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Synthesizing Unequally Spaced Pattern-Reconfigurable Linear Arrays With Minimum Interspacing Control

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ABSTRACT Previously, the alternating convex optimization (ACO) was used to reduce the number of elements in the single-pattern linear array. This work extends the ACO method to synthesize the unequally spaced sparse linear arrays with reconfigurable multiple patterns. In this extended ACO, the minimum interspacing constraint can be easily incorporated in the sparse array synthesis by performing a set of constrained alternating convex optimizations. Three examples for synthesizing sparse linear array with different multiple-pattern requirements are conducted to validate the effectiveness, robustness, and advantages of the proposed method. The synthesis results show that the proposed method can effectively reduce the number of elements in the reconfigurable multiple-pattern linear arrays with good control of the sidelobe levels and minimum interspacing. The comparisons with other methods are also given in the examples.

INDEX TERMS Unequally spaced linear array, alternating convex optimization, pattern-reconfigurable array, minimum interspacing control.

I. INTRODUCTION

Antenna arrays with reconfigurable multiple shaped patterns achieved by varying only excitation distributions have been applied in many applications including multi-functional radars, remote sensing and wireless communications [1]–[4]. The capability of generating reconfigurable multiple patterns provides us a possibility of replacing a multiple-antenna system with a single pattern-reconfigurable antenna. Obviously, the usage of pattern-reconfigurable antenna arrays can reduce the number of total antennas and consequently save the weight, space as well as the cost of the whole system hardware.

In the past decades, many advanced techniques have been developed to synthesize the array with reconfigurable multiple patterns. These techniques mainly includes alternating projection approaches [5], [6], the modified Woodward-Lawson technique [7], stochastic optimization algorithms [8]–[11], and some other techniques [12]–[15]. For most of them, multiple patterns are generated by varying the excitation phases with the common prefixed or optimized amplitudes based on a prescribed element positions with a uniform spacing in general. Such synthesis can reduce the complexity of designing the feeding network, but meantime reduce the freedoms of degrees in the point of view of pattern synthesis. In particular, when multiple complicated patterns with different shapes are required, the array synthesis with a uniform spacing may require a large number of elements to simultaneously achieve the multiple pattern characteristics.

Optimizing antenna element positions can provide additional degrees of freedoms to improve the array synthesis performance, for example, reducing the total number of elements required for the desired pattern characteristics. Plenty of synthesis methods for unequally spaced arrays have been developed, and however, most of them are presented to design single-beam unequally spaced arrays [16]–[23]. These single-beam sparse array synthesis techniques cannot be directly extended to deal with the case of reconfigurable multiple patterns since the best element positions usually change with different pattern requirements. Nevertheless, several recent techniques have been successfully generalized to find the best common element positions for multiple-

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pattern nonuniformly spaced arrays, such as the extended matrix pencil methods (MPM) [24] and enhanced unitary MPM [25], the multiple measurement vectors FOCal underdetermined system solver (M-FOCUSS) [26], and the joint sparse recovery techniques [27], [28]. Among them, the first three methods including the extended MPM, the enhanced unitary MPM and the M-FOCUSS, can be considered as a kind of sparse array reconstruction by matching the synthesized patterns to multiple reference ones in both mainlobe and sidelobe regions. Clearly, artificially presetting the sidelobe distributions for multiple references patterns is not easy, and the help of some other synthesis methods would be required to generate the reference pattern. In addition, these techniques usually measure the pattern matching accuracy in terms of ℓ_2-norm error, and consequently the pattern reconstruction accuracy may deteriorate at low sidelobe and null region. The joint sparse recovery technique in [27] formulates the multiple-pattern synthesis problem as a mixed ℓ_2/ℓ_1-norm optimization under multiple convex constraints, and both the mainlobe shape and sidelobe level for multiple patterns can be easily controlled by using multiple pattern constraints. However, this technique cannot constrain the minimum interspacing, just like many other reweighted ℓ_1-norm optimization techniques used in the single-beam case [29], [30]. The synthesized array positions may be impractical due to physical size limitation of antenna elements.

