Dual Pulse Shaping Transmission with Complementary Nyquist Pulses

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***Abstract*—The concept of complementary Nyquist pulse is introduced in this paper. Making use of a half rate Nyquist pulse and its complementary one, a dual pulse shaping transmission scheme is proposed, which achieves full Nyquist rate transmission with only a half of the sampling rate required by conventional Nyquist pulse shaping. This is essential for realizing high-speed digital communication systems with available and affordable data conversion devices. The condition for cross-symbol interference free transmission with the proposed dual pulse shaping is proved in theory, and two classes of ideal complementary Nyquist pulses are formulated assuming raised-cosine pulse shaping. Simulation results are also presented to demonstrate the improved spectral efficiency with dual pulse shaping and compare other system performance against conventional Nyquist pulse shaping.**

***Index Terms*—Nyquist pulse, complementary Nyquist pulse, high-speed digital communications, and data conversion devices.**

I. Introduction

In conventional single carrier communication systems, Nyquist pulse shaping at the transmitter and matched-filtering at the receiver are used to achieve inter-symbol interference (ISI) free transmission and maximize the received signal-to-noise ratio (SNR)[1]. These are the two fundamental principles for digital communications. As a class of Nyquist pulses, raised-cosine pulses are widely used for such single pulse shaping transmission by which a root raised-cosine pulse is used as the spectral shaping pulse at the transmitter and the same root raised-cosine pulse is used as the matched-filter impulse response. A roll-off factor associated with the raised-cosine pulse determines the actual transmitted signal bandwidth which is often wider than the data symbol rate, resulting in reduced spectral efficiency.

With the ever growing demand for high-speed wireless communications, it is necessary to increase the signal bandwidth and improve the spectral efficiency at the same time. For wideband wireless applications such as high-speed point-to-point wireless backhauls and aerial backbone links [2], millimeter wave (mm-wave) communication technology has been proven to be a feasible solution since there is more bandwidth to be used for wireless transmission in mm-wave bands such as the E-band (71-76 and 81-86 GHz) which has two 5 GHz contiguous spectra [3-4]. On the other hand, the devices for digital-to-analog conversion (D/A) and analog-to-digital conversion (A/D) at high sampling rate such as 10 Giga samples per second (Gsps) or higher are lacking or very expensive. In order to achieve high-speed mm-wave communications with 5 GHz or wider signal bandwidth, higher than 5 Gsps sampling rate D/A and A/D devices must be used and the systems have to operate with lower spectral efficiency if the transmitted signals have slow roll-off and hence require excess bandwidth. Adopting frequency division multiplexing (FDM) to divide a wide bandwidth into multiple narrow bands will inevitably result in increased system complexity as well as reduced system performance. Faster-than-Nyquist (FTN) signalling[5-7] has recently attracted significant attention for achieving higher spectral efficiency, but its implementation complexity makes it infeasible to be adopted in high-speed communication systems with commercially available digital signal processing hardware.

This paper proposes a dual pulse shaping (DPS) transmission scheme by which the data symbols to be transmitted are split into two half rate data streams, each passing through a respective pulse shaping filter. This allows for half symbol rate D/A and A/D devices to be used to achieve full rate transmission. The proposed DPS is different from FDM as the two data streams have overlapped signal spectra with DPS whereas a guard band is necessary between two narrow band channels with FDM. Criterion for spectral shaping pulse selection to achieve cross-symbol interference (CSI) free between the two data streams is also revealed and proved in theory. Based on the concept of complementary Nyquist pulse and assuming raised-cosine pulse shaping, two classes of ideal complementary Nyquist spectral shaping pulses satisfying the CSI-free condition are given, i.e., the root complementary raised-cosine pulses and their 90 degree phase shifted versions (or Hilbert transforms). Simulations of the proposed dual pulse shaping transmission system under Gaussian and multipath fading channels are also performed to demonstrate the improved spectral efficiency and characterize other system performance such as peak-to-average power ratio (PAPR) and bit error rate (BER).

The rest of this paper is organized as follows. In Section II, the Nyquist theorem for ISI-free transmission is revisited and the feasibility of FTN is commented. In Section III, the concept of complementary Nyquist pulse is introduced and the dual pulse shaping transmission is proposed. In Section IV, the CSI-free condition is proved and two classes of complementary Nyquist pulses satisfying this condition are formulated. Simulation results are given in Section V to compare the performance between dual pulse shaping and single pulse shaping. Finally, conclusions are drawn in Section VI.

