# Virtual-Subarray based Angle-of-arrival Estimation in Analog Antenna Arrays

Lowen Qin, J. Andrew Zhang, Xiaojing Huang and Y. Jay Guo

***Abstract*—Angle-of-arrival (AoA) estimation is a challenging problem for analog antenna arrays. Typical algorithms are based on beam scanning which can be time-consuming. In this letter, we propose a virtual-subarray based recursive AoA estimation scheme that can get an AoA estimate from every two measurements and recursively improve the performance by updating beamforming weights with soft probability-based information. Simulation results validate the high efficiency of the proposed scheme, demonstrating its superiority for initial AoA estimation in analog arrays.**

***Index Terms*—Millimeter wave, Beamforming, Angle of arrival estimation**

## I. Introduction

Millimeter wave (mmWave) communication is a candidate technology that can boost the capacity of 5G cellular networks and vehicular networks. In order to support mobility, mmWave systems need to use beam-steerable antenna array to achieve sufficient received signal power, due to the large propagation loss. Hybrid array is an excellent option that can balance the cost and the performance of diversity and multiplexing gain. While hybrid array is an excellent option for base-stations, a beam-steerable analog array may be more practical for user terminals, in terms of size and cost.

Angle-of-arrival (AoA) estimation is a challenging problem for both analog or hybrid mmWave arrays. For hybrid array, AoA estimation technologies are progressing significantly, but there are still only limited solutions for the special localized hybrid arrays due to the phase ambiguity problem [1]. For analog arrays, existing estimation schemes are mostly based on beam scanning and is time-consuming [2]. Beam scanning methods [3] use narrow beams to scan the area of interest, for example, the Discrete Fourier Transform (DFT) codebook-based beam scanning is defined in the IEEE 802.11ad standard. Advanced techniques such as hierarchical multi-resolution sweeping [4] and random sweeping using orthogonal matching pursuit (OMP) compressive sensing (CS) techniques (CS-OMP) [5] have also been developed. However, these methods, with a good review available from [2], still rely on multiple scans and can only get the estimate up to a quantized value.

In this letter, we propose a virtual-subarray based recursive AoA estimation scheme for analog antenna arrays. This scheme can generate an estimate from every two measurements, and recursively update the beamforming (BF) weights using the "soft" probability-based information and generate new improved estimates. Hence it can enable super-fast AoA acquisition, which is critical for communication and localization with high-mobility nodes. The main contributions are as follows: (1) We propose for the first time, to the best of the authors' knowledge, a method that enables direct estimation of non-quantized AoA values using only two measurements, by dividing the single analog array into two virtual subarrays; (2) We formulate and introduce novel "soft decision" estimate via probability density function for the originally non-linear AoA estimation problem and provide a novel way of mapping the estimation accuracy to BF construction; and (3) We propose two methods for updating the probability function of the estimate, and an iterative least square method for updating the BF weights using the soft decision. Simulation results demonstrate that the proposed scheme is capable of achieving quick and good initial estimation for the dominating AoA, even in the presence of other small multipath signals.

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## II. System Model

For simplicity, we consider a uniform linear array (ULA) with $N$ antenna elements, where each element has an adjustable gain and phase shift. The interval between two neighbouring antenna elements is half wavelength.

Typical mmWave channels consists of one dominating line-of-sight (LOS) path and a limited number of non-line-of-sight (NLOS) paths. Let $L$ denote the total number of multipath signals, and let the first multipath $\ell = 1$ be the LOS path. The received signal at the time $t$ is given by

$$y(t) = \sum_{\ell=1}^{L} d_\ell \mathbf{w}^T(t)\mathbf{a}(\theta_\ell)s(t) + z(t) \tag{1}$$