Recently, an alternating convex optimization (ACO) method is firstly proposed in [31] to synthesize single-beam sparse linear arrays with minimum interspacing control. In this method, a set of alternating weighted ℓ_1-norm optimizations are performed and the element excitation vector and weighting vector are alternately chosen as the optimization variables. In particular, the weight vector is obtained by performing a constrained convex optimization problem instead of being simply assigned according to the excitation vector at the previous step in the reweighted ℓ_1-norm optimization technique. Consequently, the minimum interspacing control can be easily implemented by imposing constraints in the optimization of the weight vector. In this work, we further extend the ACO method to synthesize a sparse linear array with reconfigurable multiple patterns. The extended ACO method can obtain a sparse linear array generating satisfactory multiple pattern results with accurate sidelobe level and nulling region control, and the minimum interspacing can be also constrained as expected. Several synthesis experiments for different reconfigurable pattern requirements are provided to validate the effectiveness and advantages of the proposed method. The comparisons with some other methods are also provided in the examples.

II. FORMULATION AND ALGORITHM

A. MULTIPLE-PATTERN UNEQUALLY SPACED LINEAR ARRAY SYNTHESIS PROBLEM

The problem of synthesizing a unequally spaced sparse linear array with reconfigurable patterns can be formulated as that of finding the best common element positions with optimized multiple excitation distributions for the desired multiple pattern characteristics. Let us consider a linear array with N initial elements which are located at Z-axis with a uniform spacing of d. Assume the initial positions are closely distributed as to provide enough position candidates for selection. That is, we assume $d \ll \lambda$ for the initial array. The mth ($m = 1, 2, \ldots, M$) array pattern under isotropic element assumption can be given by

$$F^{(m)}(\theta) = \sum_{n=1}^{N} w_n^{(m)} e^{-j\beta nd\cos\theta} \tag{1}$$

where $j = \sqrt{-1}$, $\beta = 2\pi/\lambda$ is the free space wave number, and $w_n^{(m)}$ is the complex excitation of the nth element for the mth array pattern. This pattern can be rewritten in the form of matrix product,

$$F^{(m)}(\theta) = \mathbf{a}^T(\theta)\mathbf{w}^{(m)} \tag{2}$$

where

$$\mathbf{a}(\theta) = [e^{-j\beta d\cos\theta}, e^{-j2\beta d\cos\theta}, \ldots, e^{-jN\beta d\cos\theta}]^T \tag{3}$$
$$\mathbf{w}^{(m)} = [w_1^{(m)}, w_2^{(m)}, \ldots, w_N^{(m)}]^T \tag{4}$$

In the sparse array synthesis problem, most elements must be discarded from the predefined densely spaced array. However, in the reconfigurable multiple-pattern case, one element is related to multiple excitations for multiple different patterns. It is discarded only if all the related excitations are minimized to zeros. This is different from the single-beam sparse array synthesis case. To tackle this problem, we introduce an auxiliary variable η_n to constrain the maximum energy bound of all the coefficients related to the nth element. That is given by

$$\eta_n \geq \max\left\{|w_n^{(1)}|^2, |w_n^{(2)}|^2, \ldots, |w_n^{(M)}|^2\right\}. \tag{5}$$

Then, we can formulate the synthesis of a interspacing-constrained sparse linear array with reconfigurable multiple focused and/or shaped patterns as the following optimization problem

$$\min_{\left\{w_n^{(m)}, \eta_n; \left| \substack{m=1,\ldots,M \\ n=1,\ldots,N} \right.\right\}} \|\boldsymbol{\eta}\|_0$$
$$\text{s.t.} \begin{cases} \eta_n \geq \max\left\{|w_n^{(1)}|^2, |w_n^{(2)}|^2, \ldots, |w_n^{(M)}|^2\right\}, \\ (n = 1, 2, \ldots, N) \\ d_{\min} \geq Qd \\ (F.B.C) \begin{cases} \mathbf{a}^T(\theta_{\text{look}})\mathbf{w}^{(m)} = 1 \\ |\mathbf{a}^T(\theta)\mathbf{w}^{(m)}| \leq U_{\text{SL}}^{(m)}(\theta), \quad \forall\theta \in \Omega_{SL}^{(m)} \end{cases} \\ or \\ (S.B.C) \begin{cases} \frac{|\mathbf{a}^T(\theta)\mathbf{w}^{(m)} - f_d^{(m)}(\theta)|}{|f_d^{(m)}(\theta)|} \leq \epsilon, \quad \forall\theta \in \Omega_{ML}^{(m)} \\ |\mathbf{a}^T(\theta)\mathbf{w}^{(m)}| \leq U_{\text{SL}}^{(m)}(\theta), \quad \forall\theta \in \Omega_{SL}^{(m)} \end{cases} \\ (m = 1, 2, \ldots, M). \end{cases} \tag{6}$$

