Is the Slope of the Phillips Curve Time-Varying? Evidence from Unobserved Components Models

August 8, 2019

Abstract

This paper formally tests for time variation in the slope of the Phillips curve using a variety of measures of inflation expectations and real economic slack. We find that time variation in the slope of the Phillips curve depends on the measure of inflation expectations rather than the measure of real economic slack. We find strong evidence in support of the time-varying slopes of the Phillips curve with different measures of inflation expectations. Thus, we conclude that the slope of the Phillips curve is time-varying.

Keywords: Bayesian estimation, The slope of the Phillips curve, Unobserved Components Model.

JEL codes: C11, C32, E31.

1 Introduction

The original Phillips curve describes the empirical relationship between inflation and unemployment rate (Phillips, 1958). Other versions that use related measures of real economic activity are later considered. Estimating this relationship is important for a number of reasons. For example, many central banks need to maintain both price stability and full employment. But these two goals might be not consistent. Understanding the trade-off between these two goals is therefore important. In addition, at the aftermath of the financial crisis of 2008-2009, inflation remained stable while there was a surge in unemployment rate. This is often referred to as the "missing disinflation" puzzle. One explanation for the puzzle is that the slope of the Phillips curve has become flatter (e.g Bean, 2006; Gaiotti, 2008; Ihrig et al., 2010; Kuttner and Robinson, 2010), which calls into question of the stability of the Phillips curve.

Many papers have documented changes in the slope of the Phillips curve. Examples include Ball and Mazumder (2011), Roberts (2006), Atkeson and Ohanian (2001), and Mishkin (2007). To test for time variation in the slope of the Phillips curve, these papers estimate constant-coefficient Phillips curve using split samples and check whether the slope changes considerably across different samples. Rather than model the slope of the Phillips curve as constant and compare the estimated slope of the Phillips curve in different samples, some studies model the slope as time-varying. Examples include Stella and Stock (2012), Chan et al. (2016), and Kim et al. (2014).

However, there are two issues for assuming the slope of the Phillips curve as time-varying. First, the conclusion that the slope of the Phillips curve changes are challenged by some recent studies. For example, Gordon (2013) finds that the slope of the Phillips curve is stable by estimating a model with a hybrid Phillips curve. Coibion and Gorodnichenko (2015) estimate many models with standard expectation-augmented Phillips curve using a variety of measures of inflation expectations and find that evidence for changes in the slope of the Phillips curve is mixed. Second, the time-varying parameter specification might lead to over-parameterization compared to the constant-coefficient specification, as pointed out by Chan et al. (2012), Nakajima and West (2013), and Belmonte et al.

(2014). Therefore, one should be cautious about modeling the slope of the Phillips curve as time-varying without testing whether this specification is relevant.

Given these considerations, we consider a range of models with an embedded Phillips curve using a variety of measures of inflation expectations and real economic slack. We then test for time variation in the slope of these Phillips curves using the method proposed by Chan (2018). We find that the Bayes factors prefer models with time variation in the slope of the Phillips curve relationship than the restricted constant slope case. In addition, we find that the posterior mass of the variance that governs the time variation in the slope of the Phillips curve does not center around zero. Based on these strong evidence in favor of the time-varying slope of the Phillips curve from unobserved components models, we conclude that the slope of the Phillips curve is time-varying.

Formal tests of time variation in the slope of the Phillips curve are recently implemented by Berger et al. (2016) and Karlsson et al. (2018). Karlsson et al. (2018) test for time variation within the framework of a time-varying parameter Bayesian VAR using new tools for model selection proposed by Chan and Eisenstat (2018). By comparing a bivariate VAR with constant coefficients with a time-varying VAR, Karlsson et al. (2018) find strong evidence in favor of the latter and conclude that the slope of the Phillips curve is unstable. Instead of jointly testing time variation in all the parameters, our approach is more specific and tests only if the slope coefficient of the Phillips curve is time-varying.

Our paper is most related to Berger et al. (2016). They estimate a model with a New Keynesian Phillips Curve in which the trend inflation is interpreted as long-run inflation expectations. They then test for time variation in the slope of the Phillips curves using the stochastic model specification search approach proposed by in Frühwirth-Schnatter and Wagner (2010). Berger et al. (2016) find that the time-varying slope specification is rejected by the stochastic model specification search and conclude that the slope of the Phillips curve is not time-varying.

Our paper is different from Berger et al. (2016) in three aspects. First, we consider a wider range of measures of inflation expectations and economic slack. In particular, we do not only consider the trend inflation as a measure of inflation expectations, but also consider survey-based inflation expectations and a variety of measures of real economic slack. Second, we directly compute the Bayes factor in favor of the model with a time-varying Phillips curve via the method proposed by Chan (2018) rather than stochastic model specification search as in Berger et al. (2016). Finally, unlike Berger et al. (2016), we find strong evidence in favor of the time-varying slope of the Phillips curve.

The remainder of this paper is organized as the follows: in Section 2, we describe the models with different specifications for the Phillips Curve. In Section 3 we describe how we test the time variation in the slope of the Phillips curve. Section 4 describes the results of the test for the time variation of the slope of the Phillips curve. In Section 5, we conclude that the slope of the Phillips curve is time-varying.

2 Specifications for the Phillips Curve

We consider two classes of models for modeling the Phillips Curve: the univariate unobserved components models with stochastic volatility and the bivariate unobserved components models with stochastic volatility. For each model, we need a measure of inflation expectations and a measure of economic slack. For the univariate models, the unobserved component of real economic activities is the trend of real economic activities, z_t, denoted as z_t^*. Then, we use the deviation from the trend, $x_t = z_t - z_t^*$, as a measure of economic slack. We will use observable measures for the inflation expectations, $\mathrm{E}_t\pi_{t+1}$, such as the average of the past four quarters inflation or Survey of Professional Forecasters(SPF) inflation expectations.

A rapidly growing literature highlights that trend inflation has important implications for the specification of the NKPC (e.g. Kozicki and Tinsley; Ascari, 2004; Cogley and Sbordone, 2008). Thus, we also consider bivariate unobserved components models to jointly model real economic activities and inflation. In the bivariate case, the additional unobserved component is the trend inflation, denoted as τ_t. In the spirit of Beveridge and Nelson (1981), τ_t can be interpreted as the long-run inflation expectations. The estimated trend inflation usually has substantial variance. To reduce the variance of the estimated trend inflation, Chan et al. (2018) estimate trend inflation by linking Blue Chip ten years inflation forecasts to trend inflation. With the additional information from the Blue Chip ten years inflation forecasts, the variance of τ_t decrease substantially. Thus, in addition, we also consider the models with Phillips curve linking Blue Chip ten years inflation forecasts, q_t, to trend inflation, τ_t.

We will estimate altogether eight models from these two classes of models, using Blue Chip ten years inflation forecasts, q_t, different measures of real economic slack, x_t, and different measures of inflation expectations, $\mathrm{E}_t\pi_{t+1}$. we will give the details of the univariate and bivariate unobserved components models in Section 2.1 and Section 2.2.

2.1 Univariate Unobserved Components Model with Stochastic Volatility

Let π_t and z_t denote the inflation rate and level of economic activities respectively. And let z_t^* denote the trend of real activities. Then $x_t = z_t - z_t^*$ is a measure of the economic slack such as unemployment gap or the output gap. Considering the following class of univariate unobserved components models with stochastic volatility:

$$\pi_t - \mathrm{E}_t\pi_{t+1} = \lambda_t(z_t - z_t^*) + \varepsilon_t^{\pi}, \qquad \varepsilon_t^{\pi} \sim \mathcal{N}(0, e^{h_t}), \tag{1}$$

$$\lambda_t = \lambda_{t-1} + \varepsilon_t^{\lambda}, \qquad \varepsilon_t^{\lambda} \sim \mathcal{N}(0, \omega_{\lambda}^2), \tag{2}$$

$$z_t = z_t^* + e_t, \tag{3}$$

$$e_t = \phi_1 e_{t-1} + \phi_2 e_{t-2} + \varepsilon_t^{e}, \qquad \varepsilon_t^{e} \sim \mathcal{N}(0, \omega_e^2), \tag{4}$$

where λ_t is the slope of the Phillips curve, $\mathrm{E}_t\pi_{t+1}$ represents different measures for expectations of inflation. λ_t and τ_t are modeled as random walk. e_t follows an AR(2) process. We consider two different specifications for z_t^*: when z_t represents unemployment rate, z_t^* is modeled as random walk:

$$z_t^* = z_{t-1}^* + \varepsilon_t^{z_t^*}, \qquad \varepsilon_t^{z_t^*} \sim \mathcal{N}(0, \omega_{z^*}^2). \tag{5}$$

when z_t represents output level, the growth of z_t, Δz_t^*, is modeled as random walk:

$$\Delta z_t^* = \Delta z_{t-1}^* + \varepsilon_t^{z_t^*}, \qquad \varepsilon_t^{z_t^*} \sim \mathcal{N}(0, \omega_{z^*}^2). \tag{6}$$

2.2 Bivariate Unobserved Components Model with Stochastic Volatility

Next, we augment the univariate unobserved componets models to also model trend inflation. More specifically, the class of bivariate unobserved components models with stochastic volatility can be specified as

$$\pi_t - \tau_t = \lambda_t(z_t - z_t^*) + \varepsilon_t^{\pi}, \qquad \varepsilon_t^{\pi} \sim \mathcal{N}(0, e^{h_t}), \tag{7}$$

$$\lambda_t = \lambda_{t-1} + \varepsilon_t^{\lambda}, \qquad \varepsilon_t^{\lambda} \sim \mathcal{N}(0, \omega_{\lambda}^2), \tag{8}$$

$$\tau_t = \tau_{t-1} + \varepsilon_t^{\tau}, \qquad \varepsilon_t^{\tau} \sim \mathcal{N}(0, e^{g_t}), \tag{9}$$

$$z_t = z_t^* + e_t, \tag{10}$$

$$e_t = \phi_1 e_{t-1} + \phi_2 e_{t-2} + \varepsilon_t^{e}, \qquad \varepsilon_t^{e} \sim \mathcal{N}(0, \omega_e^2), \tag{11}$$

As before, either z_t^* or Δz_t^* is modeled as a random walk,

$$z_t^* = z_{t-1}^* + \varepsilon_t^{z_t^*}, \qquad \varepsilon_t^{z_t^*} \sim \mathcal{N}(0, \omega_{z^*}^2), \tag{12}$$

$$\Delta z_t^* = \Delta z_{t-1}^* + \varepsilon_t^{z_t^*}, \qquad \varepsilon_t^{z_t^*} \sim \mathcal{N}(0, \omega_{z^*}^2). \tag{13}$$

In addition, we also link the trend inflation τ_t to the Blue Chip inflation forecasts by adding the following equation:

$$q_t = d_0 + d_1\tau_t + \varepsilon_t^{q}, \qquad \varepsilon_t^{q} \sim \mathcal{N}(0, \omega_q^2), \tag{14}$$

where q_t is Blue Chip ten years forecasts. Following Chan et al. (2018), we allow the possibility that the forecasts are unrelated to the trend inflation by introducing the intercept d_0 and slope coefficient d_1. When $d_0 = 0$ and $d_1 = 1$, the Blue Chip forecasts are an unbiased measure of the trend inflation.