where $(\cdot)^T$ denotes transpose, $z(t)$ is the additive white Gaussian noise (AWGN) with mean zero and variance $\sigma_z^2$; $s(t)$ is the transmitted signal; $d_\ell$ and $\theta_\ell \in [\theta_a, \theta_b]$ are the complex amplitude and the AoA of the $\ell$-th multipath, respectively; $\mathbf{w} = [w_1, w_2, ....., w_N]^T$, $\mathbf{w}^H\mathbf{w} = 1$, is the BF vector with $w_n$ being the BF weight for each antenna element; and $\mathbf{a}(\theta_\ell) = [1, e^{ju_\ell}, \ldots, e^{j(N-1)u_\ell}]^T$ is the array response vector (steering vector) corresponding to the $\ell$-th multipath, with $u_\ell = \pi \sin(\theta_\ell) \in (-\pi, \pi]$. For a ULA, the range of $\theta_\ell$ satisfies $[\theta_a, \theta_b] \subseteq [-\pi/2, \pi/2]$. We assume that $s(t)$ is known, which could happen during the tracking, or during the initial AoA estimation where transceivers are synchronized using, e.g., GPS signals, and a known pseudo random noise (PN) sequence $s(t)$ is transmitted for initial AoA estimation. Our scheme is also directly applicable to the case when $s(t)$ is unknown but repeated over a time period of $T$, although this may not be a common situation.

## III. Virtual-Subarray based AoA Estimation

The basic idea of the proposed scheme is to divide the single analog array into two interleaved virtual subarrays. Assume that the number of antennas $N$ is an even number. The first and second virtual subarrays consist of antenna elements with indexes $(0, 2, \cdots, N-2)$ and $(1, 3, \cdots, N-1)$ respectively, and are called as *even subarray* and *odd subarray* accordingly. We denote the BF vectors for the even and odd subarrays as $\mathbf{w}_e(t)$ and $\mathbf{w}_o(t)$ respectively. Accordingly, we divide the array response vector $\mathbf{a}(\theta_\ell)$ to the even and odd pairs and denote them as $\mathbf{a}_e(\theta_\ell)$ and $\mathbf{a}_o(\theta_\ell)$.

It is easy to see that $\mathbf{a}_o(\theta_\ell) = e^{ju_\ell}\mathbf{a}_e(\theta_\ell)$. This relationship will be exploited for estimating the dominating LOS AoA.

The received signal in (1) can now be represented as

$$\begin{aligned} y(t) &= \sum_{\ell=1}^{L} d_\ell (e^{ju_\ell}\mathbf{w}_o^T(t) + \mathbf{w}_e^T(t))\mathbf{a}_e(\theta_\ell)s(t) + z(t) \\ &= d_1(e^{ju_1}\mathbf{w}_o^T(t) + \mathbf{w}_e^T(t))\mathbf{a}_e(\theta_1)s(t) + I(t) + z(t) \end{aligned}$$

where $I(t) = \sum_{\ell=2}^{L} d_\ell(e^{ju_\ell}\mathbf{w}_o^T(t)+\mathbf{w}_e^T(t))\mathbf{a}_e(\theta_\ell)s(t)$ denotes the combined signals from NLOS paths, and it will be treated as interference in the proposed scheme in this paper.

Next, we propose an iterative scheme for estimating $e^{ju_1}$ and then the AoA $\theta_1$. Each iteration consists of operations over two measurements. For convenience, we let the $k$-th iteration correspond to the measurement times at $t = (2k-1)T$ and $t = 2kT$, $k = 1, \cdots, K$. In the case of known $s(t)$, $T$ can be as small as the symbol period. For simplicity, we will drop $T$ in the signal notations. In the $k$-th iteration , we obtain a pair of measurements $y(2k-1)$ and $y(2k)$ by using correlated BF vectors, and derive an estimate $\hat{u}_{1,k}$ for $u_1$; we then update the BF vectors based on $y(2k-1)$, $y(2k)$ and $\hat{u}_{1,k}$. The process is repeated until a pre-selected stopping condition is met.

### A. Estimate the AoA Term $u_1 = \pi\sin(\theta_1)$

To get $y(2k-1)$ and $y(2k)$, we set the BF vectors

$$\begin{aligned} &\mathbf{w}_o(2k-1) = \mathbf{w}_e(2k-1), \text{ and} \\ &\mathbf{w}_o(2k) = -\mathbf{w}_e(2k) = -\mathbf{w}_e(2k-1). \end{aligned} \tag{2}$$

The actual values of these BF vectors will be optimized later.