where $\boldsymbol{\eta} = [\eta_1, \eta_2, \ldots, \eta_N]^T$, $\|\boldsymbol{\eta}\|_0$ denotes the number of non-zero components of $\boldsymbol{\eta}$ (i.e., the number of selected elements), $d_{\min}$ is the minimum interspacing between the selected elements and Q is a positive integer. The constraint (F.B.C) or (S.B.C) is used to control the radiation characteristics on the mth pattern with either a focused or shaped mainlobe. $U_{\mathrm{SL}}^{(m)}(\theta)$ denotes a prescribed upper bound in the sidelobe region $\Omega_{SL}^{(m)}$, the θ_{look} is the look direction for a focused pattern, and $f_d^{(m)}(\theta)$ is the desired pattern shape in the mainlobe region $\Omega_{ML}^{(m)}$ for a shaped pattern. ϵ is the pattern matching tolerance in the mainlobe region.

B. THE EXTENDED ALTERNATING CONVEX OPTIMIZATION METHOD

The problem in (6) is extremely computationally expensive due to the ℓ_0-norm optimization objective function as well as the minimum interspacing constraint. In [27], a joint sparse recovery technique based on mixed ℓ_2/ℓ_1-norm minimization is proposed to deal with the multiple-pattern sparse linear array synthesis problem. This technique is formulated as

$$\min_{\left\{w_n^{(m)},\eta_n;\left|\substack{m=1,\ldots,M\\ n=1,\ldots,N}\right.\right\}} \sum_{n=1}^{N} \eta_n$$
$$\text{s.t.} \begin{cases} \eta_n \geq \left\| w_n^{(1)}, w_n^{(2)}, \ldots, w_n^{(M)} \right\|_2, \quad (n=1,2,\ldots,N) \\ (F.B.C) \begin{cases} \mathbf{a}^T(\theta_{\mathrm{look}})\mathbf{w}^{(m)} = 1 \\ |\mathbf{a}^T(\theta)\mathbf{w}^{(m)}| \leq U_{\mathrm{SL}}^{(m)}(\theta), \quad \forall\theta \in \Omega_{SL}^{(m)} \end{cases} \\ or \\ (S.B.C) \begin{cases} \dfrac{|\mathbf{a}^T(\theta)\mathbf{w}^{(m)} - f_d^{(m)}(\theta)|}{|f_d^{(m)}(\theta)|} \leq \epsilon, \quad \forall\theta \in \Omega_{ML}^{(m)} \\ |\mathbf{a}^T(\theta)\mathbf{w}^{(m)}| \leq U_{\mathrm{SL}}^{(m)}(\theta), \quad \forall\theta \in \Omega_{SL}^{(m)} \end{cases} \\ (m = 1, 2, \ldots, M). \end{cases} \tag{7}$$

Clearly, the above optimization problem can be easily solved by convex optimization. However, the main problem is that the minimum interspacing constraint $d_{\min} \geq Qd$ cannot be incorporated in this optimization programm.

Recently we presented an ACO method in [31] for the single-beam sparse linear array synthesis. In this method, an additional weighting vector $\mathbf{g}$ is introduced, and the excitation vector and the weighting vector are chosen in turn as optimization variables. When the weighting vector $\mathbf{g}$ is chosen as the optimization variable, a sequence of constraints can be used to control the distribution of elements in the optimized $\mathbf{g}$. This finally affects the distribution of selected antenna elements when we optimize the excitation vector with the obtained $\mathbf{g}$. By using appropriate constraints in the optimization of $\mathbf{g}$, the minimum interspacing between the selected elements can be controlled in the final synthesized result. This idea can be further extended to control the minimum interspacing for the reconfigurable multiple-pattern sparse linear array synthesis. The extended ACO method can be formulated as alternatively solving two optimization problems:

$$\min_{\left\{w_n^{(m)},\eta_n;\left|\substack{m=1,\ldots,M\\ n=1,\ldots,N}\right.\right\}} \mathbf{g}_*^T \boldsymbol{\eta}$$
$$\text{s.t.} \begin{cases} \eta_n \geq \max\left\{|w_n^{(1)}|^2, |w_n^{(2)}|^2, \ldots, |w_n^{(M)}|^2\right\}, \\ (n = 1, 2, \ldots, N) \\ (F.B.C)\ or\ (S.B.C), \quad (m = 1, 2, \ldots, M); \end{cases} \tag{8a}$$

$$\min_{\mathbf{g}} \mathbf{g}^T \boldsymbol{\eta}_*$$
$$\text{s.t.} \begin{cases} \mathbf{0} \leq \mathbf{g} \leq \mathbf{1} \\ \mathbf{1}^T \mathbf{g}(n : n + Q - 1) \geq Q - 1 \\ \quad (\text{for } n = 1, \ldots, N - Q + 1). \end{cases} \tag{8b}$$