2.3 Specific Models

We provide a brief summary of the Philips curve models in Table 1. The details of these models are provided in Appendix A. In Table 1, we denote y_t and u_t as the output and the unemployment rate, respectively. y_t^* is the potential output and ν_t represents the natural unemployment rate. We have two measures of economic slack: the output gap, $y_t - y_t^*$ and the unemployment gap, $u_t - \nu_t$. We have four measures of inflation expectations: $\pi^e_{t|t-1}$, $\pi^e_{t|t+1}$, τ_t, and q_t. $\pi^e_{t|t-1}$ represent backward-looking inflation expectations, measured as the average of past four quarter inflation. $\pi^e_{t|t+1}$ represent forward-looking inflation expectations, measured as the SPF one year inflation forecasts. τ_t are the trend inflation. q_t are the Blue Chip ten years inflation forecasts. In M7 and M8, τ_t are estimated with the additional information of Blue Chip ten years inflation forecasts, q_t.

Table 1: Summary of the Phillips Curve Models.

Model	Unobserved Component	Economic Slack	Inflation Expectations
M1	ν_t	$u_t - \nu_t$	$\pi^e_{t\|t-1}$
M2	ν_t	$u_t - \nu_t$	$\pi^e_{t\|t+1}$
M3	y_t^*	$y_t - y_t^*$	$\pi^e_{t\|t-1}$
M4	y_t^*	$y_t - y_t^*$	$\pi^e_{t\|t+1}$
M5	ν_t, τ_t	$u_t - \nu_t$	τ_t
M6	y_t^*, τ_t	$y_t - y_t^*$	τ_t
M7	ν_t, τ_t	$u_t - \nu_t$	τ_t, q_t
M8	y_t^*, τ_t	$y_t - y_t^*$	τ_t, q_t

3 Testing for Time Variation

In this section, we outline the methodology to test for time-variation. We first give an overview of the Bayes factor and Savage-Dickey density ratio and then introduce a new method of calculating the Bayes factor proposed by Chan (2018).

3.1 Bayes Factor and Savage-Dickey Density Ratio

To demonstrate the method of testing for time variation in the slope of the Phillips curve, we first consider the following unobserved components model with stochastic volatility:

$$\pi_t - \mathrm{E}_t\pi_{t+1} = \lambda_t x_t + \varepsilon_t^{\pi}, \qquad \varepsilon_t^{\pi} \sim \mathcal{N}(0, e^{h_t}), \tag{15}$$

$$\lambda_t = \lambda_{t-1} + \varepsilon_t^{\lambda}, \qquad \varepsilon_t^{\lambda} \sim \mathcal{N}(0, \omega_\lambda^2), \tag{16}$$

where π_t is the inflation rate at time t, $\mathrm{E}_t\pi_{t+1}$ is a measure of expected inflation at time $t+1$ given the information at time t, x_t is a measure of real economic slack, λ_t is the slope of the Phillips curve. We model the slope, λ_t, as a random walk process instead of a stationary AR(1).[1] To test whether the slope, λ_t, is time-varying, we can compare the model (15)-(16) to a restricted version where the slope is constant, i.e., $\omega_\lambda^2 = 0$. Denote the former model as Model 1 and the restricted version Model 2. One popular model comparison criterion for comparing these two models is the Bayes factor in favor of Model 1 against Model 2, defined as

$$\mathrm{BF}_{12} = \frac{p(\mathbf{y} \mid \text{Model 1})}{p(\mathbf{y} \mid \text{Model 2})},$$

where $p(\mathbf{y} \mid \text{Model i})$ is the marginal likelihood for Model_i. The corresponding posterior odds ratio is defined as

$$\frac{p(\text{Model 1} \mid \mathbf{y})}{p(\text{Model 2} \mid \mathbf{y})} = \frac{p(\text{Model 1})}{p(\text{Model 2})} \times \mathrm{BF}_{12}.$$

Assume that the prior model probabilities are equal, i.e., $p(\text{Model 1}) = p(\text{Model 2})$, the posterior odds ratio in favor of Model 1 reduces to the Bayes factor BF_{12}. For example,

[1] Eisenstat and Strachan (2016) argue that the random walk assumption has two main advantages for macroeconomic applications. First, the random walk specification can be a parsimonious approximation to a stationary specification with high persistence. Second, random walk specification implies greater smoothness than the stationary model with low persistence.

$\mathrm{BF}_{12} = 10$ means that model Model 1 is 10 times more likely than model Model 2 given the data.

The Bayes factor is commonly used to compare models. However, the main challenge here is that it is often difficult to compute the marginal likelihood of models with time-varying parameters.

Fortunately, one simpler method is available when we need to compute the Bayes factor for nested models. Specifically, the Bayes factor can be calculated by using the Savage-Dickey density ratio (Verdinelli and Wasserman, 1995). This approach requires only the estimation of the unrestricted model. For example, the Bayes factor in favor of Model 1 against Model 2 can be obtained using the Savage-Dickey density ratio as

$$\mathrm{BF}_{12} = \frac{p(\omega_\lambda^2 = 0)}{p(\omega_\lambda^2 = 0 \mid \mathbf{y})},$$

where the numerator is the marginal prior density of ω_λ^2 evaluated at 0, and the denominator is the marginal posterior of ω_λ^2 evaluated at 0. Intuitively, if ω_λ^2 is more likely to be 0 under the prior density relative to the posterior density, this can be viewed as evidence in favor for the time-varying slope of the Phillips curve. However, this easier method cannot be directly applied in our setting due to two related issues. First, the value 0 is at the boundary of the parameter space of ω_λ^2. Therefore, the Savage-Dickey density ratio approach is not applicable. Second, ω_λ^2 is often assumed to have an inverse-gamma prior, which has zero density at zero. To deal with these two difficulties, we follow the method proposed by Chan (2018). Specifically, we use the so-called non-centered parameterization discussed in Frühwirth-Schnatter and Wagner (2010)—we work with the unsigned standard deviation, ω_λ, which has support on the whole real line. Then we directly calculate the relevant Bayes factor using the Savage-Dickey density ratio.

3.2 Non-centered Parameterization

Next, we briefly discuss the non-centered parameterization. First, we define $\lambda_t = \lambda_0 + \omega_\lambda \widetilde{\lambda}_t$, then, the state space model in (15)-(16) can be written as follows:

$$\pi_t - \mathrm{E}_t \pi_{t+1} = (\lambda_0 + \omega_\lambda \widetilde{\lambda}_t) x_t + \varepsilon_t^\pi, \qquad \varepsilon_t^\pi \sim \mathcal{N}(0, e^{h_t}), \tag{17}$$

$$\widetilde{\lambda}_t = \widetilde{\lambda}_{t-1} + \varepsilon_t^{\widetilde{\lambda}}, \qquad \varepsilon_t^{\widetilde{\lambda}} \sim \mathcal{N}(0, 1), \tag{18}$$

where $\widetilde{\lambda}_0 = 0$.

In this model, we assume $\omega_\lambda \sim \mathcal{N}(0, V_{\omega_\lambda})$, which has two main advantages. First, by a change of variable (Kroese and Chan, 2014), the implied prior for ω_λ^2 is $\mathcal{G}(\frac{1}{2}, \frac{1}{2V_{\omega_\lambda}})$. This gamma prior has more mass concentrated around small values of ω_λ^2. Therefore, it provides shrinkage—*a priori* it favors the more parsimonious constant—coefficient model. Second, it is a conjugate prior for ω_λ, under the non-centered parameterization it therefore facilitates computation. The sign of ω_λ is not identified, but alteration of the sign dose not change the likelihood value. After the non-centered parameterization of model (15)-(16), the Bayes factor $\mathrm{BF}_{12} = p(\omega_\lambda = 0)/p(\omega_\lambda = 0 \mid \mathbf{y})$, obtained by using Savage-Dickey density ratio, can be directly calculated by method proposed by Chan (2018).

4 Results

In this section, we first estimate six different models embedded with the Phillips curve, M1-M6, and test the time variation in the slopes. In addition, we estimate two additional model, M7 and M8, that use additional information of Blue Chip ten years inflation forecasts. The details of estimation are provided in Appendix B. We then test the time variation in the slopes of the Phillips curve under these two models.

Our data consist of quarterly CPI inflation rate, (civilian seasonally adjusted) unemployment rate from 1955Q1 to 2013Q1, SPF one year inflation forecasts from 1982Q1 to 2013Q1,[2] and Blue Chip ten years inflation forecasts from 1982Q1 to 2013Q1.

To formally test if there is substantial time variation in the slope of the Phillips curve λ_t, we compute the Bayes factor in favor of the six different unrestricted models, M1-M6, against their corresponding restricted versions where λ_t is constant ($\omega_\lambda = 0$). The test results for time variation of slopes of different models with the Phillips curve are shown in Table 2.

Table 2: Estimated Log Bayes Factors and the Numerical Standard Errors.

	M1	M2	M3	M4	M5	M6
Log BF	4.8 (0.08)	4.1 (0.12)	8.4 (2.05)	57.2 (3.57)	0.2 (0.03)	0.7 (0.06)

Overall for most models, the data prefer the version with time variation. Specifically, the log Bayes factors associated with M1, M2, M3, M4 are all larger than 4, indicating substantial time variation in the slope, λ_t. On the other hand, the log Bayes factors associated with M5 and M6 are small but positive, suggesting slight evidence in favor of time variation in λ_t.

To corroborate these model comparison results, we plot the posterior estimates of λ_t and ω_λ in Figures 1 and 3. First, Figure 1 shows the results for the Phillips curve specified as the univariate unobserved components model with stochastic volatility. Figure 3 shows the corresponding results for the bivariate unobserved components model with stochastic volatility.

[2] We also consider median expected price change next 12 months covering the period 1982Q1-2013Q1 from University of Michigan inflation expectation survey data. In Appendix C, we show that the results are similar to those of using SPF one year inflation forecasts.

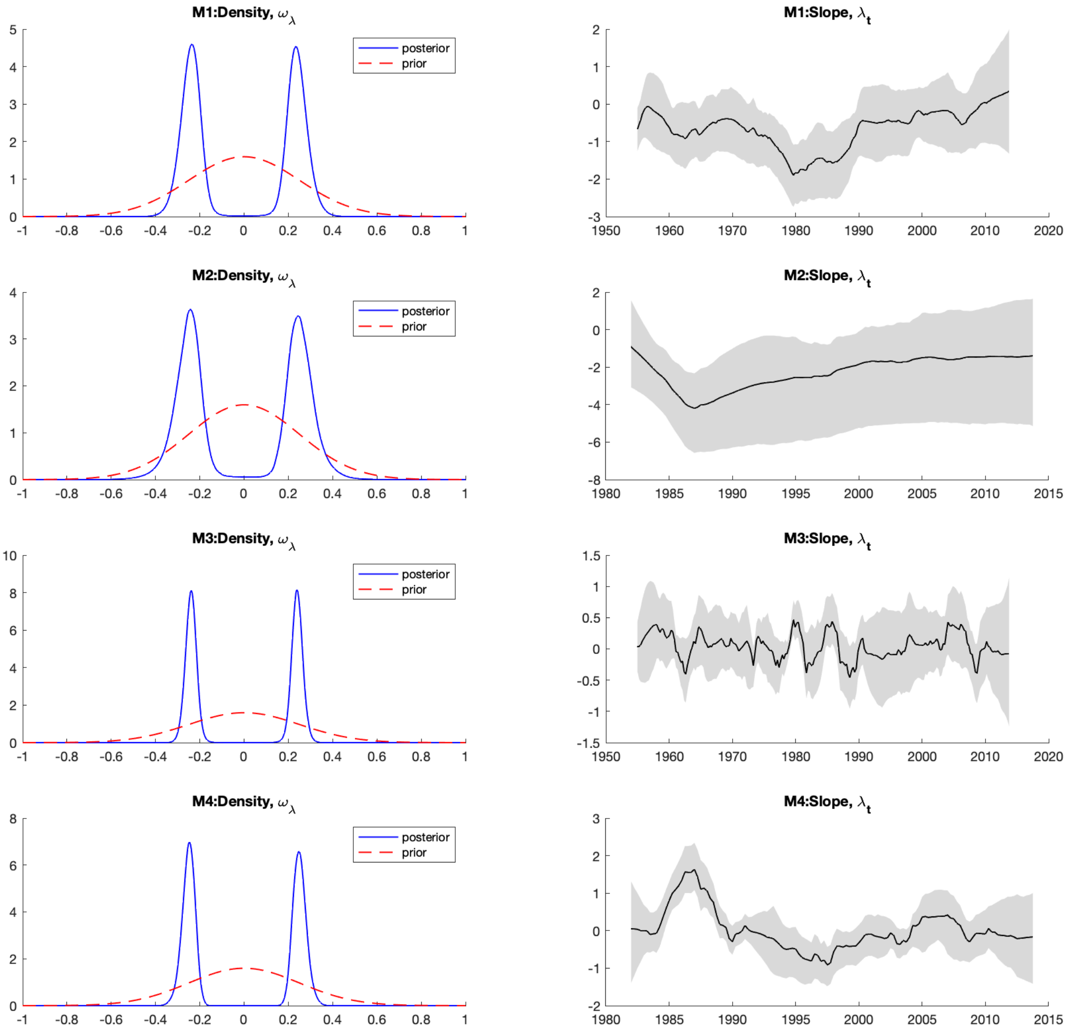

Figure 1: Estimated Slope λ_t and Density of ω_λ for the Phillips Curves Specified as Univariate Unobserved Components Model with Stochastic Volatility.