II. Nyquist Pulse Shaping and Spectral Efficiency

A. Nyquist Theorem

Nyquist pulse shaping is one of the fundamental techniques widely used in digital communications, by which the signal pulse $x(t)$, a combination of the transmitter filter, the transmission channel, and the receiver filter (ideally a matched filter), satisfies the ISI-free condition

$$x(nT_s) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \tag{1}$$

where T_s is the symbol duration. In the frequency domain, its Fourier transform $X(f)$ satisfies

$$\frac{1}{T_s}\sum_{k=-\infty}^{\infty} X\left(f - k\frac{1}{T_s}\right) = 1. \tag{2}$$

Suppose that the digital communication system has a bandwidth (single sided) B. It can be shown that only if $T_s \geq \frac{1}{2B}$ there exists numerous choices for $X(f)$ such that the superposition of the overlapping replications of $X(f)$ separated by $\frac{1}{T_s}$ is a constant as indicated by (2).

The well-known Nyquist pulse is the raised-cosine (RC) pulse given in the frequency domain as

$$X_{RC}(f) = \begin{cases} T_s, & |f| \leq \frac{1-\beta}{2T_s} \\ \frac{T_s}{2}\left[2 + cos\frac{\pi T_s}{\beta}\left(|f| - \frac{1-\beta}{2T_s}\right)\right], & \frac{1-\beta}{2T_s} < |f| \leq \frac{1+\beta}{2T_s} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where β is called the roll-off factor and takes values in the range $0 \leq \beta \leq 1$. In the time domain, the pulse, having the raised-cosine spectrum, is

$$x_{RC}(t) = \begin{cases} 1, & t = 0 \\ \frac{sin\pi\frac{1}{2\beta}}{\frac{2}{\beta}}, & |t| = \frac{T_s}{2\beta} \\ \frac{sin\pi\frac{t}{T_s}}{\pi\frac{t}{T_s}}\frac{cos\pi\frac{\beta t}{T_s}}{1-\left(\frac{2\beta t}{T_s}\right)^2}, & \text{otherwise} \end{cases}. \tag{4}$$

Furthermore, in the case where the channel is ideal, if the receiver filter is matched to the transmitter filter, both filters will have the root raised-cosine (RRC) frequency response

$$\begin{aligned} X_{RRC}(f) &= \sqrt{X_{RC}(f)} \\ &= \begin{cases} \sqrt{T_s}, & |f| \leq \frac{1-\beta}{2T_s} \\ \sqrt{T_s} cos\frac{\pi T_s}{2\beta}\left(|f| - \frac{1-\beta}{2T_s}\right), & \frac{1-\beta}{2T_s} < |f| \leq \frac{1+\beta}{2T_s} \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{5}$$

and hence the RRC impulse response

$$\begin{aligned} &x_{RRC}(t) \\ &= \begin{cases} \frac{1}{\sqrt{T_s}}\left(1 - \beta + \frac{4\beta}{\pi}\right), & t = 0 \\ \frac{\beta}{\sqrt{2T_s}}\left[\left(1 + \frac{2}{\pi}\right) sin\pi\frac{1}{4\beta} + \left(1 - \frac{2}{\pi}\right) cos\pi\frac{1}{4\beta}\right], & |t| = \frac{T_s}{4\beta} \\ \frac{1}{\sqrt{T_s}}\frac{sin\pi\frac{t}{T_s}(1-\beta) + 4\beta\frac{t}{T_s} cos\pi\frac{t}{T_s}(1+\beta)}{\pi\frac{t}{T_s}\left[1-\left(\frac{4\beta t}{T_s}\right)^2\right]}, & \text{otherwise} \end{cases}. \end{aligned} \tag{6}$$

B. Spectral Efficiency of Nyquist Pulse Shaping

Apparently, when $\beta \neq 0$, there is an excess bandwidth beyond the symbol rate $\frac{1}{T_s}$. Defining the spectral efficiency (SE) as the symbol rate versus the occupied signal bandwidth (double sided), i.e.,

$$SE = \frac{\frac{1}{T_s}}{\frac{1+\beta}{T_s}} = \frac{1}{1+\beta}, \tag{7}$$

we see that the SE for raised cosine Nyquist pulse shaping is always less then 1 symbol per second per Hz (s/s/Hz).

C. Fast-than-Nyquist Signalling

FTN signalling generates data symbols every ξT_s, $0 < \xi < 1$, at the transmitter. With the same bandlimited Nyquist pulse, the ISI free condition as expressed in Eq. (1) for $x(n\xi T_s)$ will no longer be satisfied. Though the data rate can be increased, additional complexity at the receiver will be needed to deal with the interference between symbols and a sequence of received samples must be processed to make decisions. Moreover, the signal PAPR at transmitter will normally increase, and signal synchronization and channel estimation at the receiver will become more difficult. Obviously, for achieving high-speed low-cost wireless communications, the FTN technique may not be a feasible solution due to the likely unaffordable hardware implementation cost.

III. Complementary Nyquist Pulse

In order to improve the spectral efficiency and achieve Nyquist rate transmission with low complexity, we introduce the concept of *complementary Nyquist pulse* in this Section as follows.