Obviously, Problem (8b) and (8b) are both weighted ℓ_1-norm minimization problems, and they can be solved by applying convex optimization. In Problem (8b), $\mathbf{g}_*$ has been already known by solving Problem (8b) at the previous step, and $w_n^{(m)}$ $(m = 1, \ldots, M, n = 1, \ldots, N)$ and $\boldsymbol{\eta}$ are the variables to be optimized. In Problem (8b), $\boldsymbol{\eta}_*$ has been already known, and $\mathbf{g}$ is the optimization variable. At the beginning, $\mathbf{g}_*$ should be initialized. For example, we can choose $\mathbf{g}_*$ as random numbers or just equal to ones. The whole synthesis procedure can be implemented by alternatively solving the two minimization problems of (8b) and (8b). To understand why the minimum interspacing can be controlled by this method, we can at first consider the solution to Problem (8b). We assume that there are no identical elements in every Q-length segment of $\boldsymbol{\eta}_*$ obtained from Problem (8b). Then, the solution to Problem (8b) will have '1' for at the least $Q-1$ entries and '0' for the left in each Q-length segment of $\mathbf{g}(n : n+Q-1)$, due to the constraints $\mathbf{1}^T \mathbf{g}(n : n+Q-1) \geq Q-1$ for $n = 1, \ldots, N - Q + 1$. Then we return to Problem (8b). Clearly, the elements in $\boldsymbol{\eta}$ corresponding to the entries of '1' in $\mathbf{g}_*$ will be significantly penalized in Problem (8b), while the elements in $\boldsymbol{\eta}$ corresponding to '0' entries of $\mathbf{g}_*$ will be no doubt retained. Since every Q-length segment of $\mathbf{g}_*$ obtained from Problem (8b) has at the most one entry of '0', each Q-length segment of $\boldsymbol{\eta}$ obtained from Problem (8b) will have at the most one element left and all other elements in this segment will go to zeros. This is the principle of the extended ACO method to control the minimum interspacing in the framework of convex optimization.

C. THE PROPOSED EXTENDED ACO SYNTHESIS PROCEDURE

The proposed extended ACO procedure for synthesizing unequally spaced multiple-pattern sparse linear arrays with minimum interspacing constraint is given as follows.

1) Set initial array: the parameters N and d.
2) Set the minimum interspacing constraint: $d_{\min}$ and Q.
3) Give prescribed multiple patterns: the focused beam direction $\theta_{look}^{(m)}$, the shaped pattern $f_d^{(m)}$ in mainlobe region $\Omega_{ML}^{(m)}$, the sidelobe level $U_{SL}^{(m)}$ within the sidelobe region $\Omega_{SL}^{(m)}$ for all patterns $(m = 1, 2, \ldots, M)$.

4) Set the initial weighting vector $\mathbf{g}_*^{(0)}$ to be $\mathbf{1}$ (initialize the $\mathbf{g}_*^{(0)}$ randomly if required).
5) Set the allowed number of iterations P.
6) **for** $p = 1 : 1 : P$ **do**
   (i) Perform the convex optimization in problem (8b) to obtain $\mathbf{w}$ and $\boldsymbol{\eta}$, and set $\boldsymbol{\eta}_*^{(p)} = \boldsymbol{\eta}$.
   (ii) Perform the convex optimization in problem (8b) with the interspacing constraint, and set $\mathbf{g}_*^{(p)} = \mathbf{g}$.
   (iii) If $\mathbf{g}$ remains the same for multiple iterations, then re-initialize $\mathbf{g}$.
   (iv) Sort the elements in $\boldsymbol{\eta}$ in descent order, and determine the element number $K = \min\{k; |\eta_{k+1}/\eta_k \leq 10^{-2}\}$.
   (v) Compute the multiple patterns by picking up the K element positions and multiple sets of excitations associated with the K largest elements in the ordered $\boldsymbol{\eta}$.
   (vi) If all the constraints in problem (6) are satisfied, jump out of the loop.
   end for
7) Return the selected element positions and the associated multiple sets of excitations for all patterns.

