Synthesis of Frequency-invariant Beam Patterns under Accurate Sidelobe Control by Second-order Cone Programming

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Abstract— It is shown in this work that the FI pattern synthesis can be treated as an optimization problem for minimizing the mainlobe frequency variation. To control both the mainlobe and sidelobe regions, we introduce several constraints imposed on the broadband pattern, called the look-direction constraint, the spatial response variation constraint and the sidelobe constraint, respectively. The whole optimization process needs to perform the SOCP solver. A synthesis of FI pattern with low sidelobe level (SLL) is given to validate the accuracy and effectiveness of the proposed method.

1. INTRODUCTION

Frequency-invariant (FI) arrays can receive broadband signals of interest without waveform distortion, and they have been widely applied in sonar, radar, communication, imaging systems and other fields. In general, the FI array is implemented by a filter-and-sum beamforming structure. The finite-impulse-response filter coefficients for each antenna channel can be optimized to obtain a broadband frequency invariant beam pattern. In the past decades, many practical methods have been developed to design the FI beamformers. They include, analytical robust synthesis method [1, 2], Fourier transform method [3, 4], stochastic optimization method [5] and convex optimization method [6–9]. Relatively speaking, both the analytical robust synthesis method and Fourier transform method can synthesize FI patterns with high efficiency. However, due to the usage of the reference pattern or function, the solution space can be reduced, which causes that the synthesized broadband patterns cannot be accurate. Besides, it is not easy to artificially preset a suitable reference pattern or function. On the contrary, both the stochastic optimization method and convex optimization method can synthesize broadband patterns with excellent performance in terms of both mainlobe FI property and sidelobe distribution control. Although these methods can take much more time than the previous two, this time cost is worthwhile when accurate FI patterns are required. In the convex optimization method, the FI pattern synthesis problem is formulated as minimizing the mainlobe pattern variation over the interested frequency band. In addition, multiple constraints can be imposed on the broadband pattern to individually control the mainlobe and sidelobe regions. In this work, we will present the required constraints, including the look-direction constraint, the spatial response variation constraint and the sidelobe constraint. Then, we can solve the whole optimization problem by means of second-order cone programming (SOCP). Some powerful SOCP solvers have been proposed, and we choose the self-dual-minimization (SeDuMi) tool for the synthesis. An example for synthesizing FI patterns with low uniform sidelobe level (SLL) is conducted to verify the effectiveness and advantages of the proposed method. Synthesis results show that this method can indeed obtain accurate FI pattern performance in terms of both mainlobe FI property and sidelobe control.

2. MATHEMATICAL FORMULATION

Consider a uniformly-spaced linear array, which is composed of N elements allocated at $x_n = -(N-1)\Delta_x/2 + n\Delta_x$ $(n = 0, 1, \ldots, N-1)$ where Δ_x is the element spacing. Besides, each element is connected to a L-length FIR filter used to provide the required frequency-dependent excitation. Assume that a broadband signal with angle frequency $\omega \in \Omega$ illuminates this array from the direction $\theta \in \Theta$ measured from the array broadside, and the array pattern is given as

$$F(\omega, \theta) = \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} h_{l,n} e^{-j\omega l T_s} e^{-j\omega \tau_n(\theta)} \tag{1}$$

where $h_{l,n}$ is the lth coefficient of the nth FIR filter, T_s is the temporal sampling interval, and $\tau_n(\theta) = [-(N-1)/2 + n]\Delta_x \sin\theta/c$ is the time delay where c is the wave propagation velocity in the medium. We can transform (1) into a form of matrix product

$$F(\omega,\theta) = \mathbf{s}^T(\omega,\theta)\breve{\mathbf{h}} \tag{2}$$

where

$$\begin{aligned}
\mathbf{s}(\omega,\theta) &= \mathbf{s}_\tau(\omega,\theta) \otimes \mathbf{s}_t(\omega) &(3)\\
\mathbf{s}_t(\omega) &= [1, e^{-j\omega T_s}, \ldots, e^{-j\omega(L-1)T_s}]^T &(4)\\
\mathbf{s}_\tau(\omega,\theta) &= [e^{-j\omega\tau_0(\theta)}, e^{-j\omega\tau_1(\theta)}, \ldots, e^{-j\omega\tau_{N-1}(\theta)}]^T &(5)\\
\breve{\mathbf{h}} &= \{[\mathbf{h}_0]^T, [\mathbf{h}_1]^T, \ldots, [\mathbf{h}_{N-1}]^T\}^T &(6)\\
\mathbf{h}_n &= [h_{0,n}, h_{1,n}, \ldots, h_{L-1,n}]^T &(7)
\end{aligned}$$

Note that $\otimes$ denotes the Kronecker product of matrix.