Consistent with model comparison results, the right panel of Figure 1 shows that estimates of the slopes of the Phillips curve, λ_t, under M1 and M2 are volatile and time-varying. They are always negative, which is consistent with the idea of the Phillips curve—there is a trade-off between inflation and the unemployment gap. Also, starting from the 1980s, λ_t becomes flatter and is closer to zero. Estimates of the slopes of the Phillips curve, λ_t, under M3 and M4 are also volatile but they mostly move around 0. This suggests that real GDP gap has little effects on inflation. These results are similar to those in Berger et al. (2016) and Chan and Grant (2017). They both find the magnitude of λ_t is small when the economic slack is measured as the output gap. Comparing models with different measures of inflation expectations, we find that the λ_t with the SPF one year forecasts is smoother than the λ_t with the average of past four quarter inflation. To understand this difference, we plot the SPF one year forecasts (solid line) and the average of past four quarter inflation (dash line) in Figure 2.

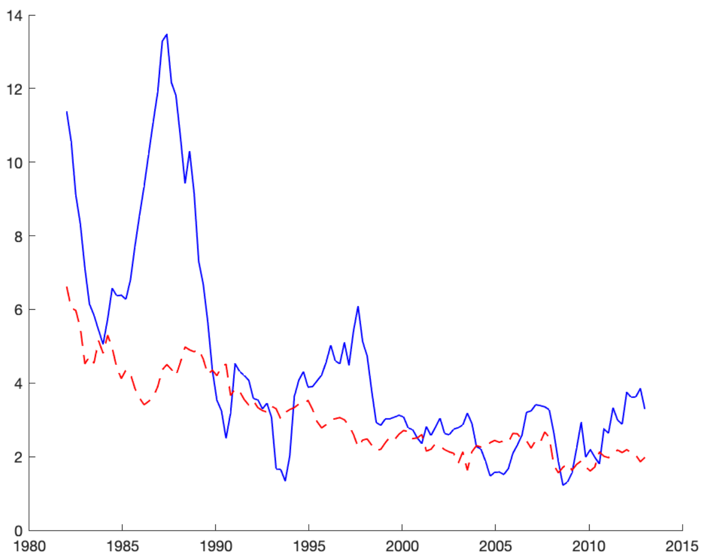

Figure 2: Inflation Expectations, $\pi^e_{t|t-1}$ and $\pi^e_{t|t+1}$.

Figure 2 shows that the SPF one year forecasts are smoother than the average of past four quarter inflation. Thus, the SPF one year forecasts lead to a smoother λ_t than the average of the past four quarters of inflation.

The left panel of Figure 1 shows that the posterior densities of ω_λ under M1, M2, M3, and M4 are all bimodal and have almost no mass around 0. This can be viewed as strong evidence in support of the time-varying λ_t.

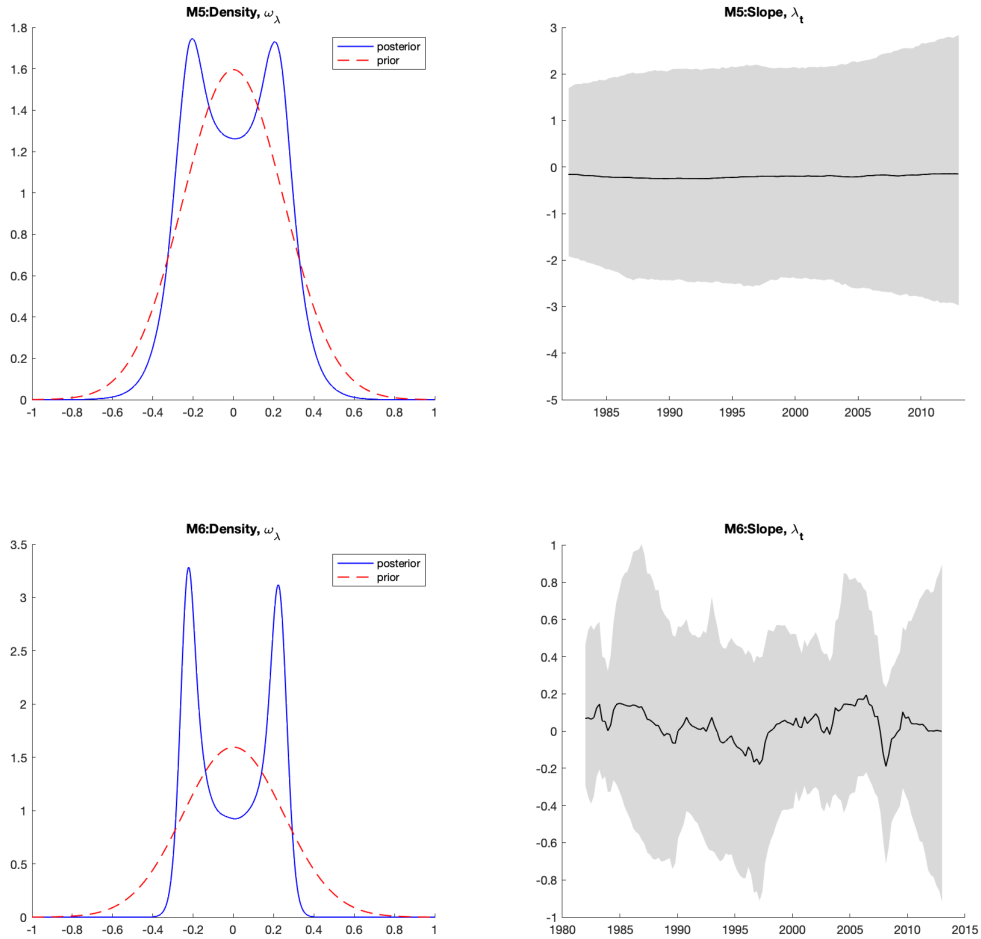

Figure 3: Estimated Slope λ_t and Density of ω_λ for the Phillips Curves with Bivariate Unobserved Components Model with Stochastic Volatility.

The right panel of Figure 3 shows that estimates of the time-varying slopes of the Phillips curve, λ_t, of M5 and M6 are insignificant and stable around 0. The left panel of Figure

3 shows that the posterior densities of ω_λ under M5 and M6 are bimodal, but have a considerable mass around 0. However, compared to the prior density, the posterior density at 0 is lower, suggesting ω_λ is less likely to be 0 given the data. This is consistent with the model comparison result that shows moderate evidence on time variation of λ_t.

To summarize our results so far, in the univariate case, the slope of the Phillips curve is conclusively time-varying. In the bivariate case, the evidence on the time variation on the slope of the Phillips curve is suggestive but not conclusive.

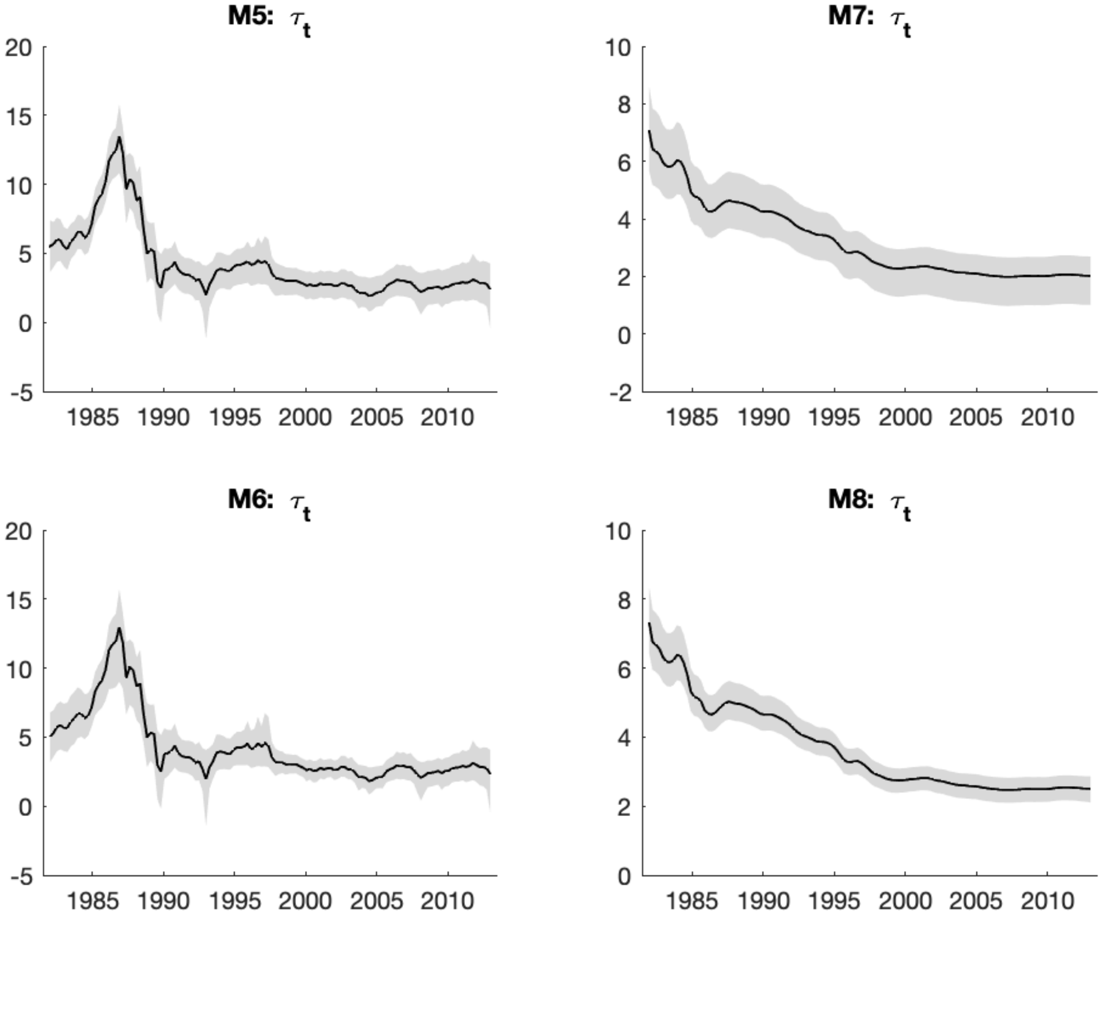

Figure 4: Trend inflation: τ_t.

