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Estimating Design Flood Magnitude for a Vietnamese Catchment

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Abstract

Flood is a common phenomenon in many tropical countries. Estimation of design flood flow has been a concern for many years in both hydrologic research and in hydrologic practice. Design flood magnitudes provide a basis for sustainable flood management which has the aims of reducing flood risk, and protecting people's lives and property. Design flood magnitudes for a given location can be estimated by a number of approaches including analysis of past flood statistics or the use of catchment modelling approaches like design storm methods or continuous simulation. The aim of this paper is to apply Annual Maximum Series fitting method for design flood estimation in continuous simulation with particular reference to a monsoon catchment. In this aspect, the annual maximum series was used as a performance measure rather than reproduction of individual hydrographs. This approach was used as the focus was on reproducing the observed frequency curve. For this purpose, a case study is performed for a large catchment, namely the Ba River, located in central Vietnam. This catchment is subject to a monsoonal climate and also to tropical cyclones.

Keywords: Flood flow, hydrology, flood frequency analysis, continuous simulation.

1. INTRODUCTION

Flood risk assessment is widely used in water resources management including, for example, design of hydraulic structures, flood plain management, and environmental and ecological studies (Haddad and Rahman 2012; Haddad et al. 2012). Techniques for design flood estimation have been developed in many countries. General approach for this estimation includes statistical analyses of observed peak discharges and modelling catchment system using rainfall-runoff simulation. The overview of approaches for flood estimation was developed in some studies for example (Ball 2011; Beven and Binley 2014; Pilgrim 1987; Руководств 1973 and Ha 2008). For Vietnamese catchments Ha, 2008 summarized methods used and classified as 3 main approaches that may be adopted as situations where long records of gauged streamflow data are available; situations where no data are available; and situations where there is inadequate data.

Recently, Ball (2011) summarized the methods used for flood estimation as being divided into 2 cases: sufficient historical information is available and insufficient historical information is available. In the case where sufficient flood data is available, flood frequency analysis (FFA) methods can be used. Application of this method requires the use of recorded data to select and fit a probability model for flood peaks. Guidelines for application of the method are provided by Pilgrim (1987) and Bulletin 17B (Interagency Advisory Committee on Water Data, 1982). An alternative approach for ungauged and poorly gauged catchments is the application of hydrological models whereby the transformation of rainfall into runoff is simulated. Catchment simulation can be performed as a single burst simulation, a Monte-Carlo burst simulation, or a continuous simulation.

The degree of belief in predictions will normally depend on how well they can reproduce observations; the reproduction of observations usually is assessed by a performance measure. The Nash-Sutcliffe efficiency (a most widely used performance measure in hydrology) has been used in many studies for both single event based simulations and for continuous simulations. How appropriate this criterion is for measuring goodness of fit, as well as what is an acceptable value, has been debated in the literature (see Criss and Winston 2008; Gupta et al. 2009; Krause, Boyle and Bäse 2005; Legates and McCabe 1999; Seibert 2001). Additionally, modified versions of the Nash-Sutcliffe criterion have been proposed by, for example, Garrick et al. (1978), Krause et al. (2005), Legates and McCabe (1999), McMillan and Clark (2009), Refsgaard and Knudsen (1996), Schaefli and Gupta (2007), and Seibert (2001); the conclusion derived from this discussion is that the measure often is influenced by the performance at specific flow magnitudes. With continuous simulation, Westerberg et al. (2011) claims the suitability of performance measures is more challenging due to the sensitivity of different performance measures to different flow magnitudes and to the need for overlapping of discharge and model input data. This later issue has been addressed by the use of a time series approach where the annual maxima series (AMS) (Cameron et al. 2000; Lamb 1999) or a duration curve (Westerberg et al. 2011) are used as the performance measure for calibration purposes. Furthermore, the concept of using combined multi-criteria approaches has been developed in Lamb (1999), where the best parameter set was selected by analysis of four alternative measures which included the Nash Sutcliffe coefficient (1), sum of weighted absolute errors (2), error of the ranked pair of peak series combining the timing and magnitudes (3) and errors in the magnitudes of the ranked pairs of peak series, irrespective of timing (4).