Consider a half Nyquist rate system where the data symbols are transmitted at symbol rate $\frac{1}{2T_s}$. Denote $H_N(f)$ as the frequency domain representation of a Nyquist pulse satisfying the condition $\sum_{k=-\infty}^{\infty} H_N\left(f - k\frac{1}{2T_s}\right) = 1$, where the the scaling factor $\frac{1}{2T_s}$ is ignored for convenience. The complementary Nyquist pulse is defined as

$$H_{CN}(f) = 1 - H_N(f), \text{for} - \frac{1}{2T_s} \leq f \leq \frac{1}{2T_s}. \tag{8}$$

With raised-cosine pulse shaping, $H_N(f)$ and its time domain pulse can have the raised-cosine spectra and raised-cosine waveforms given by

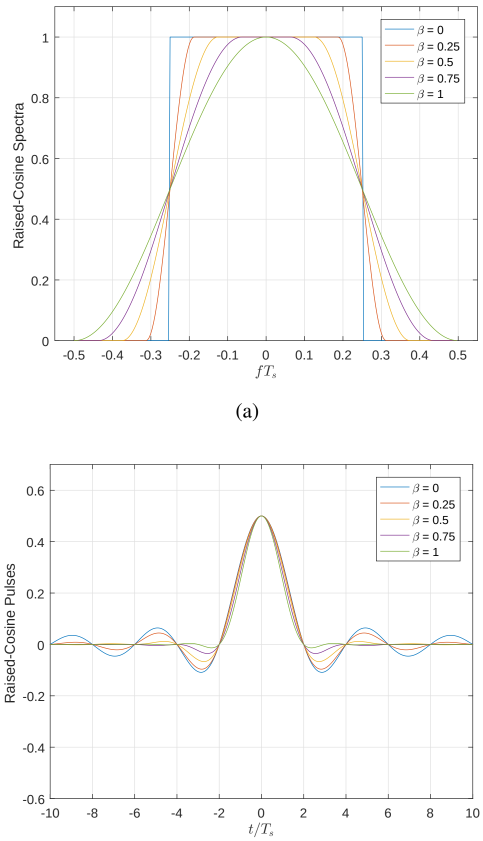
$$H_{RC}(f) = \begin{cases} 1, & |f| \leq \frac{1-\beta}{4T_s} \\ \frac{1}{2}\left[1 + cos\frac{2\pi T_s}{\beta}\left(|f| - \frac{1-\beta}{4T_s}\right)\right], & \frac{1-\beta}{4T_s} < |f| \leq \frac{1+\beta}{4T_s} \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

and

$$h_{RC}(t) = \begin{cases} \frac{1}{2T_s}, & t = 0 \\ \frac{sin\pi\frac{1}{2\beta}}{4\frac{T_s}{\beta}}, & |t| = \frac{T_s}{\beta} \\ \frac{sin\pi\frac{t}{2T_s}}{\pi t}\frac{cos\pi\frac{\beta t}{2T_s}}{1-\left(\frac{\beta t}{T_s}\right)^2}, & \text{otherwise} \end{cases} \tag{10}$$

respectively, where $0 \leq \beta \leq 1$ is the roll-off factor.

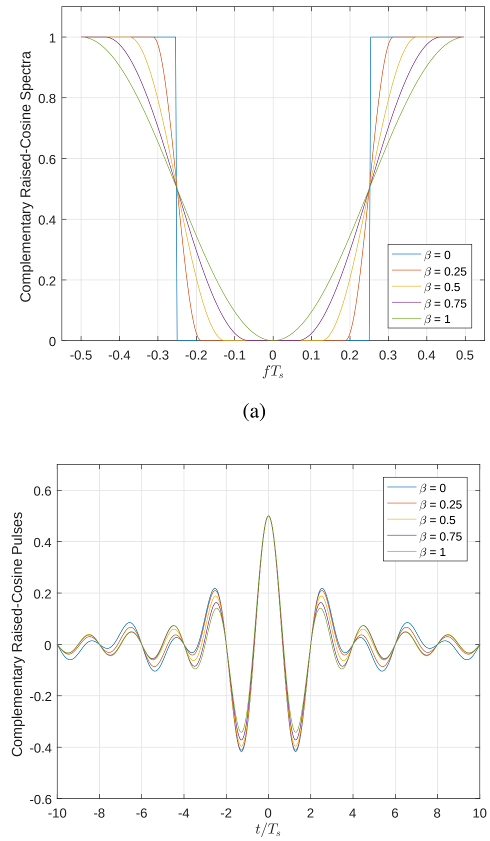
The corresponding complementary raised-cosine spectra and complementary raised-cosine waveforms are then have



(a)

(b)

Fig. 1. Half rate raised-cosine spectra (a) and their corresponding time-domain pulses (b).



