figure 3. Reconstructed impact forces at (a) location 1, (b) location 2, (c) location 3 and (d) location 4, using dynamic signals captured by sensor 1.

The abovementioned impact identification is now automated through wavelet transform based on time-frequency analysis. The 2nd order Daubechies wavelet (db2) is adopted as the mother wavelet and the wavelet coefficients are obtained using Eq. (9) over scales 1 to 32 a step of 1. Basically, the scale parameter controls the width of the mother wavelet, so that the wavelets are compressed for $|a|<1$ and are dilated for $|a|>1$. The positions of the dilated or compressed wavelets are controlled by the translation parameter *b*. The effect of this shifting and scaling process is to produce a graphic time-scale representation of the wavelet coefficients, as depicted in Figure 4. In Figure 4 (left), the horizontal axis shows the location of the wavelet over the signal and the vertical axis indicates the scale values.

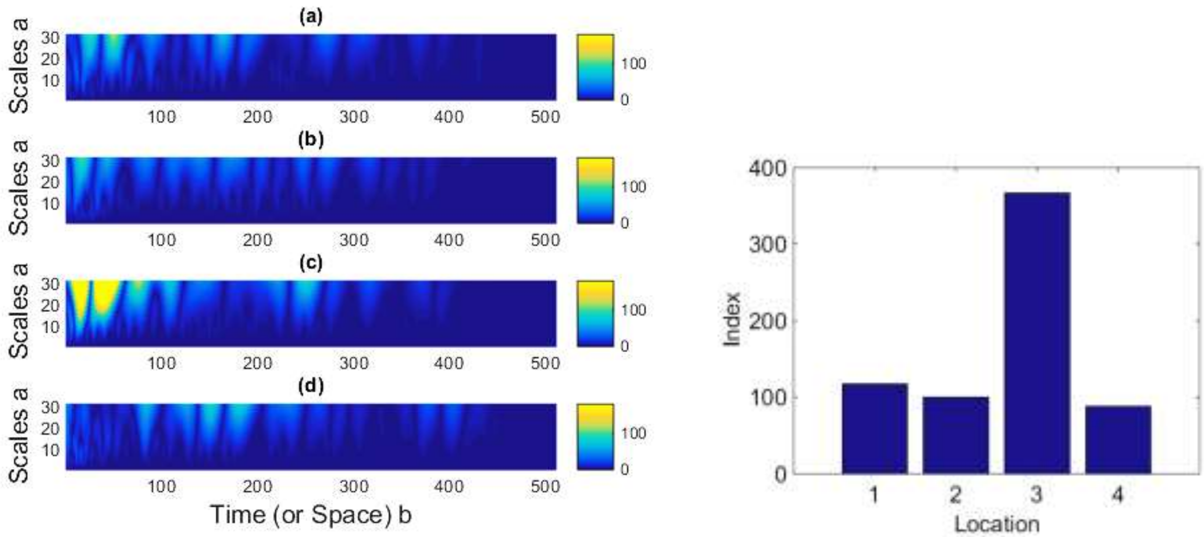

Figure 4. The left figure indicates the wavelet coefficients (a) to (d), respectively, referring to locations 1 to 4; the right figure illustrates the localisation index.

The CWT coefficients plot illustrates patterns among different scales and reveals how the frequency content of the signal changes over time. The higher scales correspond to the most stretched wavelets. The more stretched the wavelet, the longer the portion of the signal with which it is being compared, and thus the coarser the signal features being measured by the wavelet coefficients. Basically, CWT coefficients at lower scales represent the energy of the input signal at higher frequencies, whereas CWT coefficients at higher scales represent energy of the input signal at lower frequencies. Thus, it is advisable to use the advantages of the wavelet transform to track dominant spectral events in the time history of impact forces, i.e. events with high-energy spectra incorporating low-frequency patterns.

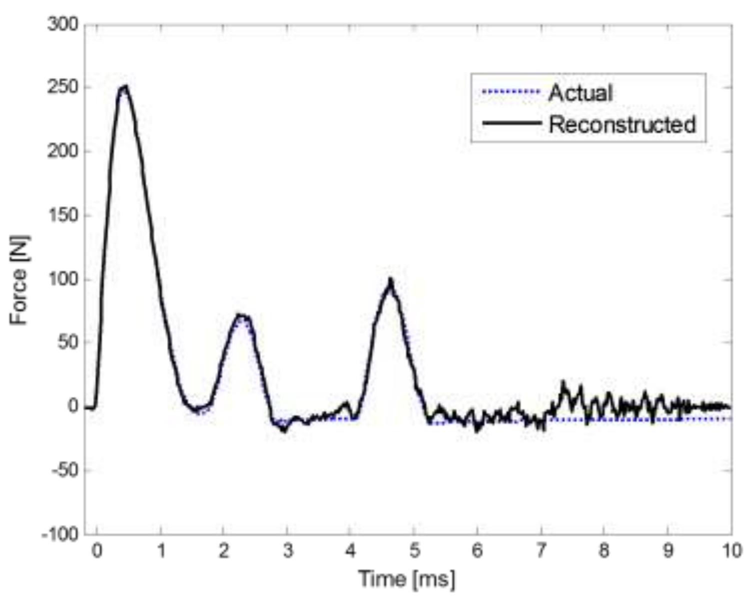

Figure 5. Reconstructed force at location 3 using sensor 1.

It can be seen that the wavelet coefficients for the source impact location produce a high-energy spectral event. These high-energy components are more obvious at higher scales, which correspond to lower frequency components of the signal. In addition, it can be observed that these high-energy events are localised in the time domain and appear at the beginning of the signal, as expected. Figure 4 (right) compares the localisation indices (see Eq. (11)) for the reconstructed forces at locations 1 to 4. As illustrated, the maximum index corresponds with the location of the actual impact, i.e., location 3.

Given the actual impact location, addressing the inverse problem now proceeded to fine-tune the reconstruction of the impact force. Further investigation, reconstructing the impact force history at location 3 through an even-determined problem (a single impact location, i.e. location 3, and a single measurement point), resulted in better resolution in the identified impact force, as illustrated in Figure 5.

6. Concluding remarks

Synchronised identification of location and magnitude of an impact force applied at a location among a set of given potential locations on a composite panel, employing a single piezoelectric sensor, was achieved through solving a regularisation problem. The algorithm was based on simultaneously reconstructing the forces at all potential locations and then choosing the location with a force having normal characteristics. The wavelet transform was adopted to automate the identification process on the basis of calculating the wavelet coefficients for the entire range of the scale and translation parameters. It was illustrated that the reconstructed force at the actual location encompassed the highest wavelet coefficient and energy index. A case study was performed, with four numbers of unknown potential locations. The results demonstrated successful identification of the impact location. Furthermore, the effect of using different mother wavelets on the performance of the algorithm was investigated. It was revealed that all the mother wavelets from both the orthogonal and non-orthogonal families were capable of identifying the location of the actual impact force, indicating the robustness of the proposed technique.

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