The importance of the metric used to assess the suitability of parameter values for design flood estimation was discussed by Ball (2013) who suggested that the rationale for catchment simulation is the development of a relationship between the flood hydrograph peak and AEP. Highlighted by Ball (2013) was this relationship derived from both recorded data and simulated data. For the simulated data, event-based and continuous simulation approaches calibrated by assessing against individual hydrographs were considered. It was found that, for both modelling approaches, there was no coincidence of the quantile flow from the modelling with that from the recorded data. The conclusion was the measure of fit (hydrograph fitting) was not appropriate for reliable prediction of the flood quantiles from continuous simulation, even thought the catchment modelling system generated flow sequences were calibrated and validated.

An alternative fitting metric for design flood magnitude estimation with a continuous simulation approach is proposed herein; this metric is based on the Annual Maxima Series (AMS) and is tested in a Vietnamese catchment subject to monsoonal climatic conditions. The likelihood function of this AMS fitting method is related to the three parameters of defining the distribution fitted to the AMS; namely the location; scale and shape parameters. The most probable values and acceptable ranges of the parameters for the catchment modelling system were estimated by a modified Bayesian technique.

2. METHODOLOGY

The design flood magnitudes are estimated by a modified Bayesian method based on Kuczera et al. (2006). The design flood magnitudes are estimated by FFA which is based on analysis of annual maxima series. The method considers a set of AMSs (D), hypothesized and a random realization from the probability model M, with probability density function pdf p(D|β,M) where β is an unknown finite-dimensioned parameter vector. In Bayesian inference, the parameter vector β is considered to be a random vector, the probability distribution of which describes the true value of β and the prior pdf p(β|M) for given probability model M. However, known β can be used as subjective to refine β. In this aspect, the AMSs are treated as a set of data, the true value β of which can be described by the density function. The posterior distribution $p(\beta|D, M)$ fully defines the parameter uncertainty and is sampled by the importance sampling method described in (Kuczera et al. 2006).

The modified Bayesian approach applied this importance sampling method for sampling procedure of this posterior distribution p(β|D,M). The method identifies the most probable values of parameter sets β and confidence limits describing the uncertainty arising from uncertainty in the fitted parameters. The method involves in three steps: Find most probable posterior parameters; Multinormal approximation to posterior Distribution and Importance sampling of posterior distribution and Plotting the curves including: expected probability curve; expected parameter curve and; quantile confidence limits. The expected parameter curve is derived from the most probable values of the parameter β and the expected probability curve is drawn from most probable quantiles for each exceedence probability (1 in Y AEP).

The method is applied at 2 gauges Ankhe and Cungson to estimate most probable values and confidence limits of parameter β. These ranges will be used as a likelihood function to calibrate and validate the catchment modelling system.

3. CASE STUDY- TEST CATCHMENT

3.1. Location

The Ba river, Vietnam was chosen as a test catchment for the analysis being presented herein. The River is one of the largest river systems in central Vietnam located in South Central Vietnam. The catchment area of the river is 13,900km^2. The total length of the river is 347km with its headwaters in Kon Tum province and ultimately flowing into the South Sea at Tuy Hoa in Phu Yen Province (KTTV&MT 2010). Major tributaries of the Ba River include Hinh River and Ayunpa River.

3.2. Climate Conditions and flood characteristics

The catchment is located in a tropical monsoonal climatic regime. The main features of this climate regime are extraordinarily rainy wet seasons and pronounced dry seasons. The wet season consists of 5-6 months from May to October or November with about 90% of the total rainfall occurring in this period. The average number of wet days in this season is 22-24 days/month. Foehn wind and tropical cyclones strongly affect the area during the wet season. A distinct cyclone season occurs later in the summer period from September to December, sharply peaking in October (KTTV&MT 2010). During a thunderstorm, the maximum 24 hour rainfall can be as much as 228mm (19/11/1987) at Pleiku station, 628.9mm (03/10/1993) at Tuy Hoa station and 579mm (04/10/1993) at Son Hoa station. Flooding is a common phenomenon in the Ba River catchment. The largest recorded flood peak at the Cungson gauging station is 20,700m^3/s.