Let $b_{\ell,k} = \mathbf{w}_e^T(2k-1)\,\mathbf{a}_e(\theta_\ell)$. The two received signals in the $k$-th iteration are given by

$$\begin{aligned} y(2k-1) &= \sum_{\ell=1}^{L}(1+e^{ju_\ell})b_{\ell,k}d_\ell s(2k-1) + z(2k-1), \\ y(2k) &= \sum_{\ell=1}^{L}(1-e^{ju_\ell})b_{\ell,k}d_\ell s(2k) + z(2k). \end{aligned} \tag{3}$$

When $s(t)$ is known, it can be simply removed from the received signal. We hence consider the situation when $s(2k-1) = s(2k)$ but is unknown. Let

$$\begin{aligned} r_{d,k} &= y(2k-1) - y(2k), \text{ and} \\ r_{s,k} &= y(2k-1) + y(2k). \end{aligned} \tag{4}$$

We then compute the correlation between them as

$$\begin{aligned} &r_{d,k}r_{s,k}^* = 4|b_{1,k}d_1 s(2k)|^2 e^{ju_1} \\ &+2(b_{1,k}d_1 s(2k))^*(2\sum_{\ell=2}^{L} e^{ju_\ell}d_\ell b_{\ell,k}s(2k) + z(2k-1) - z(2k)) \\ &+2e^{ju_1}b_{1,k}d_1 s(2k)(2\sum_{\ell=2}^{L} d_\ell b_{\ell,k}s(2k) + z(2k-1) + z(2k))^* \\ &+(2\sum_{\ell=2}^{L} e^{ju_\ell}d_\ell b_{\ell,k}s(2k) + z(2k-1) - z(2k))\cdot \\ &\quad(2\sum_{\ell=2}^{L} d_\ell b_{\ell,k}s(2k) + z(2k-1) + z(2k))^*, \end{aligned} \tag{5}$$

where $(\cdot)^*$ denote the conjugate operation. In (5), the first term contains the effective signal for the estimation, and the rest terms represent interference and noise.

This calculation is repeated in each of a total of $K$ iterations. During the iterations, we use a forgetting factor $g_{r,k}$ to regulate the contribution of the received signals to the estimation. The averaged signal in the $k$-th iteration is then given by

$$\rho_k = (1-g_{r,k})\rho_{k-1} + g_{r,k}r_{d,k}r_{s,k}^*. \tag{6}$$

Note that $|r_{d,k}r_{s,k}^*|$ reflects the BF gain achieved by the used BF vector $b_{\ell,k}$. A larger value generally indicates the higher likelihood that the BF vector is close to the right AoA. Hence we suggest to use an adaptive and time-varying $g_{r,k}$, given by

$$g_{r,k} = \frac{|r_{d,k}r_{s,k}^*|}{|r_{d,k}r_{s,k}^*| + |\rho_{k-1}|}. \tag{7}$$

When the SNR is not very small and the LOS path has dominating energy, we can get the estimate for $u_1$ from $\rho_k$ as

$$\hat{u}_{1,k} = \angle(\rho_k). \tag{8}$$

### B. Design and Update BF Vector $\mathbf{w}_e(t)$

We define a grid of angles within the area of interest as $\theta_a + m\delta_\theta, m = 1, \cdots, M$, $\delta_\theta = (\theta_b - \theta_a)/(M+1)$. The value of $M$ may be decided according to the ratio between $\theta_b - \theta_a$ and the 3dB BF width of the even subarray. A typical value we propose to use is $M = 2N$ for $\theta_b = -\theta_a = \pi/2$.