III. NUMERICAL EXAMPLE

A. SYNTHESIS OF A SPARSE LINEAR ARRAY WITH RECONFIGURABLE DUAL-PATTERNS

As the first example, we will apply the proposed method to synthesize a sparse linear array with reconfigurable dual-patterns. In [32], dual-patterns including a flat-top pattern and a cosecant-squared pattern were obtained by optimizing the amplitudes and phases of a 20-element $\lambda/2$-spaced array. The dual-patterns are re-plotted in Fig. 1. Now, we try to apply the proposed method to re-synthesize the dual patterns with reduced number of elements. Assume that the desired two mainlobe shapes are used as the same as those in [32]. The SLL bound for the flat-top pattern is set as $U_{SL}(\theta) = -40$ dB for $\theta \in [65°, 71°] \cup [109°, 115°]$ and $U_{SL}(\theta) = -20$ dB for $\theta \in [0°, 65°) \cup (115°, 180°]$. The SLL for the cosecant-squared pattern is set as $U_{SL}(\theta) = -30$ dB for $\theta \in [96°, 116°]$ and $U_{SL}(\theta) = -20$ dB for others in the sidelobe region. A linear array consisting of 191 potential elements with a spacing of $\lambda/20$ is used as the initial array. The minimum interspacing constraint is set as $d_{\min} = 0.5\lambda$, and accordingly Q equals to 10. We now perform the extended ACO method to find the best common element positions for the desired dual-patterns. Finally, 16 unequally spaced elements are selected from the initial element positions, and 175 elements have been discarded. The synthesized dual-patterns are shown in Fig. 1. As can be seen, both of the obtained mainlobe shapes of the dual-patterns agree well with the desired ones while the sidelobe distributions for the both patterns meet the prescribed bound. Fig. 2 shows the selected 16 element positions by the proposed method and the 20 $\lambda/2$-spaced elements used in [32] for comparison. For the synthesized array, the minimum and maximum interspacing is 0.50λ and 0.85λ, respectively. This means the obtained array exactly satisfies the minimum interspacing constraint. Compared with the 20-element $\lambda/2$-spaced array, the synthesized array saved 20% elements. Fig. 3(a) and (b) show the obtained excitation amplitudes and phases for the dual-patterns, respectively.

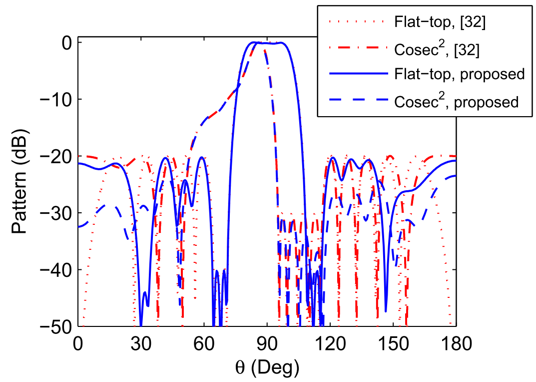

FIGURE 1. The synthesized dual-patterns by the proposed method with 16 elements and the patterns obtained in [32] with 20 uniformly spaced elements.

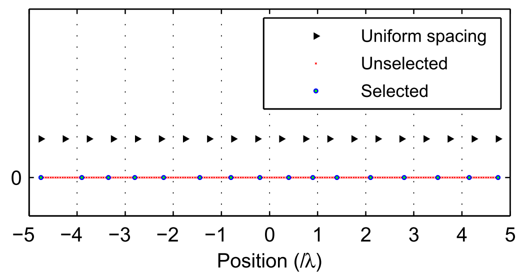

FIGURE 2. The synthesized element positions by the proposed method and the uniformly spaced positions used in [32] for the dual-patterns.

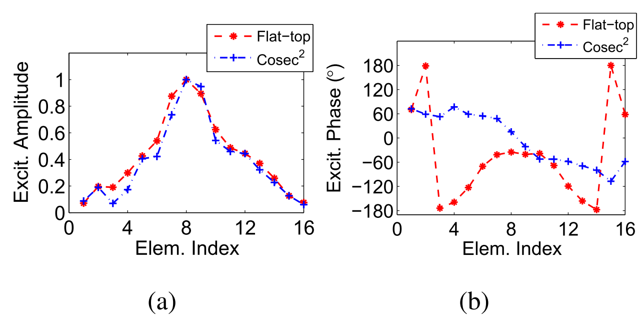

FIGURE 3. The synthesized excitation distributions by the proposed method for the reconfigurable dual-patterns. (a) Amplitude and (b) phase.