3.3. Data availability

Rainfall data: Daily rainfall data are available at 26 stations across the catchment (see Figure 1 for locations of these gauges). The daily rainfall records at almost all of these stations are available for more than 30 years covering the period 1980 – 2011. However, there are only 12 stations recording hourly rainfall with periods of record ranging from 14 to 33 years. Only 4 gauges have hourly rainfall records more than 30 years from 1976 to 2011.

Flood flow data: Flow data are observed at 3 stations Cung Son, An Khe and Song Hinh. Two gauges, Cung Son and Ankhe have discharge data in hourly and 6-hourly intervals for more than 30 years. This 30 year period is from 1980 to 2011 (see Table 1). At the Song Hinh gauge, flood flows were available only for 13 years from the period from 1980 to 1992 with discontinuous measurement.

Table 1: Flood flow observation period of Ba river

TT	Gauge	River	Flv (km^2)	Observation years	Observation period
1	An Khe	Ba	1350	35	1977-2011
2	Cung Son	Ba	12410	35	1977-2011
3	Song Hinh	Hinh	747	13	1980-1992

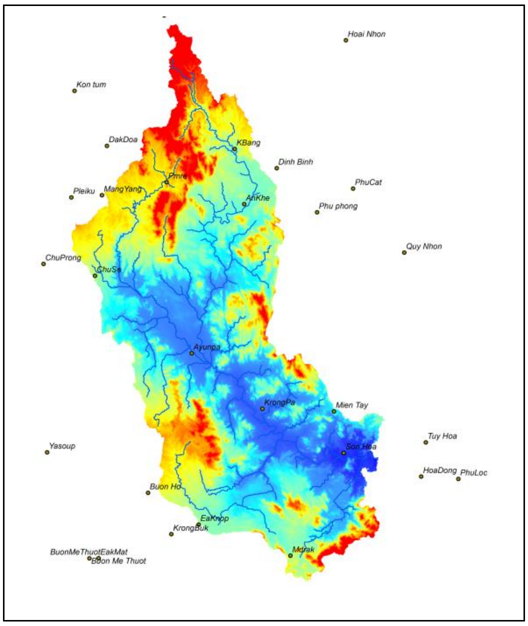

Figure 1: Distribution of meteorological stations across Ba basin.

4. CATCHMENT MODELLING SYSTEM

The deterministic catchment modelling system used for the simulations was HEC-HMS which is described by US Army Corps of Engineers (2010) as being a physically based, semi-distributed parameter model. Application of this software used gridded rainfall with a 2000m resolution while the SCS curve number method was used as the loss model. For rainfall runoff transform, a kinematic wave approach was applied with flood translation along reach elements simulated by a Muskingum-Cunge technique.

4.1. Rainfall Model and Method of Fragments for Rainfall Disaggregation

To generate the gridded rainfall data necessary for implementation of the model, an Inverse Distance Weight method using rainfall data at 19 stations across the catchment was used. Unfortunately, there was a limitation of available observed rainfall data as presented earlier. The Method of Fragments was used to generate data at the rainfall gauges at the desired time increment, namely 1 hour. Details of the method are presented by Cu and Ball (2014).

4.2. Catchment Stream Network and Preliminary Subcatchment Parameters

Application of a distributed modelling system requires the subdivision of a catchment into a number of subcatchments. For this study, the catchment was divided into 155 subcatchments as shown in Figure 2. These delineations are based on a DEM with a horizontal resolution of 90m and the assumption that each subcatchment should be small so that application of the kinematic wave model was feasible; a small catchment according to Vietnamese practice is less than 100km^2. At the same time, there is a need to ensure that the number of parameters to be determined during calibration of the model is not excessive; as shown in (Table 2), each subcatchment requires the value of 10 parameters.

Initial model parameter values were estimated using various sources such as land use and land cover map, the DEM, and soil maps. In this case, applying SCS method (US Army Corps of Engineers 2010), the curve number map was developed by combination of soil map and land cover map. Roughness coefficients of sub-catchments were defined by the slope and land cover maps. All these parameter maps were gridded at a horizontal resolution of 2000m which was consistent with the rainfall grid data. Once the value of parameters had been estimated for individual grids, the average value for each sub-catchment was determined.