(a)

(b)

Fig. 2. Complementary raised-cosine spectra (a) and their corresponding time-domain pulses (b).

the following mathematical expressions

$$\begin{aligned} H_{CRC}(f) &= 1 - H_{RC}(f) \\ &= \begin{cases} 0, & \text{otherwise} \\ \frac{1}{2}\left[1 - cos\frac{2\pi T_s}{\beta}\left(|f| - \frac{1-\beta}{4T_s}\right)\right], & \frac{1-\beta}{4T_s} < |f| \le \frac{1+\beta}{4T_s} \\ 1, & \frac{1+\beta}{4T_s} < |f| \le \frac{1}{2T_s} \end{cases} \end{aligned} \tag{11}$$

and

$$\begin{aligned} h_{CRC}(t) &= \frac{sin\pi\frac{t}{T_s}}{\pi t} - h_{RC}(t) \\ &= \begin{cases} \frac{1}{2T_s}, & t = 0 \\ \frac{sin\pi\frac{1}{\beta}}{\pi\frac{T_s}{\beta}} - \frac{sin\pi\frac{1}{2\beta}}{4\frac{T_s}{\beta}}, & |t| = \frac{T_s}{\beta} \\ \frac{sin\pi\frac{t}{T_s}}{\pi t} - \frac{sin\pi\frac{t}{2T_s}}{\pi t}\frac{cos\pi\frac{\beta t}{2T_s}}{1-\left(\frac{\beta t}{T_s}\right)^2}, & \text{otherwise} \end{cases} \end{aligned} \tag{12}$$

respectively. Fig. 1 and Fig. 2 show $H_{RC}(f)$ and $h_{RC}(t)$ as well as $H_{CRC}(f)$ and $h_{CRC}(t)$ respectively for $\beta = 0,\ 0.25,\ 0.5,\ 0.75,\ \text{and}\ 1$.

We see that the Nyquist spectrum and its complementary one for any given roll-off factor demonstrate a very interesting property, that is, the sum of the two spectra is always a constant within the signal bandwidth equivalent to a full rate Nyquist spectrum with zero roll-off factor. This implies that if two independent half rate data streams are transmitted with the Nyquist pulse shaping and complementary Nyquist pulse shaping respectively, called *dual pulse shaping transmission*, full Nyquist rate can be achieved without excess bandwidth.

IV. CSI-Free Condition for Dual Pulse Shaping Transmission

In general, a dual pulse shaping transmission system can transmit two independent data streams with any pair of spectral shaping pulses as long as the pair of spectral shaping pulses satisfy certain ISI-free and cross-symbol interference (CSI) free conditions.

Suppose that the symbol rate for each parallel data stream is $\frac{1}{2T_s}$ and the symbol interval is $2T_s$. Denote the frequency representations of the two spectral shaping pulses as $H_1(f)$ and $H_2(f)$ respectively. Obviously, after matched filtering, the pulse frequency responses $|H_1(f)|^2$ and $|H_2(f)|^2$ must satisfy the Nyquist ISI-free condition

$$\sum_{k=-\infty}^{\infty}\left|H_1\left(f - k\frac{1}{2T_s}\right)\right|^2 = 1 \tag{13}$$

and

$$\sum_{k=-\infty}^{\infty}\left|H_2\left(f - k\frac{1}{2T_s}\right)\right|^2 = 1 \tag{14}$$

respectively, where a scaling factor $\frac{1}{2T_s}$ is ignored for convenience. In addition, to prevent the two data streams from interfering each other, $H_1(f)$ and $H_2(f)$ also need to satisfy a further CSI-free condition. Hence, we have the following theorem.

Theorem (CSI-free condition): The necessary and sufficient condition for a dual pulse shaping transmission system with pulse shaping spectra $|H_1(f)|^2$ and $|H_2(f)|^2$ to satisfy cross-symbol interference free is that

$$\sum_{k=-\infty}^{\infty} H_1\left(f-k\frac{1}{2T_s}\right) H_2^*\left(f-k\frac{1}{2T_s}\right)=0. \quad (15)$$

Proof: Consider the interference from the first data stream to the second one. Since the matched filter used for receiving the second data stream is $H_2^*(f)$, the interference caused by the signal pulse in the first data stream can be expressed as

$$z(t)=\int_{-\infty}^{\infty} H_1(f) H_2^*(f) e^{j2\pi ft} df. \quad (16)$$

At any sampling instant $t=n\times 2T_s$, the interference is

$$z(n\times 2T_s)=\int_{-\infty}^{\infty} H_1(f) H_2^*(f) e^{j4\pi nfT_s} df. \quad (17)$$

Breaking up the integral into integrals covering the finite range of $\frac{1}{2T_s}$, we obtain

$$\begin{aligned} z(n\times 2T_s) &= \sum_{k=-\infty}^{\infty}\int_{(2k-1)/4T_s}^{(2k+1)/4T_s} H_1(f) H_2^*(f) e^{j4\pi nfT_s} df \\ &= \sum_{k=-\infty}^{\infty}\int_{-1/4T_s}^{1/4T_s} H_1\left(f-k\frac{1}{2T_s}\right) H_2^*\left(f-k\frac{1}{2T_s}\right) e^{j4\pi nfT_s} df \\ &= \int_{-1/4T_s}^{1/4T_s}\left[\sum_{k=-\infty}^{\infty} H_1\left(f-k\frac{1}{2T_s}\right) H_2^*\left(f-k\frac{1}{2T_s}\right)\right] e^{j4\pi nfT_s} df. \end{aligned} \quad (18)$$

To ensure $z(n\times 2T_s)=0$ for any n, the necessary and sufficient condition is for (15) to be satisfied.