The inconclusive evidence in the bivariate case could be due to the substantial variance of the estimated trend inflation. To investigate this possibility, we follow Chan et al. (2018) who link trend inflation τ_t to the Blue Chip inflation forecasts, which substantially reduces the variance of the estimated trend inflation. Following them, we add additional measurement equation linking the trend inflation to the Blue Chip ten years inflation forecasts to M5 and M6 respectively, and then we have M7 and M8. Figure 4 shows that with the additional information from the Blue Chip ten years inflation forecasts, M7 and M8 have a substantially smaller variance of the estimated trend inflation than M5 and M6.

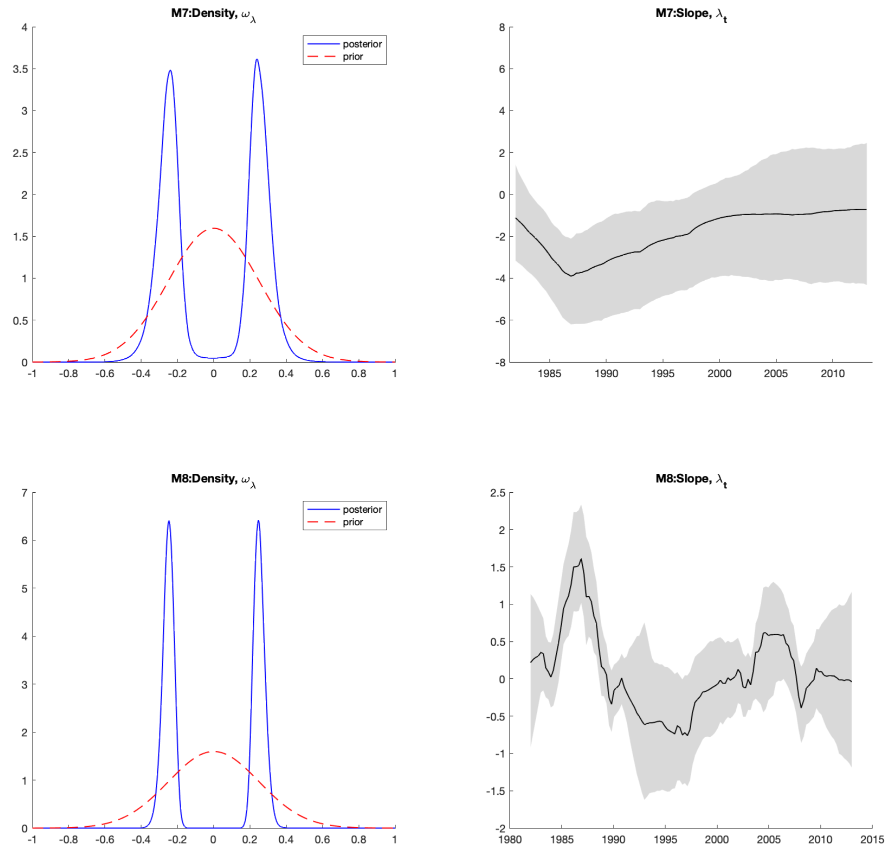

Figure 5: Estimated slope λ_t and density of ω_λ of M7 and M8

Next, we test the time variation in the slopes under M7 and M8. The log Bayes factors associated with M7 and M8 are 3.5 (0.24) and 51.0 (3.46), respectively. These values are large, indicating substantial time variation in the slope, λ_t. Consistent with model comparison results of M7 and M8, Figure 5 shows that estimates of the slopes of the Phillips curve, λ_t, of M7 and M8 are volatile and the posterior densities of ω_λ under M7 and M8 are bimodal and have almost no mass around 0. This shows strong evidence in favor of time variation in the slope.

In summary, in the univariate case, the slope of the Phillips curve is conclusively time-varying. Moreover, in the bivariate case, the slope of the Phillips curve is also conclusively time-varying with more precise estimates of the trend inflation, τ_t.

5 Conclusion

In this paper, we estimate eight Phillips curve models and test for time variation in the slopes of the Phillips curve under these models. The test shows that the Bayes factors favor models with time variation in the slope of the Phillips curve relationship than the restricted constant slope case. Further, we find that the posterior mass of the variance that governs the time variation in the slope of the Phillips curve does not center around 0. Our formal test provides strong evidence in favor of the time-varying slope of the Phillips curve from unobserved components models. Therefore, we conclude that the slope of the Phillips curve is time-varying.

References

Ascari, G. (2004). Staggered prices and trend inflation: some nuisances. *Review of Economic dynamics*, 7(3):642–667.

Atkeson, A. and Ohanian, L. E. (2001). Are Phillips curves useful for forecasting inflation? *Quarterly Review*, (Winter):2–11.

Ball, L. M. and Mazumder, S. (2011). Inflation dynamics and the great recession. Technical report, National Bureau of Economic Research.

Bean, C. R. (2006). Globalisation and inflation. Retrieved from `https://www.bankofengland.co.uk/-/media/boe/files/speech/2006/globalisation-and-inflation`.

Belmonte, M. A., Koop, G., and Korobilis, D. (2014). Hierarchical shrinkage in time-varying parameter models. *Journal of Forecasting*, 33(1):80–94.

Berger, T., Everaert, G., and Vierke, H. (2016). Testing for time variation in an unobserved components model for the U.S. economy. *Journal of Economic Dynamics and Control*, 69:179–208.

Beveridge, S. and Nelson, C. R. (1981). A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycle'. *Journal of Monetary Economics*, 7(2):151–174.

Chan, J. C., Koop, G., Leon-Gonzalez, R., and Strachan, R. W. (2012). Time varying dimension models. *Journal of Business & Economic Statistics*, 30(3):358–367.

Chan, J. C. C. (2018). Specification tests for time-varying parameter models with stochastic volatility. *Econometric Reviews*, 37(8):807–823.

Chan, J. C. C., Clark, T. E., and Koop, G. (2018). A new model of inflation, trend inflation, and long-run inflation expectations. *Journal of Money, Credit and Banking*, 50(1):5–53.

Chan, J. C. C. and Eisenstat, E. (2018). Bayesian model comparison for time-varying parameter VARs with stochastic volatility. *Journal of Applied Econometrics*, 33(4):509–532.

Chan, J. C. C. and Grant, A. (2017). Measuring the output gap using stochastic model specification search. (Working Paper 2/2017). Retrieved from https://cama.crawford.anu.edu.au/sites/default/files/publication/cama_crawford_anu_edu_au/2017-01/2_2017_chan_grant.pdf.

Chan, J. C. C. and Jeliazkov, I. (2009). Efficient simulation and integrated likelihood estimation in state space models. *International Journal of Mathematical Modelling and Numerical Optimisation*, 1(1–2):101–120.

Chan, J. C. C., Koop, G., and Potter, S. M. (2016). A bounded model of time variation in trend inflation, NAIRU and the Phillips curve. *Journal of Applied Econometrics*, 31(3):551–565.

Cogley, T. and Sbordone, A. M. (2008). Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips curve. *American Economic Review*, 98(5):2101–2126.

Coibion, O. and Gorodnichenko, Y. (2015). Is the Phillips curve alive and well after all? Inflation expectations and the missing disinflation. *American Economic Journal: Macroeconomics*, 7(1):197–232.

Eisenstat, E. and Strachan, R. W. (2016). Modelling inflation volatility. *Journal of Applied Econometrics*, 31(5):805–820.

Frühwirth-Schnatter, S. and Wagner, H. (2010). Stochastic model specification search for Gaussian and partial non-Gaussian state space models. *Journal of Econometrics*, 154(1):85–100.

Gaiotti, E. (2008). Has globalisation changed the Phillips curve? Firm-level evidence on the effect of activity on prices. 676. (Working Paper No. 676). Rome, IT: Bank of Italy.

Gordon, R. J. (2013). The Phillips curve is alive and well: Inflation and the NAIRU during the slow recovery. Technical report, National Bureau of Economic Research.

Ihrig, J., Kamin, S. B., Lindner, D., and Marquez, J. (2010). Some simple tests of the globalization and inflation hypothesis. *International Finance*, 13(3):343–375.

Karlsson, S., Österholm, P., et al. (2018). Is the US Phillips curve stable? Evidence from bayesian VARs. (Working Paper 2018:5). Retrieved from `https://ideas.repec.org/p/hhs/oruesi/2018_005.html`.

Kim, C.-j., Manopimoke, P., and Nelson, C. R. (2014). Trend inflation and the nature of structural breaks in the New Keynesian Phillips curve. *Journal of Money, Credit and Banking*, 46(2-3):253–266.

Kim, S., Shephard, N., and Chib, S. (1998). Stochastic volatility: likelihood inference and comparison with arch models. *The Review of Economic studies*, 65(3):361–393.

Kozicki, S. and Tinsley, P. A. Alternative sources of the lag dynamics of inflation. (Research Working Paper RWP 02-12). Kansas, TX: Federal Reserve Bank of Kansas City.

Kroese, D. P. and Chan, J. C. C. (2014). *Statistical modeling and computation.* New York, NY: Springer.

Kuttner, K. and Robinson, T. (2010). Understanding the flattening Phillips curve. *The North American Journal of Economics and Finance*, 21(2):110–125.

Mishkin, F. S. (2007). Inflation dynamics. *International Finance*, 10(3):317–334.

Nakajima, J. and West, M. (2013). Bayesian analysis of latent threshold dynamic models. *Journal of Business & Economic Statistics*, 31(2):151–164.

Phillips, A. W. (1958). The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861–1957. *Economica*, 25(100):283–299.

Roberts, J. M. (2006). Monetary policy and inflation dynamics. *International Journal of Central Banking*, 2(3):193–230.

Stella, A. and Stock, J. H. (2012). A state-dependent model for inflation forecasting. (Discussion Paper, International Finance). Retrieved from `https://www.federalreserve.gov/pubs/ifdp/2012/1062/ifdp1062.pdf`.

Verdinelli, I. and Wasserman, L. (1995). Computing Bayes factors using a generalization of the Savage-Dickey density ratio. *Journal of the American Statistical Association*, 90(430):614–618.

A Appendix: Details of the Specific Models

In this section, we outline the eight model M1-M8 in detail. In general, we have two classes of models: univariate and bivariate unobserved components models. The former includes M1-M4 which use different measures of economic slack and inflation expectations. The latter includes M5-M8. The specifications for each model are discussed below.

A.1 M1

M1 is a univariate unobserved components model where π_t is inflation. λ_t is the slope of the Phillips curve. $\mathrm{E}_t\pi_{t+1}$ is measured as the average of past four quarter inflation, $\pi^e_{t|t-1}$=$(\pi_{t|t-1} + \pi_{t|t-2} + \pi_{t|t-3} + \pi_{t|t-4})/4$. λ_t is modeled as random walk. u_t represents unemployment rate. e_t follows an AR(2) process. NAIRU, ν_t, is modeled as random walk. Log of stochastic volatility, h_t, is modeled as random walk.

$$\pi_t - \pi^e_{t|t-1} = \lambda_t(u_t - \nu_t) + \varepsilon^\pi_t, \qquad \varepsilon^\pi_t \sim \mathcal{N}(0, e^{h_t}), \tag{19}$$

$$u_t = \nu_t + e_t, \tag{20}$$

$$e_t = \phi_1 e_{t-1} + \phi_2 e_{t-2} + \varepsilon^e_t, \qquad \varepsilon^e_t \sim \mathcal{N}(0, \omega^2_e), \tag{21}$$

$$\nu_t = \nu_{t-1} + \varepsilon^\nu_t, \qquad \varepsilon^\nu_t \sim \mathcal{N}(0, \omega^2_\nu), \tag{22}$$

$$\lambda_t = \lambda_{t-1} + \varepsilon^\lambda_t, \qquad \varepsilon^\lambda_t \sim \mathcal{N}(0, \omega^2_\lambda), \tag{23}$$

$$h_t = h_{t-1} + \varepsilon^h_t, \qquad \varepsilon^h_t \sim \mathcal{N}(0, \omega^2_h). \tag{24}$$

.