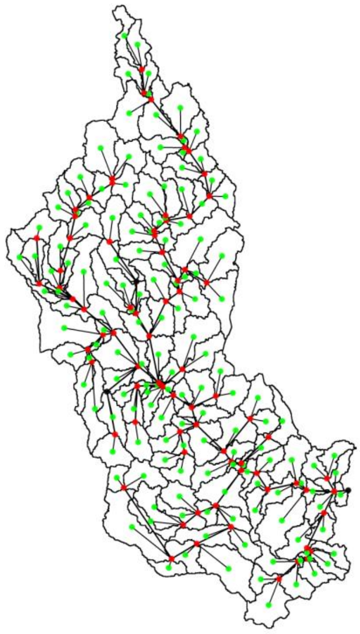

Figure 2: Catchment delineation and stream network

Table 2: Model parameters and their available ranges

Models	Parameters	Range
Loss models	Curve number	20 – 90
Kinematic wave *(Overland flow planes)*	Typical length	
	Representative slope	0.0001 – 1
	Overland-flow roughness coefficient	0.35 - 0.8
	Area represented by plane	
Musking-Cunge routing *(The main channel)*	Main channel length	
	Description of main channel shape	Rectangular
	Channel slope	0.0001 – 1
	Channel width	
	Representative Manning's roughness coefficient	0.035 – 0.08

4.3. FFA and modelling system measure of fit

A flood frequency analysis (FFA) of the basin using the observed flow was conducted for the two stream flow gauges located in upstream and downstream areas of the catchment: Ankhe and Cungson. Ankhe gauge is located in the upstream area of the catchment with a contributing area of 1350km^2, and the Cung Son gauge is located in the downstream area of the catchment with a contributing area of 12,410 km^2.

Choosing annual series: This flood frequency analysis used an AMS as recommended by Kuczera and Franks (2005) for a gauge with more than 10 years of record. As a preliminary step, the periods for the AMS were chosen based on catchment regulation and the quality of data. Thus, only the period without major influences of regulation and hydropower were examined. The selected time period for analysis was 33 years for Ankhe station from 1978 to 2010 and 23 years for Cungson station from 1978 to 2000.

Censored data: Detection of outliers for the LP-III distribution was undertaken using the methodology described by Interagency Advisory Committee on Water Data, (1982). The adopted high and low threshold for the 2 gauges are shown in (Table 3). As can be seen from this table, peak flow of 20700 m³/s (in 1993) at Cungson gauge was considered as censored data, peak flow of 326 m³/s (in 1978), 250 m3/s (in 1989) and 275 m3/s (in 2006) at Ankhe gauge were treated as censored data.

Table 3: High outlier and low outlier for annual maximum series

Gauge	Mean	Cv	N	K_N	Peak flow Logarithms (m3/s)	
					High threshold	Low threshold
Cung Son	8.69	0.48	22	2.43	9.8514 (18984)	7.5287 (1860)
An Khe	7.03	0.46	29	2.55	8.2089 (3673)	5.8448 (345)

Frequency curves: Flood quantiles were estimated by Bayesian approach using the FLIKE software (Kuczera, 1999) with an LP-III distribution and Bayesian parameter estimation. No prior information was used. In general, it was found that use of the LP-III distribution produced consistent results; in the majority of cases, the observed data were within the confidence limits as shown in Figure 3,4). The derived parameters for the LP-III distribution (mean, standard deviation and skewness) are presented in Table 4. The table showed most probable values of the parameters and the standard deviation of these parameters which identify the confidence (acceptable) ranges of these values.