Similarly, to ensure interference free from the second data stream to the first one, the same condition $\sum_{k=-\infty}^{\infty} H_1^*\left(f-k\frac{1}{2T_s}\right) H_2\left(f-k\frac{1}{2T_s}\right) = \sum_{k=-\infty}^{\infty} H_1\left(f-k\frac{1}{2T_s}\right) H_2^*\left(f-k\frac{1}{2T_s}\right) = 0$ should be satisfied. ■

When the Nyquist pulse shaped and complementary Nyquist pulse shaped spectra $H_N(f)$ and $H_{CN}(f)$ are used for the two parallel symbol streams in the dual pulse shaping transmission system, the two signal pulses can be selected as a root Nequist pulse and a T_s-delayed root complementary Nyquist pulse, i.e., $H_1(f)=\sqrt{H_N(f)}$ and $H_2(f)=\sqrt{H_{CN}(f)}e^{-j2\pi fT_s}$. It can be verified that these two pulses satisfy the conditions specified by Eqs. (13), (14), and (15).

Alternatively, the two pulses can be selected as $H_1(f)=\sqrt{H_N(f)}$ and $H_2(f)=\begin{cases} -j\sqrt{H_{CN}(f)}, f\geq 0 \\ j\sqrt{H_{CN}(f)}, \quad f<0 \end{cases}$ which is a 90 degree phase shifted version (or Hilbert transform) of $\sqrt{H_{CN}(f)}$. It can be also verified that these two pulses satisfy the conditions specified by Eqs. (13), (14), and (15).

For the root raised-cosine spectra and pulses given by

$$\begin{aligned} H_{RRC}(f) &= \sqrt{H_{RC}(f)} \\ &= \begin{cases} 1, & |f|\leq\frac{1-\beta}{4T_s} \\ cos\frac{\pi T_s}{\beta}\left(|f|-\frac{1-\beta}{4T_s}\right), & \frac{1-\beta}{4T_s}<|f|\leq\frac{1+\beta}{4T_s} \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad (19)$$

and

$$\begin{aligned} &h_{RRC}(t) \\ &= \begin{cases} \frac{1}{2T_s}-\frac{\beta}{2T_s}+\frac{2\beta}{\pi T_s}, & t=0 \\ \frac{\beta}{2\sqrt{2}T_s}\left[\left(1+\frac{2}{\pi}\right)sin\pi\frac{1}{4\beta}+\left(1-\frac{2}{\pi}\right)cos\pi\frac{1}{4\beta}\right], & |t|=\frac{T_s}{2\beta} \\ \frac{sin\pi\frac{t}{2T_s}(1+\beta)-2\beta\frac{t}{T_s}cos\pi\frac{t}{2T_s}(1-\beta)}{\pi t\left[1-\left(\frac{2\beta t}{T_s}\right)^2\right]}, & \text{otherwise} \end{cases} \end{aligned} \quad (20)$$

respectively, the corresponding root complementary raised-cosine spectra and pulses are given by

$$\begin{aligned} H_{RCRC}(f) &= \sqrt{H_{CRC}(f)} \\ &= \begin{cases} 0, & \text{otherwise} \\ cos\frac{\pi T_s}{\beta}\left(|f|-\frac{1+\beta}{4T_s}\right), & \frac{1-\beta}{4T_s}<|f|\leq\frac{1+\beta}{4T_s} \\ 1, & \frac{1+\beta}{4T_s}<|f|\leq\frac{1}{2T_s} \end{cases} \end{aligned} \quad (21)$$

and

$$\begin{aligned} &h_{RCRC}(t)= \\ &\begin{cases} \frac{1}{2T_s}-\frac{\beta}{2T_s}+\frac{2\beta}{\pi T_s}, & t=0 \\ \frac{sin\pi\frac{1}{2\beta}}{\pi\frac{T_s}{2\beta}}-\frac{\beta}{2\sqrt{2}T_s}\left[\left(1+\frac{2}{\pi}\right)sin\pi\frac{1}{4\beta}-\left(1-\frac{2}{\pi}\right)cos\pi\frac{1}{4\beta}\right], & |t|=\frac{T_s}{2\beta} \\ \frac{sin\pi\frac{t}{T_s}}{\pi t}-\frac{sin\pi\frac{t}{2T_s}(1+\beta)-2\beta\frac{t}{T_s}cos\pi\frac{t}{2T_s}(1-\beta)}{\pi t\left[1-\left(\frac{2\beta t}{T_s}\right)^2\right]}, & \text{otherwise} \end{cases} \end{aligned} \quad (22)$$

respectively. The root complementary raised-cosine pulses $h_{RCRC}(t)$ with different roll-off factors are shown in Fig. 3. We see that they are all even functions of time.