A.2 M2

M2 is a univariate unobserved components model where π_t is inflation. λ_t is the slope of the Phillips curve. $\mathrm{E}_t\pi_{t+1}$ is measured as SPF one year inflation forecasts, $\pi^e_{t+1|t}$. λ_t is modeled as random walk. u_t represents unemployment rate. e_t follows an AR(2) process. NAIRU, ν_t, is modeled as random walk. Log of stochastic volatility, h_t, is modeled as random walk.

$$\pi_t - \pi^e_{t+1|t} = \lambda_t(u_t - \nu_t) + \varepsilon^\pi_t, \qquad \varepsilon^\pi_t \sim \mathcal{N}(0, e^{h_t}), \tag{25}$$

$$u_t = \nu_t + e_t, \tag{26}$$

$$e_t = \phi_1 e_{t-1} + \phi_2 e_{t-2} + \varepsilon^e_t, \qquad \varepsilon^e_t \sim \mathcal{N}(0, \omega^2_e), \tag{27}$$

$$\nu_t = \nu_{t-1} + \varepsilon^\nu_t, \qquad \varepsilon^\nu_t \sim \mathcal{N}(0, \omega^2_\nu), \tag{28}$$

$$\lambda_t = \lambda_{t-1} + \varepsilon^\lambda_t, \qquad \varepsilon^\lambda_t \sim \mathcal{N}(0, \omega^2_\lambda), \tag{29}$$

$$h_t = h_{t-1} + \varepsilon^h_t, \qquad \varepsilon^h_t \sim \mathcal{N}(0, \omega^2_h). \tag{30}$$

A.3 M3

M3 is a univariate unobserved components model where π_t is inflation. λ_t is the slope of the Phillips curve. $E_t\pi_{t+1}$ is measured as the average of past four quarter inflation, $\pi^e_{t|t-1}$=$(\pi_{t|t-1} + \pi_{t|t-2} + \pi_{t|t-3} + \pi_{t|t-4})/4$. λ_t is modeled as random walk. Cyclical component, c_t, follows an AR(2) process. y_t represents real output level. Underlying output trend growth, Δy^*_t, is modeled as random walk. Log of stochastic volatility, h_t, is modeled as random walk.

$$\pi_t - \pi^e_{t|t-1} = \lambda_t(y_t - y^*_t) + \varepsilon^\pi_t, \qquad \varepsilon^\pi_t \sim \mathcal{N}(0, e^{h_t}), \tag{31}$$

$$y_t = y^*_t + c_t, \tag{32}$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon^c_t, \qquad \varepsilon^c_t \sim \mathcal{N}(0, \omega^2_c), \tag{33}$$

$$\Delta y^*_t = \Delta y^*_{t-1} + \varepsilon^{y*}_t, \qquad \varepsilon^{y*}_t \sim \mathcal{N}(0, \omega^2_{y^*}), \tag{34}$$

$$\lambda_t = \lambda_{t-1} + \varepsilon^\lambda_t, \qquad \varepsilon^\lambda_t \sim \mathcal{N}(0, \omega^2_\lambda), \tag{35}$$

$$h_t = h_{t-1} + \varepsilon^h_t, \qquad \varepsilon^h_t \sim \mathcal{N}(0, \omega^2_h). \tag{36}$$

A.4 M4

M4 is a univariate unobserved components model where π_t is inflation. λ_t is the slope of the Phillips curve. $E_t\pi_{t+1}$ is measured SPF one year inflation forecasts, $\pi^e_{t+1|t}$. λ_t is modeled as random walk. y_t represents real output level. Cyclical component, c_t, follows an AR(2) process. Underlying output trend growth, Δy^*_t, is modeled as random walk.

Log of stochastic volatility, h_t, is modeled as random walk.

$$\pi_t - \pi^e_{t|t+1} = \lambda_t(y_t - y^*_t) + \varepsilon^\pi_t, \qquad \varepsilon^\pi_t \sim \mathcal{N}(0, e^{h_t}), \tag{37}$$

$$y_t = y^*_t + c_t, \tag{38}$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon^c_t, \qquad \varepsilon^c_t \sim \mathcal{N}(0, \omega^2_c), \tag{39}$$

$$\Delta y^*_t = \Delta y^*_{t-1} + \varepsilon^{y*}_t, \qquad \varepsilon^{y*}_t \sim \mathcal{N}(0, \omega^2_{y^*}), \tag{40}$$

$$\lambda_t = \lambda_{t-1} + \varepsilon^\lambda_t, \qquad \varepsilon^\lambda_t \sim \mathcal{N}(0, \omega^2_\lambda), \tag{41}$$

$$h_t = h_{t-1} + \varepsilon^h_t, \qquad \varepsilon^h_t \sim \mathcal{N}(0, \omega^2_h). \tag{42}$$

A.5 M5

M5 is a bivariate unobserved components model where π_t is inflation. λ_t is the slope of the Phillips curve. Long-run inflation expectations are measured as trend inflation τ_t and follow a random walk. λ_t is modeled as a random walk. u_t represents unemployment rate. e_t follows an AR(2) process. NAIRU, ν_t, is modeled as random walk. Two log of stochastic volatility variables, h_t and g_t, are modeled as a random walk.

$$\pi_t - \tau_t = \lambda_t(u_t - \nu_t) + \varepsilon^\pi_t, \qquad \varepsilon^\pi_t \sim \mathcal{N}(0, e^{h_t}), \tag{43}$$

$$\tau_t = \tau_{t-1} + \varepsilon^\tau_t, \qquad \varepsilon^\tau_t \sim \mathcal{N}(0, e^{g_t}), \tag{44}$$

$$u_t = \nu_t + e_t,$$

$$e_t = \phi_1 e_{t-1} + \phi_2 e_{t-2} + \varepsilon^e_t, \qquad \varepsilon^e_t \sim \mathcal{N}(0, \omega^2_e), \tag{45}$$

$$\nu_t = \nu_{t-1} + \varepsilon^\nu_t, \qquad \varepsilon^\nu_t \sim \mathcal{N}(0, \omega^2_\nu), \tag{46}$$

$$\lambda_t = \lambda_{t-1} + \varepsilon^\lambda_t, \qquad \varepsilon^\lambda_t \sim \mathcal{N}(0, \omega^2_\lambda), \tag{47}$$

$$h_t = h_{t-1} + \varepsilon^h_t, \qquad \varepsilon^h_t \sim \mathcal{N}(0, \omega^2_h), \tag{48}$$

$$g_t = g_{t-1} + \varepsilon^g_t, \qquad \varepsilon^g_t \sim \mathcal{N}(0, \omega^2_g). \tag{49}$$

A.6 M6

M6 is a bivariate unobserved components model where π_t is inflation. λ_t is the slope of the Phillips curve. Long-run inflation expectations are measured as trend inflation τ_t and follow a random walk. λ_t is modeled as a random walk. y_t represents the real output level. Cyclical component, c_t, follows an AR(2) process. Underlying output trend growth, Δy_t^*, is modeled as a random walk. Two log of stochastic volatility variables, h_t and g_t, are modeled as a random walk.

$$\pi_t - \tau_t = \lambda_t(y_t - y_t^*) + \varepsilon_t^\pi, \qquad \varepsilon_t^\pi \sim \mathcal{N}(0, e^{h_t}), \tag{50}$$

$$\tau_t = \tau_{t-1} + \varepsilon_t^\tau, \qquad \varepsilon_t^\tau \sim \mathcal{N}(0, e^{g_t}), \tag{51}$$

$$y_t = y_t^* + c_t, \tag{52}$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t^c, \qquad \varepsilon_t^c \sim \mathcal{N}(0, \omega_c^2), \tag{53}$$

$$\Delta y_t^* = \Delta y_{t-1}^* + \varepsilon_t^{y*}, \qquad \varepsilon_t^{y*} \sim \mathcal{N}(0, \omega_{y^*}^2), \tag{54}$$

$$\lambda_t = \lambda_{t-1} + \varepsilon_t^\lambda, \qquad \varepsilon_t^\lambda \sim \mathcal{N}(0, \omega_\lambda^2), \tag{55}$$

$$h_t = h_{t-1} + \varepsilon_t^h, \qquad \varepsilon_t^h \sim \mathcal{N}(0, \omega_h^2), \tag{56}$$

$$g_t = g_{t-1} + \varepsilon_t^g, \qquad \varepsilon_t^g \sim \mathcal{N}(0, \omega_g^2). \tag{57}$$

A.7 M7

M7 is a bivariate unobserved components model where π_t is inflation. λ_t is the slope of the Phillips curve. Long-run inflation expectations are measured as trend inflation τ_t and follow a random walk. λ_t is modeled as a random walk. u_t represents unemployment rate. e_t follows an AR(2) process. NAIRU, ν_t, is modeled as a random walk. Two log of stochastic volatility variables, h_t and g_t, are modeled as a random walk. q_t is Blue Chip ten years inflation forecasts.

$$\pi_t - \tau_t = \lambda_t(u_t - \nu_t) + \varepsilon_t^\pi, \qquad \varepsilon_t^\pi \sim \mathcal{N}(0, e^{h_t}), \tag{58}$$

$$\tau_t = \tau_{t-1} + \varepsilon_t^\tau, \qquad \varepsilon_t^\tau \sim \mathcal{N}(0, e^{g_t}), \tag{59}$$

$$u_t = \nu_t + e_t,$$

$$e_t = \phi_1 e_{t-1} + \phi_2 e_{t-2} + \varepsilon_t^e, \qquad \varepsilon_t^e \sim \mathcal{N}(0, \omega_e^2), \tag{60}$$

$$\nu_t = \nu_{t-1} + \varepsilon_t^\nu, \qquad \varepsilon_t^\nu \sim \mathcal{N}(0, \omega_\nu^2), \tag{61}$$

$$\lambda_t = \lambda_{t-1} + \varepsilon_t^\lambda, \qquad \varepsilon_t^\lambda \sim \mathcal{N}(0, \omega_\lambda^2), \tag{62}$$

$$h_t = h_{t-1} + \varepsilon_t^h, \qquad \varepsilon_t^h \sim \mathcal{N}(0, \omega_h^2), \tag{63}$$

$$g_t = g_{t-1} + \varepsilon_t^g, \qquad \varepsilon_t^g \sim \mathcal{N}(0, \omega_g^2), \tag{64}$$

$$q_t = d_0 + d_1\tau_t + \varepsilon_t^q, \qquad \varepsilon_t^q \sim \mathcal{N}(0, \omega_q^2). \tag{65}$$

A.8 M8

M8 is a bivariate unobserved components model where π_t is inflation. λ_t is the slope of the Phillips curve. Long-run inflation expectations are measured as trend inflation τ_t and follow a random walk. λ_t is modeled as a random walk. y_t represents the real output level. Cyclical component, c_t, follows an AR(2) process. Underlying output trend growth, Δy_t^*, is modeled as a random walk. Two log of stochastic volatility variables, h_t and g_t, are modeled as a random walk. q_t is Blue Chip ten years inflation forecasts.

$$\pi_t - \tau_t = \lambda_t(y_t - y_t^*) + \varepsilon_t^\pi, \qquad \varepsilon_t^\pi \sim \mathcal{N}(0, e^{h_t}), \tag{66}$$

$$\tau_t = \tau_{t-1} + \varepsilon_t^\tau, \qquad \varepsilon_t^\tau \sim \mathcal{N}(0, e^{g_t}), \tag{67}$$

$$y_t = y_t^* + c_t, \tag{68}$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t^c, \qquad \varepsilon_t^c \sim \mathcal{N}(0, \omega_c^2), \tag{69}$$

$$\Delta y_t^* = \Delta y_{t-1}^* + \varepsilon_t^{y*}, \qquad \varepsilon_t^{y*} \sim \mathcal{N}(0, \omega_{y^*}^2), \tag{70}$$

$$\lambda_t = \lambda_{t-1} + \varepsilon_t^\lambda, \qquad \varepsilon_t^\lambda \sim \mathcal{N}(0, \omega_\lambda^2), \tag{71}$$

$$h_t = h_{t-1} + \varepsilon_t^h, \qquad \varepsilon_t^h \sim \mathcal{N}(0, \omega_h^2), \tag{72}$$

$$g_t = g_{t-1} + \varepsilon_t^g, \qquad \varepsilon_t^g \sim \mathcal{N}(0, \omega_g^2), \tag{73}$$

$$q_t = d_0 + d_1\tau_t + \varepsilon_t^q, \qquad \varepsilon_t^q \sim \mathcal{N}(0, \omega_q^2). \tag{74}$$

B Appendix: Estimation Details

In this appendix we provide the details of the priors and estimation for M5, M6 and M7 are outlined in this section. Estimation for M1 and M2 is similar to M5, estimation for M3 and M4 is similar to M6, and estimation for M8 is similar to M7. Thus, for brevity, we omit estimation details for these five models.