Denote the probability mass function (PMF) of $\theta_1$ at $\theta_a + m\delta_\theta$ in the $k$-th iteration as $\bar{p}_{m,k}, m = 1, \cdots, M$. Let $\mathbf{P}_k$ denote a diagonal matrix with the $m$-th diagonal element $\bar{p}_{m,k}$ and $\mathbf{A}$ be an $M \times N/2$ array response matrix with the $m$-th row $\mathbf{a}_e^T(\theta_a + m\delta_\theta)$. Without any prior knowledge, $\bar{p}_{m,1} = 1/M$. After each iteration, we will update $\bar{p}_{m,k}$ and $\mathbf{P}_k$, as will be discussed in Section III-C. The matrix $\mathbf{A}$ can be either pre-determined, or varied on-the-fly with varying $\theta_a$ and $\theta_b$.

We propose to use the PMF function $\bar{p}_{m,k}$ as the magnitude of the desired BF response, and use the iterative least-squares (ILS) method [6] to generate the BF vector. Since we only specify the PMF function as the magnitude of the desired BF response, there are $N/2$-degrees of freedom for determining its phase vector. The ILS method iteratively finds a near-optimal solution for the phase vector of the desired BF response, while computing the LS solution for $\mathbf{w}_e(2k-1)$. This can be formulated as

$$\boldsymbol{\phi}_k^\star = \arg\min_{\boldsymbol{\phi}_k} \| (\mathbf{A}\mathbf{A}^\dagger - \mathbf{I})\mathbf{P}_k\boldsymbol{\phi}_k \|_2^2,$$
$$\mathbf{w}_e^\star(2k-1) = \mathbf{A}^\dagger\mathbf{P}_k\boldsymbol{\phi}_k^\star, \tag{9}$$

where $\mathbf{A}^\dagger$ denotes the pseudo inversion of $\mathbf{A}$, $\boldsymbol{I}$ is an identity matrix, and $\boldsymbol{\phi}_k$ denotes the $M \times 1$ phase shifting vector. A converged solution can be found within a few iterations.

### C. Update Probability Matrix $\mathbf{P}_{k+1}$ for Next Iteration

In each iteration, we estimate the PMF of the AoA $\theta_1$ from the received signals, in order to update the BF vector and achieve higher BF gain in the next iteration.

From (5), the mean SNR can be approximated as

$$\gamma_k = \frac{16|b_{1,k}d_1 s|^4}{\sigma_I^2 + 16|b_{1,k}d_1 s(2k-1)|^2\sigma_z^2 + 4\sigma_z^4}, \tag{10}$$

where in the denominator, the three terms represent the variances of (i) the sum of all terms involving NLOS paths, (ii) the cross-product between the useful signal and the noise, and (iii) the cross-product of the sum and difference of the noise samples. Detailed derivation is omitted due to page limit.Approximating the distribution of $r_{d,k}r_{s,k}^*$ in (5) as a Gaussian function, we can now formulate (5) as a problem of estimating $e^{ju_1}$ in the presence of AWGN with mean 0 and variance $1/\gamma_k$. In this case, we can get the Cramer-Rao Lower Bound (CRLB) for the estimation error of $u_1$ as $1/(2\gamma_k)$ [7]. Note that the mapping between $\theta_a + m\delta_\theta$ and $\exp(j\pi(\theta_a + m\delta_\theta))$ is one-to-one. Therefore, given the estimate $\hat{u}_{1,k}$, the relative probability of an angle $\theta_a + m\delta_\theta$ being the true AoA is given by

$$p_{m,k+1} = \frac{1}{\sqrt{2\pi/\gamma_k}}\exp\left(\frac{-(e^{j\pi\sin(\theta_a+m\delta_\theta)} - e^{j\hat{u}_{1,k}})^2}{2/\gamma_k}\right).$$

Since we do not really know $\gamma_k$, we propose the following two approximation methods. Both methods approximate $|b_{1,k}d_1 s(2k-1)|^2$ by

$$|b_{1,k}d_1 s(2k-1)|^2 \approx \underbrace{(|r_{d,k}|^2 + |r_{s,k}|^2)/2}_{\varepsilon_k}. \tag{11}$$

Approximation using the mean power is statistically better than using either of them, because $|z(2k-1) - z(2k)|$ and $|(z(2k-1) + z(2k))|$ generally have opposite values.