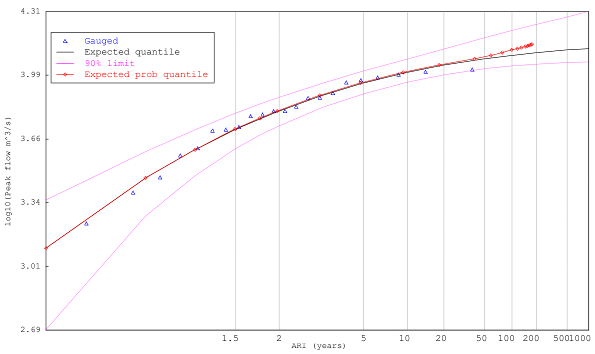

Figure 3: Flood frequency curve at Cungson gauge

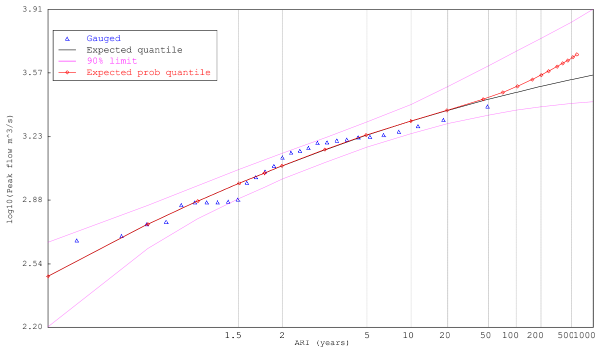

Figure 4: Flood frequency curve at Ankhe gauge

Table 4: LPIII parameters for flood frequency at Cung Son and Ankhe

N	Parameter	Most probable	Standard deviation	Maximum	Minimum
1	***An Khe***				
	Mean (loge flow)	7.0268	0.0901	7.1169	6.9367
	Loge [Std dev (loge flow)]	-0.7458	0.1661	-0.5797	-0.9119
	Skew (loge flow)	-0.9490	0.4080	-0.5410	-1.3570
2	***Cung Son***				
	Mean (loge flow)	8.6900	0.1049	8.7949	8.5851
	Loge [Std dev (loge flow)]	-0.7378	0.2466	-0.4912	-0.9844
	Skew (loge flow)	-1.4374	0.6422	-0.7952	-2.0796
	Mean (loge flow)	8.6900	0.1049	8.7949	8.5851

4.4. Model calibration

Performance measures: The calibration metric used in this study was the capacity of the predicted flows to result in a similar flood frequency curve to that of the observed flows. As a modified Bayesian analysis method was used, an outcome of the analysis was the most probable value for a parameter together with the likely range of parameter values defined through the standard deviation (see Tables 4). The calibration process, therefore, sought parameters which were in the ranges of the parameter values for the observed frequency curve.

Parameter values were determined during calibration of the modelling system. Sensitivity analyses showed that some parameters for the model had a greater influence on predicted flows than other parameters. Based on these analyses, the parameters selected for calibration were the Curve number, the subcatchment representative slope, the overland-flow roughness, the representative subcatchment width, the channel slope, the channel roughness; both the overland flow and the channel roughness were represented by a Manning's coefficient. As a result, approximately 1000 parameters were calibrated. While it is admitted that the factor of computational power has become less limiting nowadays, the calibration process can be developed by global search method with noting of constraint subjective components (parameter range allowed). However, the process can be accelerated by a monitored procedure as outlined below:

Step 1: Selection of initial random parameters; Adjustment of each parameter was conducted through consideration of two coefficients: the mean coefficient and a bias coefficient. The purpose of the mean coefficient was to increase the parameter for all sub-catchments while the purpose of the bias coefficient was to change the variation of parameters across the catchment. For example, 155 subcatchments characterized by 155 curve numbers (CN) produced a mean value $\overline{CN}_0$. An adjusted curve number for was calculated using:

$$CN_{i,j} = K_1 \left[\overline{CN}_{\mathrm{J}-1} + \left(CN_{i,\mathrm{j}-1} - \overline{CN}_{\mathrm{j}-1}\right) K_2 \right]$$

Where: $CN_{i,j}$ is the curve number of sub-catchment i at calibration step j, K_1 is mean the coefficient, and K_2 is the bias coefficient. The mean coefficient (K_1) and bias coefficient (K_2) were randomly and evenly selected within the allowed range.