To check the effectiveness of the proposed method for different minimum interspacing requirements, we set $d_{\min} = [0.40, 0.45, 0.50, 0.55, 0.60]\lambda$ ($Q = [8, 9, 10, 11, 12]$). Assume that the same dual-patterns are required, and the initial array is also used as the same as the above for all test cases. Table 1 lists the synthesized results including the minimum and maximum element spacings, the number of selected elements as well as the saving in the element count compared to the 20-element $\lambda/2$-spaced array in [32]. As can be seen,

TABLE 1. Synthesis results of the proposed method with different interspacing constraints.

Constrained $d_{\min}$ (/λ)	Q	Synthesized spacings (/λ)		Element number	Element saving
		Min	Max		
0.40	8	0.40	0.90	15	25%
0.45	9	0.45	0.80	16	20%
0.50	10	0.50	0.85	16	20%
0.55	11	0.55	0.85	15	25%
0.60	12	0.60	0.80	15	25%

although the number of selected elements varies slightly in different cases, the sparse arrays obtained by the proposed method strictly satisfy the specified minimum interspacing constraints. Hence, we can carefully draw the conclusion that the extended ACO method is robust for dealing with different minimum interspacing constraints.

B. SYNTHESIS OF A SPARSE LINEAR ARRAY WITH RECONFIGURABLE TRIPLE-PATTERNS

In this case, we consider synthesizing a sparse linear array generating reconfigurable triple-patterns. In [7], a reconfigurable triple-patterns including a focused, a flat-top, and a cosecant-squared patterns were synthesized by using the modified Woodward-Lawson method with a 20-element $\lambda/2$-spaced array. The obtained SLL is -20 dB for all the three patterns. In [24], similar triple-patterns with the same mainlobe shapes were obtained by the extended FBMPM using 16 optimized element positions. Fig. 4(a) re-plots all the pattern results obtained in [7] and [24]. As can be seen, the patterns synthesized by the extended FBMPM is not satisfactory in the sidelobe region, and the obtained SLL is considerably higher than -20 dB. In [27], the joint sparse recovery method is applied to produce the same triple-patterns. The pattern results are shown in Fig. 4(b). It is seen, the obtained patterns by the joint sparse recovery method are more accurate than the extended FBMPM. However, the required number of elements by this method is increased to 18, and the minimum interspacing is only 0.21λ. As we know, such an interspacing is not easy to implement for conventional antenna arrays.

Now, we apply the proposed method to synthesize the same triple-patterns, and the SLL bound for each pattern is set as -20 dB. The initial array is still set as 191 elements with a spacing of $\lambda/20$, and the minimum interspacing constraint is set as $d_{\min} = 0.5\lambda$ ($Q = 10$). By performing a set of alternating convex optimizations, the proposed method finally picks up 16 elements. The obtained triple-patterns have exactly the same mainlobe shapes while all the sidelobe distributions for the triple-patterns completely meet the specification. Fig. 5 shows the synthesized element positions by the proposed method as well as the element distributions obtained in [7], [24] and [27] for comparison. The obtained minimum interspacing by the proposed method is exactly equal to the specified 0.5λ. In this example, the array layouts obtained by the proposed method and the FBMPM are more practical than the result given by the joint sparse recovery method. However,