B.1 M5

B.1.1 Prior

The parameters under M5 are τ, ν, ϕ,$\widetilde{\lambda}$, λ_0, ω_λ, ω_e^2, ω_ν^2, h, g.

We assume the following priors:

$$\begin{array}{ccccc}
\tau_0 = 0, & \tau_1 \sim \mathcal{N}(\tau_0, V_\tau e^{g_t}), & \lambda_0 \sim \mathcal{N}(a_0, V_{\lambda_0}), & e_0 = 0, & e_{-1} = 0, \\
\omega_\lambda \sim \mathcal{N}(0, V_{\omega_\lambda}), & \omega_g = V_{\omega_g}^{1/2}, & \omega_h = V_{\omega_h}^{1/2}, & \omega_e^2 \sim \mathcal{IG}(\nu_e, S_{\omega_e}), & \omega_\nu^2 \sim, \mathcal{IG}(\nu_\nu, S_{\omega_\nu}), \\
V_{\omega_h} = 0.2, & V_{\omega_g} = 0.2, & V_{\omega_\lambda} = 0.25^2, & \nu_e = 3, & S_{\omega_e} = 1 * (\nu_e - 1), \\
V_{\lambda_0} = 0.25^2, & V_\tau = 10, & V_g = 10, & \nu_\nu = 3, & S_{\omega_\nu} = 1 * (\nu_\nu - 1), \\
a_0 = -0.25, & V_\beta = (V_{\lambda_0}, V_{\omega_\lambda}), & \widehat{\beta} = (a_0, 0), & \phi \sim \mathcal{N}(\phi_0, V_\phi), & V_\phi = \mathrm{I}_2, \\
\boldsymbol{\phi_0} = (0.5; 0.2). & & & &
\end{array}$$

B.1.2 Likelihood

In this section, we derive the densities of $\pi = (\pi_1, \dots, \pi_T)'$ and $u = (u_1, \dots, u_T)'$, which will be used to construct the posterior sampler.

Let

$$\Lambda_\lambda = \mathrm{diag}(\lambda_0 + \omega_\lambda \widetilde{\lambda}_1, \lambda_0 + \omega_\lambda \widetilde{\lambda}_2, \lambda_0 + \omega_\lambda \widetilde{\lambda}_3, \dots, \lambda_0 + \omega_\lambda \widetilde{\lambda}_T).$$

Then, we have

$$\pi - \tau - \Lambda_\lambda(u - \nu) = \varepsilon^\pi.$$

Then, the log conditonal density of π is

$$\log p(\pi \mid \tau, u, \nu, c, \widetilde{\lambda}, \lambda_0, \omega_h, \omega_\lambda, \omega_\nu, \omega_c, h) \propto -\frac{1}{2}(\pi - \tau - \Lambda_\lambda(u-\nu))' S_\pi^{-1}(\pi - \tau - \Lambda_\lambda(u-\nu)),$$

where

$$S_\pi = \mathrm{diag}(e^{h_1}, e^{h_2}, e^{h_3}, \dots, e^{h_T}).$$

Let

$$H_\phi = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ -\phi_1 & 1 & 0 & 0 & \dots & 0 \\ -\phi_1 & -\phi_2 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \dots & \dots & -\phi_1 & -\phi_2 & 1 \end{bmatrix}.$$

Then, we have

$$H_\phi e = \varepsilon^e.$$

Then, the log conditional density of u is

$$\log p(u \mid \nu, e, \omega_\nu, \omega_e, \phi) \propto -\frac{T}{2}\log\omega_e^2 - \frac{1}{2\omega_e^2}(u-\nu)' H_\phi' H_\phi (u-\nu).$$

B.1.3 Sampling τ

In this section, we derive the joint prior density of $\tau = (\tau_1, \dots, \tau_T)'$, which will be used to construct the posterior sampler of τ.

Let

$$S_g = (V_\tau e^{g_1}, e^{g_2}, e^{g_3}, \ldots, e^{g_T}).$$

and

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & 0 \\ -1 & 1 & 0 & 0 & \ldots & 0 \\ 0 & -1 & 1 & 0 & \ldots & 0 \\ 0 & 0 & -1 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \ldots & \ldots & 0 & -1 & 1 \end{bmatrix}.$$

Then, we have

$$H\tau = \varepsilon^\tau.$$

Then,

$$\tau \sim \mathcal{N}(0, H^{-1} S_g H'^{-1}).$$

Then, the log prior density for τ is

$$\log p(\tau) = -\frac{1}{2}\tau' H' S_g^{-1} H\tau.$$

Then, we have

$$\begin{aligned} &\log p(\tau \mid \pi, u, \nu, e, \phi, \gamma, e, \omega_\nu, \widetilde{\lambda}, \lambda_0, \omega_h, \omega_\lambda, \omega_g, h, g) \\ &\propto -\frac{1}{2}(\tau' S_\pi^{-1}\tau - 2\tau' S_\pi^{-1}(\pi - \Lambda_\lambda(u - \nu))) - \frac{1}{2}\tau' H' S_g^{-1} H\tau. \end{aligned}$$

Then, the conditional distribution of τ is

$$\tau \sim \mathcal{N}(\widehat{\tau}, K_\tau^{-1}),$$

where

$$\widehat{\tau} = K_\tau^{-1}((S_\pi^{-1})(\pi - \Lambda_\lambda(u - \nu))), \qquad K_\tau = S_\pi^{-1} + H' S_g^{-1} H.$$

Since K_τ is a band matrix, τ can be sampled using the precision sampler of Chan and Jeliazkov (2009).

B.1.4 sample h and g

we sample h and g, following Kim et al. (1998).

B.1.5 Sample ν

In this section, we construct the posterior sampler of ν.

We have

$$\nu \sim \mathcal{N}(0, H'^{-1}\omega_\nu^2 H^{-1}).$$

Then, the log prior density of ν is

$$\log p(\nu) = -\frac{T}{2}\log \omega_\nu^2 - \frac{1}{2\omega_\nu^2}\nu' H' H \nu.$$

Then, the posterior distribution of ν is

$$\mathcal{N}(\widehat{\nu}, K_\nu^{-1}),$$

where

$$K_\nu = \frac{H_\phi' H_\phi}{\omega_e^2} + \frac{H'H}{\omega_\nu^2} + \Lambda_\lambda' S_\pi^{-1} \Lambda_\lambda$$

and

$$\widehat{\nu} = K_\nu^{-1}(\frac{H_\phi' H_\phi u}{\omega_e^2} - S_\pi^{-1}\Lambda_\lambda(\pi - \tau - \Lambda_\lambda u)).$$

B.1.6 Sample ϕ

In this section, we construct the posterior sampler of ϕ.

Let

$$\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$

and

$$X_\phi = \begin{bmatrix} c_0 & c_{-1} \\ c_1 & c_0 \\ c_3 & c_2 \\ \vdots & \vdots \\ c_{T-1} & c_{T-2} \end{bmatrix},$$

then, we have

$$e = X_\phi \phi + \varepsilon^e.$$

Then, the conditional distribution of ϕ is

$$\phi \sim \mathcal{N}(\widehat{\phi}, K_\phi^{-1}) 1(\phi \in R)$$

where

$$\widehat{\phi} = K_{\phi}^{-1}(V_{\phi}^{-1}\phi_0 + \frac{X_{\phi}'e}{\omega_e^2}), \qquad K_{\phi}^{-1} = V_{\phi}^{-1} + \frac{X_{\phi}'X_{\phi}}{\omega_e^2}.$$

B.1.7 Sample $\widetilde{\lambda}$

In this section, we construct the posterior sampler of $\widetilde{\lambda}$.

We have

$$H\widetilde{\lambda} = \varepsilon^{\widetilde{\lambda}}.$$

Then, $\widetilde{\lambda}$ is distributed as

$$\mathcal{N}(0, H^{-1}H'^{-1}).$$

Then, the log prior density of $\widetilde{\lambda}$ is

$$\log p(\widetilde{\lambda}) = -\frac{1}{2}(\widetilde{\lambda}'H'H\widetilde{\lambda}).$$

Let

$$\Lambda_u = \mathrm{diag}(u_1 - \nu_1, u_2 - \nu_2, u_3 - \nu_3, \ldots, u_T - \nu_T).$$

Then, we have

$$-\frac{1}{2}(\omega_{\lambda}^2\widetilde{\lambda}'\Lambda_u S_{\pi}^{-1}\Lambda_u\widetilde{\lambda}) - 2(\widetilde{\lambda}'\Lambda_u\omega_{\lambda}S_{\pi}^{-1}(\pi - \tau - \lambda_0\Lambda_u)).$$

$\widetilde{\lambda}$ is distributed as

$$\mathcal{N}(\widehat{\widetilde{\lambda}}, K_{\widetilde{\lambda}}^{-1}),$$

where

$$\widehat{\widetilde{\lambda}} = K_{\widetilde{\lambda}}^{-1}(\Lambda_u\omega_{\lambda}S_{\pi}^{-1}(\pi - \tau - \lambda_0\Lambda_u)), \qquad K_{\widetilde{\lambda}} = H'H + \omega_{\lambda}^2\Lambda_u S_{\pi}^{-1}\Lambda_u.$$

B.1.8 Sample λ_0 and ω_λ

In this section, we construct the posterior sampler of λ_0 and ω_λ.

Let $X_\beta = (u - \nu, \Lambda_u \widetilde{\lambda})$ and $\beta = (\lambda_0, \omega_\lambda)'$,

Then, we have

$$\pi - \tau = X_\beta \beta + \varepsilon^\pi.$$

Then, β is distributed as

$$\mathcal{N}(\widehat{\beta}, K_\beta^{-1}),$$

where

$$K_\beta = V_\beta^{-1} + X_\beta' S_\pi^{-1} X_\beta, \qquad \widehat{\beta} = K_\beta^{-1}(V_\beta^{-1}\beta_0 + X_\beta' S_\pi^{-1}(\pi - \tau)).$$

B.1.9 Sample ω_e^2

In this section, we show the posterior sampler of ω_c^2.

The conditional distribution of ω_e^2 is

$$\mathcal{IG}(\nu_e + \frac{T}{2}, S_{\omega_e} + \frac{1}{2}(e - X_\phi \phi)'(e - X_\phi \phi)).$$

B.1.10 Sample ω_ν^2

In this section, we show the posterior sampler of ω_ν^2.