Step 2: Adjusting the coefficient values. In the first step, by uniform random selection of the mean and bias coefficients, all values of the parameters were treated equally. In this step, these parameter values were refined. All acceptable coefficient values were assigned a likelihood weight of 1 and plotted in a histogram. Using the histogram, a new acceptable range was defined. Generation of new parameter values for the next calibration step was undertaken using the new acceptable range for that parameter and the fitted normal distribution (mostly normal distribution).

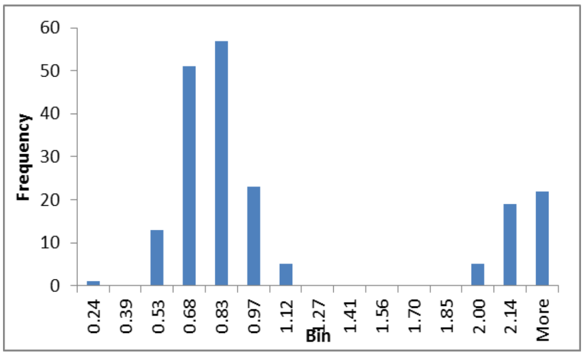

Figure 5: Histogram of distribution of mean coefficient – Catchment roughness (K1-Catchment roughness) – (step 2)

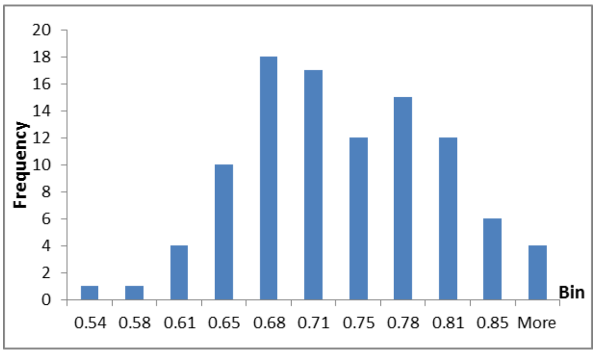

Figure 6: Histogram of distribution of mean coefficient – Catchment roughness (K1-Catchment roughness) (step 3)

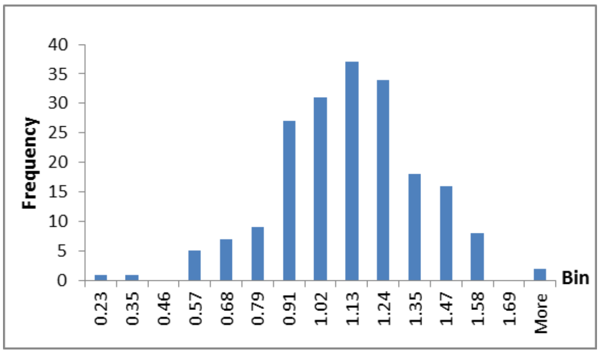

Figure 7: Histogram of distribution of bias coefficient – Catchment roughness (K2-Catchment roughness) (step 2)

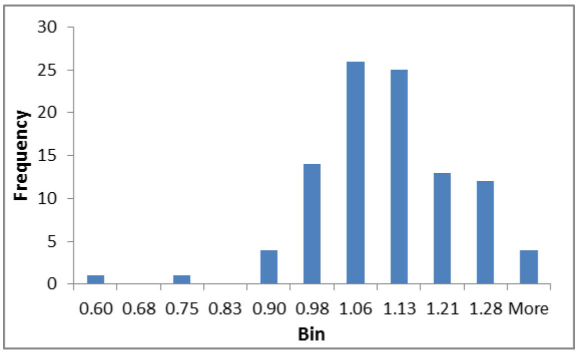

Figure 8: Histogram of distribution of bias coefficient – Catchment roughness (K2-Catchment roughness) (step 3)

For each calibration step, 600 parameter sets were generated. This process was repeated until the design results were achieved. Shown in Figure 5, 6, 7,8 are examples of the histogram obtained for the catchment roughness coefficient after completion of steps 2 and 3 of mean and bias coefficients of Catchment roughness.

5. RESULTS

The derived flood frequency curves for these catchments have been compared with the flood frequency curves defined by the observed floods, and it was found that the AMS fitting method provides a relatively precise reproduction of observed frequency curves over a wide range of flood frequencies and this can cope well with non-linearity of the rainfall and runoff process.