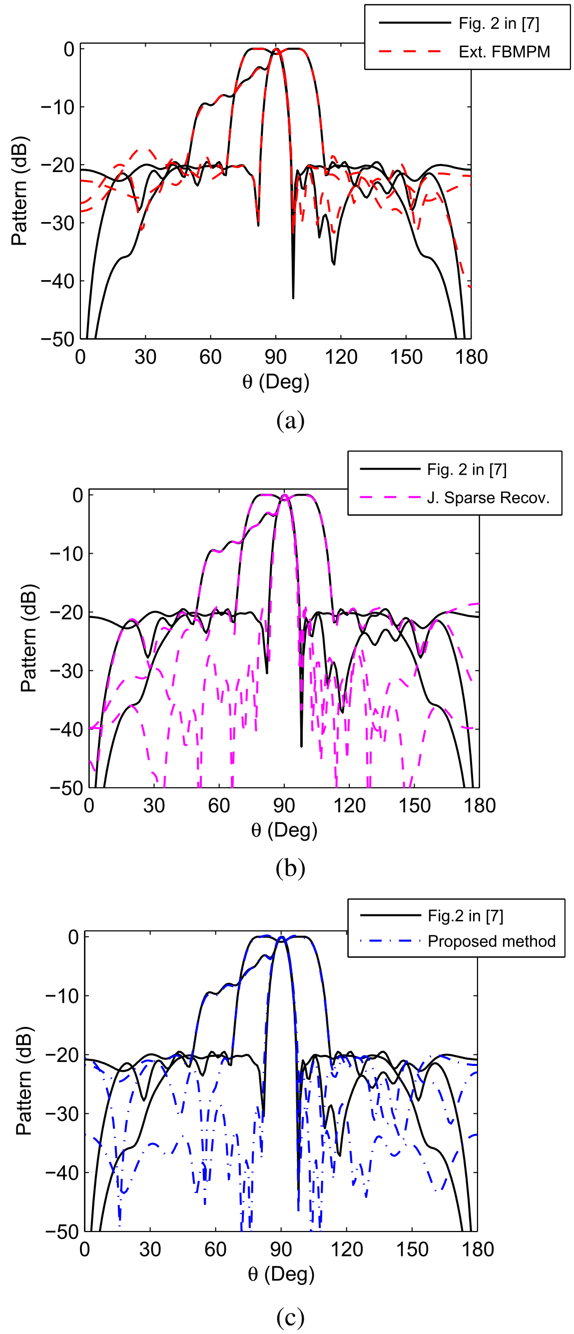

FIGURE 4. The triple-patterns synthesized in [7] with 20 elements, and the patterns synthesized by (a) the extended FBMPM in [24] with 16 elements, (b) the joint sparse recovery in [27] with 18 elements, and (c) the proposed method with 16 elements.

it should be noted that the extended FBMPM cannot accurately control the minimum interspacing, and consequently the obtained layout depending on the case cannot always meet the requirement. In addition, the extended FBMPM cannot implement accurate sidelobe control. Hence, in terms of both the pattern accuracy and minimum interspacing control ability, the proposed method is more preferable. Fig. 6(a) and (b) show the obtained excitation amplitudes and phases by the proposed method for the triple- patterns, respectively. In this

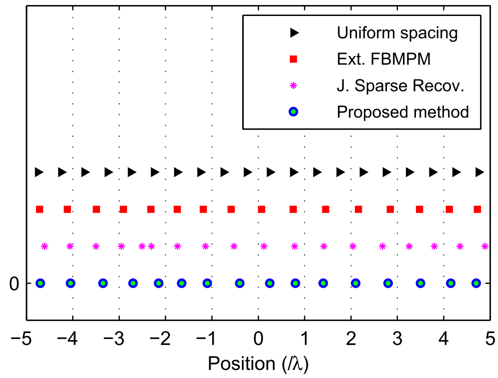

FIGURE 5. **The element positions synthesized by the extended FBMPM, the joint sparse recovery and the proposed method, as well as the uniformly spaced positions used in [7].**

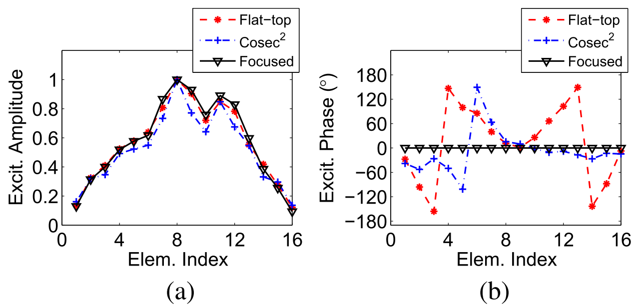

FIGURE 6. **The synthesized excitation distributions by the proposed method for the reconfigurable triple-patterns. (a) Amplitude and (b) phase.**

triple-pattern synthesis case, we still saved 20% elements if compared with the 20-element $\lambda/2$-spaced array.