The convolution between $h_{RRC}(t)$ and $h_{RCRC}(t-T_s)$ can be expressed as $h_{RRC}(t)*h_{RCRC}(t-T_s)=\frac{1}{\pi T_s}\frac{\beta}{1-\left(\beta\frac{t-T_s}{T_s}\right)^2}sin\pi\frac{t}{2T_s}cos\pi\beta\frac{t-T_s}{2T_s}$ which is zero at $t=\pm 2T_s, \pm 4T_s, \cdots$, meaning that there is no CSI between the two parallel symbol streams if $h_{RRC}(t)$ and $h_{RCRC}(t-T_s)$ are used as the transmission pulses for the two parallel symbol streams.

Corresponding to the 90 degree phase shifted version of $h_{RCRC}(t)$ with spectra $H_{ORCRC}(f)=\begin{cases} -j\sqrt{H_{CRC}(f)}, f\geq 0 \\ j\sqrt{H_{CRC}(f)}, \quad f<0 \end{cases}$, the odd root complementary raised-cosine pulses are expressed as

$$\begin{aligned} &h_{ORCRC}(t)= \\ &\begin{cases} 0, & t=0 \\ -\frac{cos\pi\frac{1}{2\beta}}{\pi\frac{T_s}{2\beta}}+\frac{\beta}{2\sqrt{2}T_s}\left[\left(1-\frac{2}{\pi}\right)sin\pi\frac{1}{4\beta}+\left(1+\frac{2}{\pi}\right)cos\pi\frac{1}{4\beta}\right], & t=\frac{T_s}{2\beta} \\ \frac{cos\pi\frac{1}{2\beta}}{\pi\frac{T_s}{2\beta}}-\frac{\beta}{2\sqrt{2}T_s}\left[\left(1-\frac{2}{\pi}\right)sin\pi\frac{1}{4\beta}+\left(1+\frac{2}{\pi}\right)cos\pi\frac{1}{4\beta}\right], & t=-\frac{T_s}{2\beta} \\ -\frac{cos\pi\frac{t}{T_s}}{\pi t}+\frac{cos\pi\frac{t}{2T_s}(1+\beta)+2\beta\frac{t}{T_s}sin\pi\frac{t}{2T_s}(1-\beta)}{\pi t\left[1-\left(\frac{2\beta t}{T_s}\right)^2\right]}, & \text{otherwise} \end{cases} \end{aligned} \quad (23)$$

which are shown in Fig. 4.

The convolution between $h_{RRC}(t)$ and $h_{ORCRC}(t)$ can be expressed as $h_{RRC}(t)*h_{ORCRC}(t)=\frac{1}{\pi T_s}\frac{\beta}{\left(\beta\frac{t}{T_s}\right)^2-1}sin\pi\frac{t}{2T_s}cos\pi\beta\frac{t}{2T_s}$ which is zero at $t=\pm 2T_s, \pm 4T_s, \cdots$, meaning that there is no CSI between the two parallel symbol streams if $h_{RRC}(t)$ and $h_{ORCRC}(t)$ are used as the transmission pulses for the two parallel symbol streams.

The above root raised-cosine and root complementary raised-cosine pulse shaping spectra are band-limited with

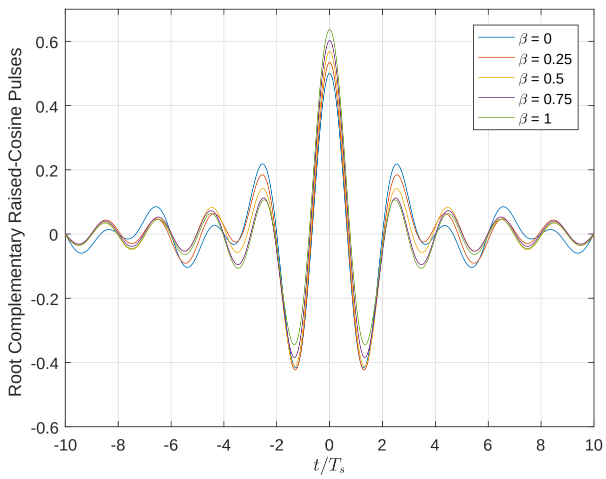


Fig. 3. Root complementary raised-cosine pulses.