The conditional distribution of ω_ν^2 is

$$\mathcal{IG}(\nu_\nu + \frac{T}{2}, S_{\omega_\nu} + \frac{1}{2}\nu' H' H \nu).$$

B.2 M6

B.2.1 Prior

The parameters under M5 are τ, y^*, γ, ϕ, $\widetilde{\lambda}$, λ_0, ω_λ, ω_c^2, $\omega_{y^*}^2$.

We assume the following priors:

$$\begin{array}{ccccc}
\tau_0 = 0, & \tau_1 \sim \mathcal{N}(\tau_0, V_\tau e^{g_t}), & \lambda_0 \sim \mathcal{N}(a_0, V_{\lambda_0}), & c_0 = 0, & c_{-1} = 0, \\
\omega_\lambda \sim \mathcal{N}(0, V_{\omega_\lambda}), & \omega_g = V_{\omega_g}^{1/2}, & \omega_h = V_{\omega_h}^{1/2}, & \omega_c^2 \sim \mathcal{IG}(\nu_c, S_{\omega_c}), & \omega_{y^*}^2 \sim \mathcal{U}(0, V_{\omega_{y^*}}), \\
V_{\omega_h} = 0.2, & V_{\omega_g} = 0.2, & V_{\omega_\lambda} = 0.25^2, & \nu_c = 3 & S_{\omega_c} = 1 * (\nu_c - 1), \\
V_{\lambda_0} = 0.25^2, & V_\tau = 10, & V_g = 10, & \gamma \sim \mathcal{N}(\gamma_0, V_\gamma) & V_{\omega_{y^*}} = 0.001, \\
a_0 = -0.25, & V_\beta = (V_{\lambda_0}, V_{\omega_\lambda}), & \widehat{\beta} = (a_0, 0), & V_\gamma = 100 * \mathrm{I}_2, & V_\phi = \mathrm{I}_2, \\
\gamma_0 = (750; 750), & \phi \sim \mathcal{N}(\phi_0, V_\phi), & \phi_0 = (1.34; -0.7). & &
\end{array}$$

B.2.2 Likelihood

In this section, we derive the densities of $\pi = (\pi_1, \ldots, \pi_T)'$ and $y = (y_1, \ldots, y_T)'$, which will be used to construct the posterior sampler.

Let

$$\Lambda_\lambda = \mathrm{diag}(\lambda_0 + \omega_\lambda \widetilde{\lambda}_1, \lambda_0 + \omega_\lambda \widetilde{\lambda}_2, \lambda_0 + \omega_\lambda \widetilde{\lambda}_3, \ldots, \lambda_0 + \omega_\lambda \widetilde{\lambda}_T).$$

Then, we have

$$\pi - \tau - \Lambda_\lambda (y - y^*) = \varepsilon^\pi.$$

Then, the log conditional density of π is

$$\log p(\pi \mid \tau, y, y^*, c, \widetilde{\lambda}, \lambda_0, \omega_h, \omega_\lambda, \omega_{y^*}, \omega_c, h) \propto -\frac{1}{2}(\pi - \tau - \Lambda_\lambda (y - y^*))' S_\pi^{-1} (\pi - \tau - \Lambda_\lambda (y - y^*)),$$

where

$$S_\pi = \mathrm{diag}(e^{h_1}, e^{h_2}, e^{h_3}, \ldots, e^{h_T}).$$

Let

$$H_\phi = \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & 0 \\ -\phi_1 & 1 & 0 & 0 & \ldots & 0 \\ -\phi_1 & -\phi_2 & 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \ldots & \ldots & -\phi_1 & -\phi_2 & 1 \end{bmatrix}.$$

Then, we have

$$y = y^* + c,$$
$$H_\phi c = \varepsilon^c.$$

Then, the log conditional density of y is

$$\log p(y \mid y^*, c, \omega_{y^*}, \omega_c, \phi, y_0^*, y_{-1}^*) \propto -\frac{T}{2}\log \omega_c^2 - \frac{1}{2\omega_c^2}(y - y^*)' H_\phi' H_\phi (y - y^*)$$

B.2.3 Sampling τ

In this section, we derive the densities of $\tau = (\tau_1, \ldots, \tau_T)'$, which will be used to construct the posterior sampler of τ.

Let

$$S_g = (V_\tau e^{g_1}, e^{g_2}, e^{g_3}, \dots, e^{g_T})$$

and

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 1 & 0 & \dots & 0 \\ 0 & 0 & -1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \dots & \dots & 0 & -1 & 1 \end{bmatrix}.$$

Then, we have

$$H\tau = \varepsilon^\tau.$$

Then,

$$\tau \sim N(0, H^{-1} S_g H'^{-1}).$$

Then, the log prior density for τ is

$$\log p(\tau) = -\frac{1}{2}\tau' H' S_g^{-1} H\tau.$$

and we have

$$\begin{aligned} &\log p(\tau \mid \pi, y, y^*, c, \phi, \gamma, \omega_c, \omega_{y^*}, \widetilde{\lambda}, \lambda_0, \omega_h, \omega_\lambda, \omega_g, h, g) \\ &\propto -\frac{1}{2}(\tau' S_\pi^{-1}\tau - 2\tau' S_\pi^{-1}(\pi - \Lambda_\lambda(y - y^*))) - \frac{1}{2}\tau' H' S_g^{-1} H\tau. \end{aligned}$$

Then, the conditional distribution of τ is

$$\tau \sim \mathcal{N}(\widehat{\tau}, K_\tau^{-1}),$$

where

$$\widehat{\tau} = K_{\tau}^{-1}((S_{\pi}^{-1})(\pi - \Lambda_{\lambda}(y - y^{*}))), \qquad K_{\tau} = S_{\pi}^{-1} + H' S_{g}^{-1} H.$$

B.2.4 Sample h and g

we sample h and g, following Kim et al. (1998).

B.2.5 Sample y^*

In this section, we construct the posterior sampler of y^*. Let

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & -2 & 0 & 0 & \dots & 0 \\ 0 & 1 & -2 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \dots & \dots & 1 & -2 & 1 \end{bmatrix}.$$

then we have

$$H_2 y^* = \widetilde{\alpha}_{y^*} + \varepsilon^{y^*}$$

where

$$\widetilde{\alpha}_{y^*} = (y_0^* + \Delta y_0^*, -y_0^*, 0, \dots, 0)'.$$

Let

$$\alpha_{y^*} = H_2^{-1} \widetilde{\alpha}_{y^*}.$$

Then,

$$y^* \sim N(\alpha_{y^*}, \omega_{y^*}^2 (H_2' H_2)^{-1}).$$

Then, the log prior density of y^* is

$$\log p(y^*) = -\frac{T}{2}\log\omega_{y^*}^2 - \frac{1}{2\omega_{y^*}^2}(y^* - \alpha_{y^*})' H_2' H_2 (y^* - \alpha_{y^*}).$$

The posterior distribution of y^* is

$$\mathcal{N}(\widehat{y^*}, K_{y^*}^{-1}),$$

where

$$K_{y^*} = \frac{H_\phi' H_\phi}{\omega_c^2} + \frac{H_2' H_2}{\omega_{y^*}^2} + \Lambda_\lambda' S_\pi^{-1} \Lambda_\lambda$$

and

$$\widehat{y^*} = K_{y^*}^{-1}\left(\frac{H_\phi' H_\phi y}{\omega_c^2} + \frac{H_2' H_2 \alpha_{y^*}}{\omega_{y^*}^2} - S_\pi^{-1}\Lambda_\lambda(\pi - \tau - \Lambda_\lambda y)\right).$$

B.2.6 Sample ϕ

In this section, we construct the posterior sampler of ϕ.

Let

$$\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$

and

$$X_\phi = \begin{bmatrix} c_0 & c_{-1} \\ c_1 & c_0 \\ c_3 & c_2 \\ \vdots & \vdots \\ c_{T-1} & c_{T-2} \end{bmatrix}$$

then, we have

$$c = X_\phi \phi + \varepsilon^c.$$

Then, the conditional distribution of ϕ is

$$\phi \sim \mathcal{N}(\widehat{\phi}, K_\phi^{-1}) 1(\phi \in R)$$

where

$$\widehat{\phi} = K_\phi^{-1}(V_\phi^{-1}\phi_0 + \frac{X'_\phi c}{\omega_c^2}), \qquad K_\phi^{-1} = V_\phi^{-1} + \frac{X'_\phi X_\phi}{\omega_c^2}$$

B.2.7 Sample $\boldsymbol{\gamma}$

In this section, we construct the posterior sampler of γ.

Let

$$\boldsymbol{\gamma} = (y_0^*, y_{-1}^*)$$

and

$$\alpha_{y^*} = \begin{bmatrix} 2y_0^* - y_{-1}^* \\ 3y_0^* - 2y_{-1}^* \\ \cdots \\ (T+1)y_0^* - Ty_{-1}^* \end{bmatrix} = X_\gamma \gamma$$

where

$$X_{\boldsymbol{\gamma}} = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ \vdots & \vdots \\ T+1 & -T \end{bmatrix}.$$

Then,

$$y^* = X_\gamma \boldsymbol{\gamma} + H_2^{-1}\varepsilon^{y^*}.$$

Then, the conditional distribution of γ is

$$\mathcal{N}(\widehat{\gamma}, K_{\boldsymbol{\gamma}}^{-1})$$

where

$$\widehat{\gamma} = K_{\boldsymbol{\gamma}}^{-1}(V_\gamma^{-1}\gamma_0 + \frac{X_\gamma' H_2' H_2 y^*}{\omega_{y*}^2}), \qquad K_\gamma = V_\gamma^{-1} + \frac{X_\gamma' H_2' H_2 X_\gamma}{\omega_{y^*}^2}.$$

B.2.8 Sample $\widetilde{\lambda}$

In this section, we construct the posterior sampler of $\widetilde{\lambda}$.

Let

$$H\widetilde{\lambda} = \varepsilon^{\widetilde{\lambda}}.$$

Then, $\widetilde{\lambda}$ is distributed as

$$N(0, H^{-1}H'^{-1}).$$

Then, the log prior density of $\widetilde{\lambda}$ is

$$\log p(\widetilde{\lambda}) = -\frac{1}{2}(\widetilde{\lambda}' H' H \widetilde{\lambda}).$$

Let

$$\Lambda_y = \mathrm{diag}(y_1 - y_1^*, y_2 - y_2^*, y_3 - y_3^*, \ldots, y_T - y_T^*).$$

Then, we have

$$-\frac{1}{2}(\omega_\lambda^2 \widetilde{\lambda}' \Lambda_y S_\pi^{-1} \Lambda_y \widetilde{\lambda}) - 2(\widetilde{\lambda}' \Lambda_y \omega_\lambda S_\pi^{-1}(\pi - \tau - \lambda_0 \Lambda_y)).$$

$\widetilde{\lambda}$ is distributed as

$$\mathcal{N}(\widehat{\widetilde{\lambda}}, K_{\widetilde{\lambda}}^{-1}),$$

where

$$\widehat{\widetilde{\lambda}} = K_{\widetilde{\lambda}}^{-1}(\Lambda_y \omega_\lambda S_\pi^{-1}(\pi - \tau - \lambda_0 \Lambda_y)), \qquad K_{\widetilde{\lambda}} = H'H + \omega_\lambda^2 \Lambda_y S_\pi^{-1} \Lambda_y.$$

B.2.9 Sample λ_0 and ω_λ

In this section, we construct the posterior sampler of λ_0 and ω_λ.