The first method, denoted as the *reconstruction method*, constructs the sum of the interference and noise terms using the estimate $\hat{u}_{1,k}$ and obtains the approximation as

$$\gamma_k \approx \frac{16\varepsilon_k^2}{|\rho_k\exp(-j\hat{u}_{1,k}) - 4\varepsilon_k|^2}. \tag{12}$$

The second method, denoted as the *pre-assumed method*, uses a pre-assumed or calibrated values for the noise variance $\hat{\sigma_z}^2$, and the approximated SNR is given by

$$\gamma_k \approx \frac{1}{\hat{\sigma_z}^2/\varepsilon_k + \frac{1}{4}\left(\hat{\sigma_z}^2/\varepsilon_k\right)^2}. \tag{13}$$

Intuitively, the first method should outperform the second one at a higher SNR since the estimate for $\hat{u}_{1,k}$ has a higher accuracy in this case. At a lower SNR, the second method is expected to perform better. These will be validated by simulation results in Section IV.

We then use a forgetting factor to average $p_{m,k+1}$ over multiple iterations:

$$\bar{p}_{m,k+1} = (1-g_p)\bar{p}_{m,k} + g_p p_{m,k+1}. \tag{14}$$

The initial $\bar{p}_{m,1} = p_{m,1} = 1/M$, and we use a constant $g_p$ here. The value of $g_p$ is chosen to be larger than 0.5 as a new estimate typically has higher accuracy thanks to the improved BF gain.

Intuitively, convergence of the proposed scheme can be guaranteed from the close-loop effect between BF gain and PMF: the BF gain is always increased at the directions for which the PMF increases in the current iteration, and the increased BF gain can then lead to higher PMF in the next iteration. Since $\bar{p}_{m,k}$ is used for allocating the BF gain to different directions, an over-estimate or under-estimate of $\gamma_k$ will generate smaller or broader beamwidth, respectively. A rule of thumb is, if the SNR is high, having an over-estimate and hence a narrow beam is preferred; otherwise a conservative under-estimate is a better option.

## IV. Simulation Results

We simulate a ULA with $N = 16$ antenna elements. The AoAs are randomly generated from a uniform distribution over $[-\pi/3, \pi/3]$. The parameter $M$ is selected as 32. The forgetting factor $g_p = 0.7$. The initial probability $\bar{p}_{m,1} = 1/M$. In the pre-assumed method for approximating $\gamma_k$, we assume that $\hat{\sigma}_z^2$ is 3dB lower than the actual $\sigma_z^2$.

The conventional exhaustive scanning method is used as a benchmark for comparison. In the scanning method, for a given $K$, the ULA forms a single beam to scan $K\times 2$ directions uniformly distributed within $[-\pi/3, \pi/3]$, and then pick up the direction with the maximum received power as the estimate for $\theta_1$. We define the mean square error (MSE) between $e^{ju_1}$ and its estimate as the performance metric.

In Fig. 1, we illustrate how the performance improves with the number of iterations, and also compare the two approximation methods for $\gamma_k$ against the actual one. The reconstruction method outperforms the pre-assumed method when the SNR is sufficiently large. We also present the results for the scanning method, as well as the CS-OMP method [5]. A random phase-only sensing matrix and a dictionary of size 48 are used for CS-OMP. The figure shows that our proposed scheme significantly outperforms the scanning and CS-OMP methods in the first several iterations (up to six here). However, its performance improvement is slower than the scanning method, with $K$ increasing. The curves for the proposed scheme actually saturate when they approach to $10^{-2}$, which are consistent with the CRLB at $\gamma_k = 8{+}10\log_{10}8 \approx 17$dB. This suggests that the proposed scheme is mostly suitable for initial acquisition of AoAs. To verify this, we also present simulation results for a new *switching method* which uses the proposed method with reconstructed $\gamma_k$ in several initial iterations, and