The synthesis for a FI pattern can be treated as an optimization problem aiming to minimize the mainlobe frequency variation of the broadband pattern. Besides, in order to control both the mainlobe and sidelobe regions, we have developed multiple constraints imposed on the broadband pattern. Therefore, the whole optimization problem can be formulated as

$$\begin{aligned}
&\min \epsilon\\
&\text{Const.}\begin{cases}
|\mathbf{s}^T(\omega_{\text{ref}},\theta)\breve{\mathbf{h}} = | = 1 \ \& \ \frac{\partial\{Re[\mathbf{s}^T(\omega_{\text{ref}},\theta)]\}}{\partial\theta}\breve{\mathbf{h}} = 0, \quad \text{for} \ \ \theta = \theta_{\text{look}};\\
\frac{\int_\Omega p(\omega)|\mathbf{s}^T(\omega,\theta)\breve{\mathbf{h}} - \mathbf{s}^T(\omega_{\text{ref}},\theta)\breve{\mathbf{h}}|^2 d\omega}{\int_\Omega g(\omega)d\omega} \le \epsilon, \quad \text{for} \ \ \omega \in \Omega, \quad \theta \in \Theta_{\text{ML}};\\
|\mathbf{s}^T(\omega,\theta)\breve{\mathbf{h}}|^2 \le \Gamma_{\text{SL}}, \quad \text{for} \ \omega \in \Omega, \quad \theta \in \Theta_{\text{SL}}.
\end{cases}
\end{aligned} \tag{8}$$

In the above objective function, $\epsilon > 0$ is an auxiliary variable which can be regarded as the tolerance of the broadband response variation respect to its value at $\omega_{\text{ref}} = \sqrt{\omega_L\omega_U}$, i.e., the reference angle frequency. Beside, the first constraint is called the look-direction constraint, where θ_{look} is the desired maximum radiation direction. The purpose of this constraint is to avoid the maximum array response deviating from the desired direction. Then, the second constraint, called the spatial response variation constraint, is designed to reduce the mainlobe frequency variation, where $p(\omega)$ is a positive frequency-domain weighting function, and Θ_{ML} is the angle range of the mainlobe region. The last constraint is the sidelobe constraint, where Γ_{SL} is the upper boundary of the broadband pattern within the sidelobe angle range Θ_{SL}. This constraint can restrict the sidelobe level (SLL), and we also can adjust the upper boundary to generate broad nulls in the synthesized broadband pattern. The whole optimization process in (8) is required to perform the SOCP solver. For solving the SOCP problem, there are some powerful optimization toolboxes, and we use the SeDuMi tool in this work.

3. NUMERICAL RESULTS

In this example, we apply the proposed method to synthesize a FI pattern with low SLL. Assume that the desired broadband pattern shows good FI property within the mainlobe region of $|\theta| \le 19°$ over the frequency band of $f \in [0.4, 1]$ GHz. The SLL of the broadband pattern is expected to less than -20 dB for $|\theta| > 19°$. We consider a linear array with 15 elements, each connected to a 16-length FIR filter. The element spacing is set to $\lambda_U/2$, where λ_U is the wavelength at $f_U = 1$ GHz ($c = 3 \times 10^8$ m/s). The temporal sampling interval is set to $T_s = 1/(2\ \text{GHz})$. In addition, the frequency range and the angle range are uniformly discretized with $\Delta_f = 3.75 \times 10^{-2}$ GHz and $\Delta_\theta = 3°$, respectively (i.e., $K = 17$ and $M = 61$ for the number of frequencies and angles, respectively). The optimization problem (8) is then solve by the SOCP. The synthesized broadband pattern is shown in the Fig. 1. As can be seen, the obtained broadband pattern presents satisfactory FI property. There are $\hat{M} = 13$ interested angles in the mainlobe region, and we define a frequency-variation factor to evaluate the frequency variation of them, that is

$$\overline{\sigma} = \sqrt{\frac{1}{K\hat{M}}\sum_m\sum_k \{20\lg|F(\omega_k,\theta_m)| - \mu(\theta_m)\}^2} \tag{9}$$