Let $X_\beta = (y - y^*, \Lambda_y \widetilde{\lambda})$ and $\beta = (\lambda_0, \omega_\lambda)'$, then, (50) can be written as

$$\pi - \tau = X_\beta \beta + \varepsilon^\pi.$$

Then, β is distributed as

$$\mathcal{N}(\widehat{\beta}, K_\beta^{-1}),$$

where

$$K_\beta = V_\beta^{-1} + X_\beta' S_\pi^{-1} X_\beta, \qquad \widehat{\beta} = K_\beta^{-1}(V_\beta^{-1}\beta_0 + X_\beta' S_\pi^{-1}(\pi - \tau)).$$

B.2.10 Sample ω_c^2

In this section, we show the posterior sampler of ω_c^2.

The conditional distribution of ω_c^2 is

$$\mathcal{IG}(\nu_c + \frac{T}{2}, S_{\omega_c} + \frac{1}{2}(c - X_\phi \phi)'(c - X_\phi \phi)).$$

B.2.11 Sample ω_{y^*}

In this section, we show the posterior sampler of ω_{y^*}.

The conditional density of ω_{y^*} is not a standard density, however, can be sampled by using Griddy-Gibbs .

B.3 M7

In equation (65) we link Blue Chip ten years inflation forecasts to trend inflation, so the differences of estimation details between M7 and M5 are that M7 have different sampler for τ_t and has two more samplers for $d = (d_0 \quad d_1)$ and ω_q^2. For brevity, we only display the estimation details for τ_t, $d = (d_0 \quad d_1)$, and ω_q^2.

B.3.1 Prior

$$
\begin{array}{ccccc}
\tau_0 = 0, & \tau_1 \sim \mathcal{N}(\tau_0, V_\tau e^{g_t}), & \lambda_0 \sim \mathcal{N}(a_0, V_{\lambda_0}), & e_0 = 0, & e_{-1} = 0, \\
\omega_\lambda \sim \mathcal{N}(0, V_{\omega_\lambda}), & \omega_g \sim \mathcal{N}(0, V_{\omega_g}), & \omega_h \sim \mathcal{N}(0, V_{\omega_h}), & \omega_e^2 \sim \mathcal{IG}(\nu_e, S_{\omega_e}), & \omega_\nu^2 \sim, \mathcal{IG}(\nu_\nu, S_{\omega_\nu}), \\
V_{\omega_h} = 0.2, & V_{\omega_g} = 0.2, & V_{\omega_\lambda} = 0.25^2, & \nu_e = 3, & S_{\omega_e} = 1 * (\nu_e - 1), \\
V_{\lambda_0} = 0.25^2, & V_\tau = 10, & V_g = 10, & \nu_\nu = 3, & S_{\omega_\nu} = 1 * (\nu_\nu - 1), \\
a_0 = -0.25, & V_\beta = (V_{\lambda_0}, V_{\omega_\lambda}), & \widehat{\beta} = (a_0, 0), & \phi \sim \mathcal{N}(\phi_0, V_\phi), & V_\phi = \mathrm{I}_2, \\
\phi_0 = (0.5; 0.2), & \omega_q^2 \sim \mathcal{IG}(\nu_q, S_{\omega_q}), & \nu_q = 3, & S_{\omega_q} = 1 * (\nu_q - 1), & \mu_d = (0 \quad 1), \\
V_d = \mathrm{I}_2. & & & &
\end{array}
$$

B.3.2 Likelihood

In this section, we derive the densities of $\pi = (\pi_1, \dots, \pi_T)'$, which will be used to construct the posterior sampler.

Let

$$\Lambda_\lambda = \mathrm{diag}(\lambda_0 + \omega_\lambda \widetilde{\lambda}_1, \lambda_0 + \omega_\lambda \widetilde{\lambda}_2, \lambda_0 + \omega_\lambda \widetilde{\lambda}_3, \dots, \lambda_0 + \omega_\lambda \widetilde{\lambda}_T).$$

Then, we have

$$\pi - \tau - \Lambda_\lambda (u - \nu) = \varepsilon^\pi.$$

Then, the log conditional density of π is

$$\log p(\pi \mid \tau, u, \nu, c, \widetilde{\lambda}, \lambda_0, \omega_h, \omega_\lambda, \omega_\nu, \omega_c, \omega_q, d, h) \propto -\frac{1}{2}(\pi - \tau - \Lambda_\lambda(u - \nu))' S_\pi^{-1} (\pi - \tau - \Lambda_\lambda(u - \nu)),$$

where

$$S_\pi = \mathrm{diag}(e^{h_1}, e^{h_2}, e^{h_3}, \dots, e^{h_T}).$$

B.3.3 Sampling τ

In this section, we construct the posterior sampler of τ.

Let

$$S_g = (V_\tau e^{g_1}, e^{g_2}, e^{g_3}, \dots, e^{g_T}).$$

and

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 1 & 0 & \dots & 0 \\ 0 & 0 & -1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \dots & \dots & 0 & -1 & 1 \end{bmatrix}.$$

Then, we have

$$H\tau = \varepsilon^\tau.$$

Then,

$$\tau \sim \mathcal{N}(0, H^{-1} S_g H'^{-1}).$$

Then, the log prior density for τ is

$$\log p(\tau) = -\frac{1}{2}\tau' H' S_g^{-1} H\tau.$$

(74) can be written as

$$q = d_0 1_T + d_1 \tau + \varepsilon^z, \qquad \varepsilon^z \sim \mathcal{N}(0, \omega_q^2 \mathrm{I}_T).$$

Therefore, we have

$$\log(p(q|d_0, d_1, \tau, \omega_q^2)) \propto -\frac{1}{2\omega_q^2 \mathrm{I}_T}(q - d_0 1_T - d_1\tau)'(q - d_0 1_T - d_1\tau).$$

Then, we have

$$\log p(\tau \mid \pi, u, \nu, e, \phi, \gamma, \omega_e, \omega_\nu, \widetilde{\lambda}, \lambda_0, \omega_h, \omega_\lambda, \omega_g, \omega_q, d, h, g)$$
$$\propto -\frac{1}{2}(\tau' S_\pi^{-1}\tau - 2\tau' S_\pi^{-1}(\pi - \Lambda_\lambda(u - \nu))) - \frac{1}{2}\tau' H' S_g^{-1} H\tau - \frac{1}{2}(\tau' \frac{d_1^2}{\omega_q^2 I_T}\tau - 2\tau' \frac{d_1(q - d_0 1_T)}{\omega_q^2}).$$

Then, the conditional distribution of τ is

$$\tau \sim \mathcal{N}(\widehat{\tau}, K_\tau^{-1}),$$

where

$$\widehat{\tau} = K_\tau^{-1}((S_\pi^{-1})(\pi - \Lambda_\lambda(u - \nu)) + \frac{d_1(q - d_0 1_T)}{\omega_q^2}), \qquad K_\tau = S_\pi^{-1} + H' S_g^{-1} H + \frac{d_1^2}{\omega_q^2 \mathrm{I}_T}.$$

B.3.4 Sampling d

In this section, we construct the posterior sampler of d. Let

$$X_\tau = (1_T \quad \tau),$$
$$d = (d_0 \quad d_1).$$

Then (74) can be written as

$$q = X_\tau d + \varepsilon^q.$$

Then we have

$$\log(p(q|d_0, d_1, \tau, \omega_q^2)) \propto -\frac{1}{2\omega_q^2}(q - X_\tau d)'(q - X_\tau d).$$

we also have

$$\log(p(d)) \propto -\frac{1}{2}(d - \mu_d)' V_d^{-1}(d - \mu_d).$$

Then, the posterior d is distributed as $\mathcal{N}(\widehat{d}, K_d^{-1})$, where

$$K_d = (\frac{X_\tau' X_\tau}{\omega_q^2} + V_d^{-1}),$$
$$\widehat{d} = K_d^{-1}(\frac{X_\tau' q}{\omega_q^2} + V_d^{-1}\mu_d).$$

B.3.5 Sampling ω_q^2

In this section, we show the posterior sampler of ω_q^2.

$$\omega_q^2 \sim \mathcal{IG}(\nu_{\omega_q^2} + \frac{T}{2}, S_{\omega_q} + \frac{1}{2}\sum_{t=1}^{T} \varepsilon_{q,t}^2).$$

C Appendix: Additional Results

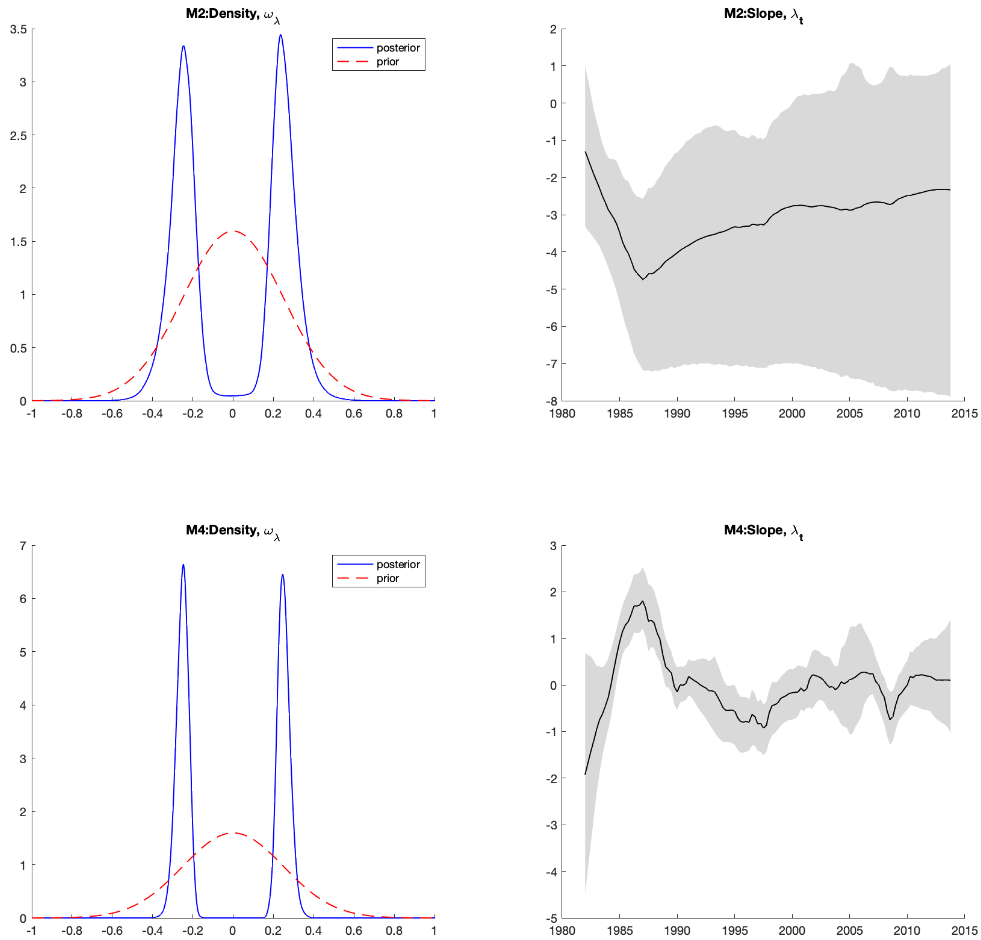

Figure 6: Estimated Slope λ_t and Density of ω_λ for M2 and M4 with University of Michigan Inflation Expectation Survey Data.

Figure 6 shows that using University of Michigan inflation expectation survey data, M2 and M4 still have similar time-varying λ_t with the case of using SPF data case in M2 and M4. Also, Figure 6 shows that the posterior densities of ω_λ under M2 and M4 are bimodal and have almost no mass around 0 in the University of Michigan inflation expectation survey data case. The log Bayes factors associated with M2 and M4 using University of

Michigan inflation expectation survey data are 4.0 (0.71) and 58.3 (3.84), respectively. They are close to 4.1 (0.12) and 57.2 (3.57), which are the log Bayes factor associated with M2 and M4 in Table 2.