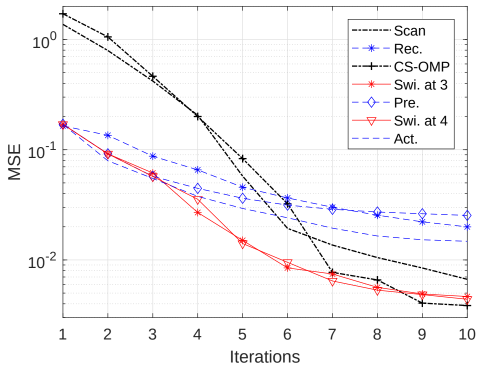


Fig. 1: Comparison between the proposed scheme with different approximations for $\gamma_k$, and several other methods, with SNR $\gamma_0 = 8$dB. Abbreviations in the legends: "Rec.", "Pre." and "Act." are for the proposed method using reconstruction, pre-assumed, and actual values of $\gamma_k$, respectively, "Swi." is for the switching method. The actual values of $\gamma_k$ are computed using the instantaneous signal and noise samples.

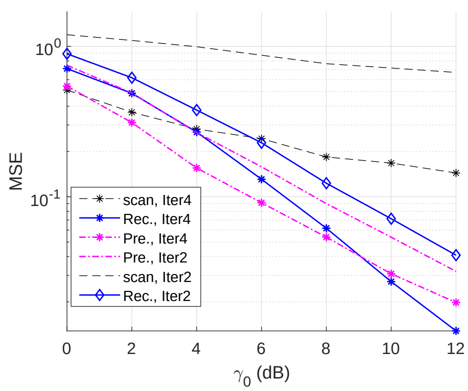


Fig. 2: MSE verus $\gamma_0$ for the scanning method, the proposed scheme with different approximations for $\gamma_k$.

then switch to the scanning method using the estimated AoA as a central direction for scanning. With the known estimated AoA, the scanning here can use a much smaller scanning interval compared to the original scanning method. For a given $K$, the interval is set as $(2/3\pi)/(2(K+5))$ in the simulation here. The two curves for switching at the 3rd and 4th iterations demonstrate that this scheme is very effective.

In Fig. 2, variation of MSE with respect to the base-SNR $\gamma_0 = |d_1|^2\mathrm{E}(|s(t)|^2)/\sigma_z^2$ is demonstrated for the outputs at iteration 2 and 4. With $\gamma_0$ increasing, the proposed scheme demonstrates higher superiority and the performance gap between it and the scanning method increases quickly.

Finally in Fig. 3, we show the MSE in the presence of 3 NLOS paths. The amplitude of each path follows a complex Gaussian distribution with mean zero and variance being 20 and 10 dB lower than the dominating LOS path. The figure indicates that the proposed scheme works well when the ratio is 20 dB, but its performance degrades significantly at 10 dB.

The proposed scheme achieves fast estimation at the cost of

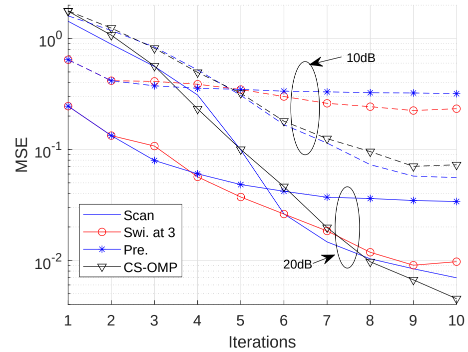


Fig. 3: MSE verus iterations with power ratios of 20dB and 10dB between the LOS and NLOS paths, SNR=8dB.

a moderately increased complexity, compared to the scanning method, but much lower than CS-OMP. The complexity is mainly resulted from the process of updating the BF vector, with the complexity $(O)(M^3)$. The complexity can be reduced by pre-generating BF vectors that correspond to different combinations of BF waveforms [8].

## V. Conclusions

We have presented a virtual subarray based AoA estimation scheme which can directly estimate non-quantized AoA using only two measurements, and recursively update the BF weights using soft probability-based information for achieving improved performance. Simulation results demonstrate that this scheme can achieve excellent initial AoA estimation, and hence is particularly suitable for being used in the first acquisition stage in beam estimation and tracking systems. It can also be extended and applied to, e.g., uniform square array by applying the estimation to each column and row, and hybrid array by forming virtual arrays in each subarray.

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