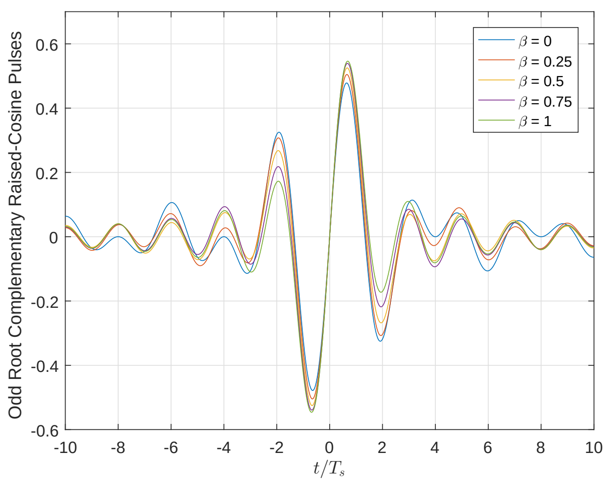


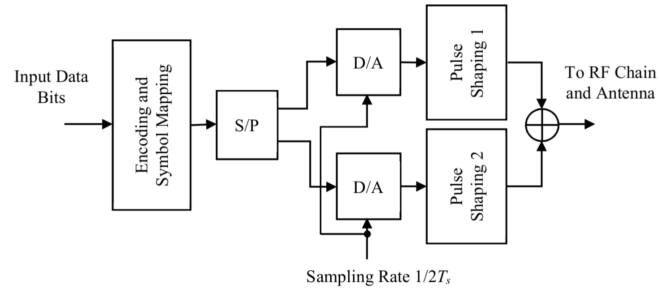
Fig. 4. Odd root complementary raised-cosine pulses.

bandwidth $\frac{1}{T_s}$. This means that they can jointly achieve Nyquist rate transmission regardless of the roll-off factor. Also note that a root complementary raised-cosine spectrum has a sharp transition at the two edges of the pass-band, which represents an ideal condition to ensure ISI and CSI free. In fact, transition from pass-band to stop-band is necessary for any practical pulse, and hence equalization is generally required to cancel both ISI and CSI.

V. A Half Sampling Rate Dual Pulse Shaping Transmission System

A. System Model

With dual pulse shaping transmission, the data rate in each parallel data stream only needs to be half of the total data rate. This allows for lower sampling rate to be used in the digital domain. Such a dual pulse shaping transmission system is shown in Fig. 5. Suppose that the data symbol rate is $\frac{1}{T_s}$. The D/A and A/D converters in the respective transmitter and receiver only require a sampling rate of $\frac{1}{2T_s}$.



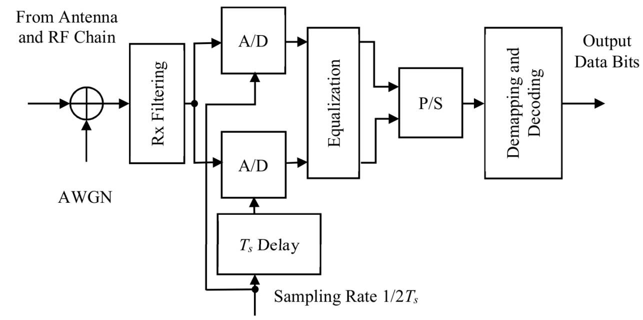


Fig. 5. Block diagrams of transmitter (upper) and receiver (lower) of half sampling rate dual pulse shaping transmission system.

The input data bits after encoding and symbol mapping are split into two streams by serial-to-parallel conversion (S/P), followed by dual pulse shaping. The spectral shaping pulses can be ideally selected as the non-causal complementary Nyquist pulse pairs. Other practical (causal) pulses can be selected as long as proper equalization techniques (which will be presented separately [8]) are used in the receiver. After equalization, the received two parallel data streams are merged into one by parallel-to-serial conversion (P/S), followed by demapping and decoding to retrieve the output data bits.

B. Performance Simulation Results

To demonstrate the feasibility and performance of the proposed dual pulse shaping transmission system for high-speed wireless applications, we now present the simulation results assuming a 25 Gbps millimeter wave communication system with 5 GHz bandwidth operating in the 71-76/81-86 GHz E-band. The sampling rate required for this system is only 2.5 Gsps which is easily available with commercial A/D and D/A devices at low cost. The 64-ary quadrature amplitude modulation (64-QAM) is used to provide 6 bps/Hz spectral efficiency over 5 GHz bandwidth, achieving 30 Gbps raw data rate including some necessary overhead for synchronization, channel estimation, and other system functionalities.

We firstly compare the spectral efficiency between the dual pulse shaping and conventional raised-cosine Nyquist pulse shaping referred to as single pulse shaping thereafter. As shown in Fig. 6, with single pulse shaping, the spectral efficiency drops from 1 to 0.5 symbols/second/Hz when the roll-off factor varies from 0 to 1, whereas the spectral efficiency is always 1 symbol/second/Hz regardless of the roll-off factor for dual pulse shaping.