Shown in table 6 are the most probable value of parameter multiples of the catchments and their acceptable range. Visual comparison of observed and simulated flow can be seen by the frequency analyses shown in figures 13 and 14. The dashed lines represent the confidence limits of the observed frequency curve while the dot points represent the simulated values. As can be seen from these figures, most of the simulated points are located within the confidence limits. In overall, there was a good fit between of the simulated AMSs with the observed AMS.

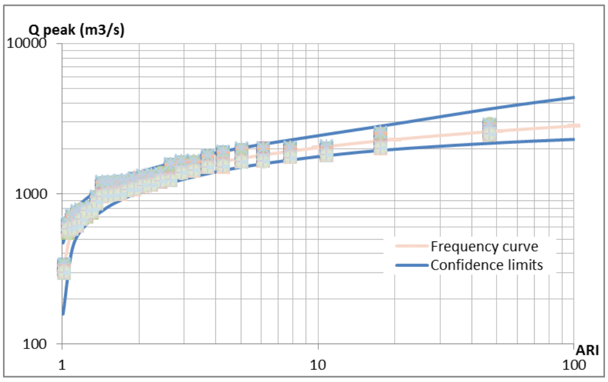

Figure 9: Frequency analysis of simulated AMS against observed frequency curve at An Khe station

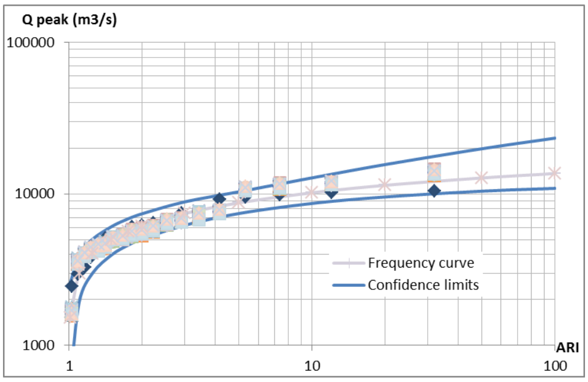

Figure 10: Frequency analysis of simulated AMS against observed frequency curve at Cung Son station

Table 6: Acceptable parameter range and distribution

Parameter	Distribution	Mean	STD
K_1 – CN	even	Range (0.65- 0.8)	
K_2 – CN	normal	1.5	0.1
		2.0	0.1
K_1 - Slope	normal	0.825	0.03

K_1 – Catchment length	normal	2.1	0.1
K_1 – Catchment roughness	normal	2.4	0.2
K_2 – Catchment roughness	normal	0.7	0.2
K_1 – Channel manning	normal	2.7	0.4
K_2 – Channel Manning	normal		

6. CONCLUSION AND DISCUSSION

The aim of catchment modelling for flood estimation is the prediction of design flood magnitudes and its likelihood. In this study, the design flood magnitudes were estimated by fitting the simulated AMSs within acceptable ranges of the observed AMS. A comparison of the predicted quantiles with recorded data for the Ba catchment at two gauges Ankhe and Cungson in Vietnam indicated that the fitting metric based on reproduction of AMS resulted in a model that consistently predicted flood quantiles with the confidence limits of the flood quantiles estimated form the recorded data. The most valuable is that the flows have been derived without resorting to gross assumptions about size, shape and duration of the design event. This makes the flows more robust and defensible than the traditional approach of using a single event to present the design case. In addition, the continuous simulation includes seasonal variation of flood flow such as base flow and antecedent condition; it eliminates the need to make assumptions about what will constitute a design event. Instead of simulation of a single event, flood events of all conceivable durations and magnitudes had been generated, with subsequent design flows being derived from these.

However, there are some limitations associated with this method. The method requires long time observation period and rainfall data record for conducting AMS estimation. This case study, Ba river, used 31 year – simulation period from 1980 to 2010. That limits the option for splitting observation period into 2 periods for calibration and validation, as a result the study was conducted only calibration process ignoring validation process.

7. ACKNOWLEDGMENTS

Support for the first author was provided by a VIED - UTS scholarship. The support of both the Australian and Vietnamese partners in this scholarship scheme is gratefully acknowledged.

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