where

$$\mu(\theta_m) = \frac{1}{K}\sum_k 20\lg|F(\omega_k, \theta_m)| \tag{10}$$

For this synthesis, the $\overline{\sigma}$ is calculated as 0.17 dB, which indicates that the synthesized broadband pattern shows excellent FI property. Besides, the achievable SLL is −20.08 dB, which satisfies the sidelobe requirement. Fig. 2(a) demonstrates the FIR filter coefficients for generating the broadband pattern, and Fig. 2(b) shows the corresponding broadband excitations defined by

$$H(\omega, n) = \sum_{l=0}^{L-1} h_{l,n} e^{-j\omega l T_s} \tag{11}$$

For a FI pattern, the corresponding broadband excitations is frequency-variant in general. This synthesis validates that the broadband pattern synthesized by the proposed method can present accurate performance in terms of both mainlobe FI property and sidelobe control.

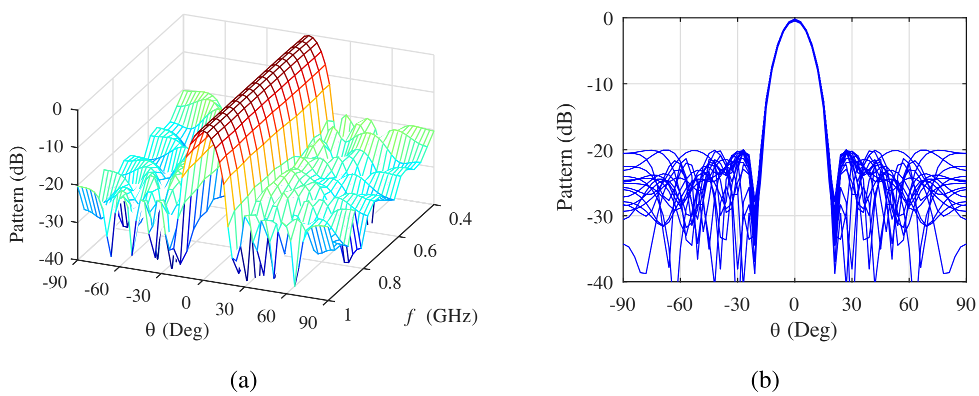

Figure 1: The synthesized FI pattern with −20 dB uniform SLL over [0.4, 1] GHz. (a) Joint space-frequency distribution. (b) Patterns at discrete frequencies.

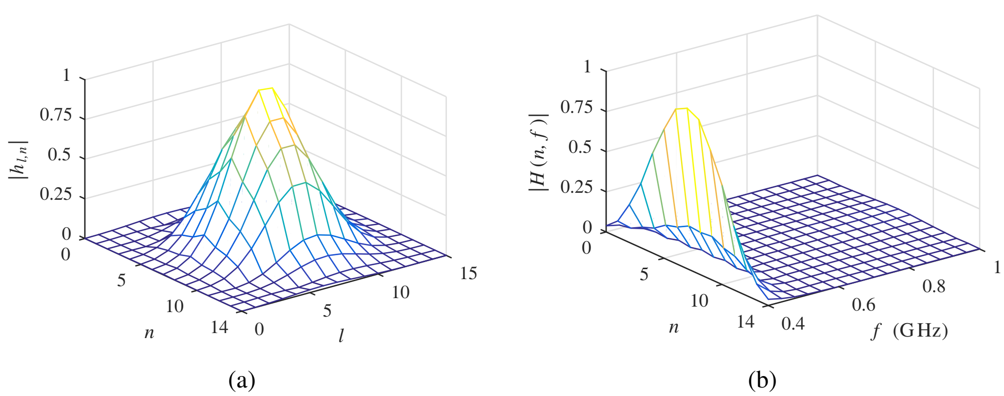

Figure 2: (a) The obtained normalized amplitudes of the FIR filter coefficients and (b) the broadband excitation distribution, corresponding to the FI pattern with low SLL shown in Fig. 1.

4. CONCLUSION

The synthesis for a FI pattern can be transformed into an optimization problem, and several constraints can be introduced. Theses constraints are designed to avoid the deflection of the maximum radiation direction, reduce the mainlobe frequency variation, and restrict the threshold of the sidelobe distribution, respectively. The optimization problem can be solved by the SOCP solver, such as the SeDuMi tool. The synthesized results validate that the proposed method can obtain accurate FI pattern performance in terms of both mainlobe FI property and sidelobe control.

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