Secondly, the peak-to-average power ratio is compared between dual pulse shaping and single pulse shaping, and

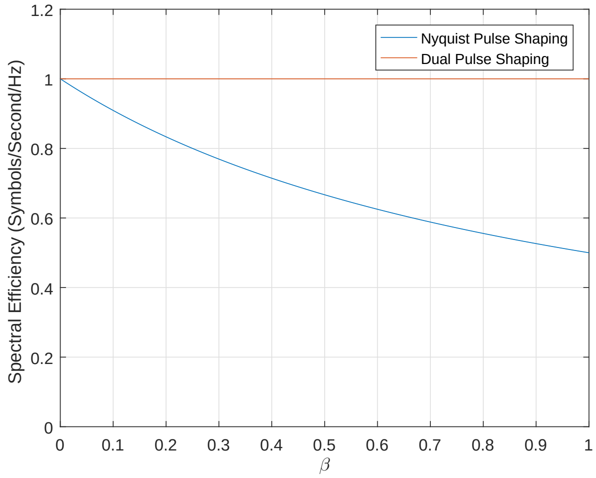


Fig. 6. Spectral efficiency (SE) comparison.

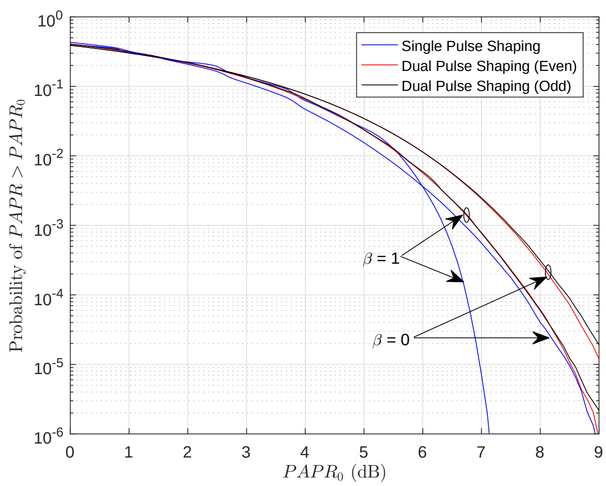


Fig. 7. Peak-to-average power ratio (PAPR) comparison.

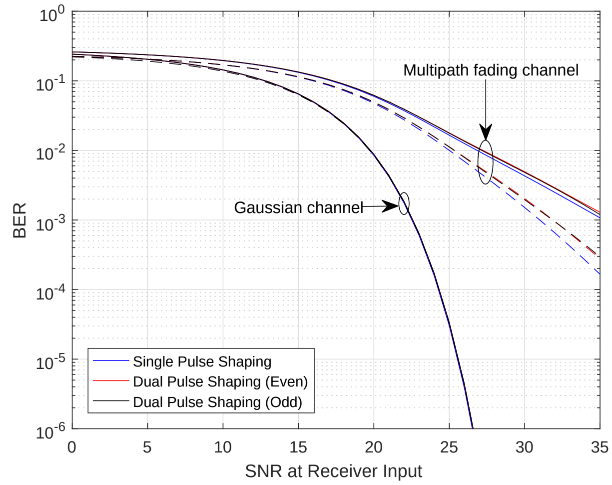


Fig. 8. Bit error rate (BER) comparison (solid lines for ZF equalization and dashed lines for MMSE equalization).

the results are shown in Fig. 7. We see that the single pulse shaping generally has better PAPR performance than dual pulse shaping. With roll-off factor of 1, both schemes demonstrate their respective best performance. However, for single pulse shaping, this comes with the maximum excess bandwidth. As we can always use larger roll-off factor for dual pulse shaping, the performance gap can be significantly reduced.

Finally, the BER performance (uncoded) is compared between the two pulse shaping schemes using zero-forcing (ZF) and minimum mean square error (MMSE) equalizations respectively. As seen from Fig. 8, both schemes demonstrate the same performance in Gaussian channel with only additive Gaussian noise. In multipath fading channel where a two-ray model is assumed and the second path has random reflection with 3 dB lower average power and a delay of 6 ns, the dual pulse shaping scheme has slight performance degradation which is almost negligible especially when the more practical ZF equalization is used.

VI. Conclusions

By introducing the concept of complementary Nyquist pulse, a dual pulse shaping transmission scheme is proposed, which can achieve Nyquist rate with half rate sampling. The cross-symbol interference free condition is proved in theory and two classes of ideal ISI and CSI free spectral shaping pulses are formulated assuming the root raised-cosine spectra. Simulation results show that the dual pulse shaping transmission system can provide higher spectral efficiency with only slight degradation on PAPR and negligible impact on BER performance as compared with conventional Nyquist pulse shaping. For achieving extremely high-speed wireless communications with tens or even hundreds of Gigabits per second data rate when high-speed data conversion devises are unattainable or costly, the proposed dual pulse shaping transmission is certainly a viable and low cost solution.

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