C. SYNTHESIS OF A SPARSE LINEAR ARRAY WITH MORE COMPLICATED RECONFIGURABLE PATTERNS

In the last example, we will check the effectiveness of the proposed method for synthesizing more complicated reconfigurable patterns. Assume that an additional common null is added into the same triple-patterns in the second example. The null level is set as $U_{SL}(\theta) = -45$ dB for $\theta \in [145°, 160°]$. Other configurations such as initial element positions and interspacing constraint are set as the same as those in the second example. The same synthesis procedure is adopted to find the best common element positions and the associated excitation coefficients. Fig. 7 shows the synthesized multiple patterns. As can be seen, all the patterns including the focused, flat-top and cosecant-squared patterns maintain their mainlobe shapes, and the additional common null is exactly produced within the required angular region. It should be noted that for this more complicated pattern requirement, 17 elements are selected from the initial array. Compared with the sparse array obtained without the additional null, one element is added. The element saving in this case is 15%. Fig. 8 shows the synthesized element positions. Fig. 9(a) and (b) show the required excitation amplitudes

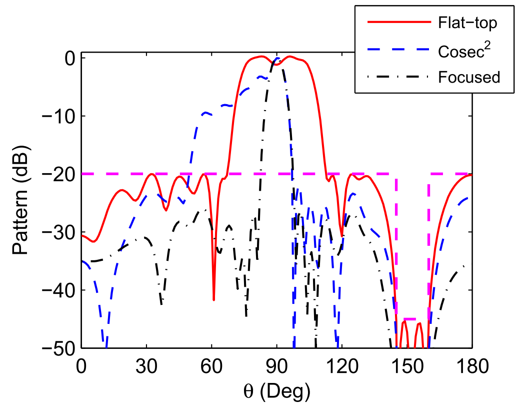

FIGURE 7. **The synthesized triple-patterns with a common null.**

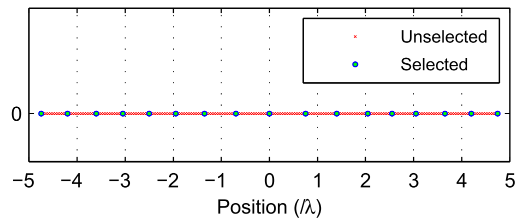

FIGURE 8. **The synthesized element positions for the triple-patterns with a common null.**

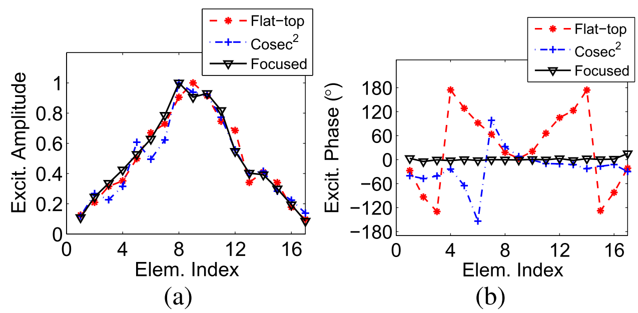

FIGURE 9. **The synthesized excitation distributions by the proposed method for the reconfigurable triple-patterns with a common null. (a) Amplitude and (b) phase.**

and phases for the triple-patterns with the additional null, respectively.

IV. CONCLUSION

An extended alternating convex optimization (ACO) method has been presented to synthesize unequally spaced sparse linear arrays with reconfigurable multiple patterns. By performing a set of alternating constrained convex optimizations, this method can accurately control both the sidelobe distribution and the minimum interspacing. A common null requirement can be also added into the reconfigurable pattern synthesis. All these properties make the proposed method more preferable in practice than some other pattern-reconfigurable sparse array synthesis techniques such as the extended FBMPM and the joint sparse recovery method. Three examples for synthesizing different reconfigurable patterns have been conducted.

The synthesis results show that the proposed method is very effective and robust for different setting in pattern shape, sidelobe distribution and minimum interspacing requirement. For test cases, the element saving is about $15\% \sim 25\%$, which is very useful for lowering the cost of fabricating pattern-reconfigurable arrays.

Finally, it should be noted that the proposed method adopts the isotropic element assumption without considering the mutual coupling. When considering real antenna array structure, the array patterns including mutual coupling may deviate from the synthesized ones. In general, since the mutual coupling effect depends on the element position arrangement, incorporating the mutual coupling into the element position optimization seems to be very hard. One possible strategy is adding a refining step to re-correct the synthesized positions and excitations. The optimal position perturbations can be found by solving a convex optimization problem based on the assumption that the position-perturbed elements have the same active element patterns except with additional phase terms associated with the position shifts. Another issue with the proposed method is that a reference pattern is still required in the mainlobe region. This problem can be overcome by employing some spectral factorization-based power pattern synthesis methods. Further research on these strategies applied to the proposed method would be very interesting but beyond the scope of this paper.

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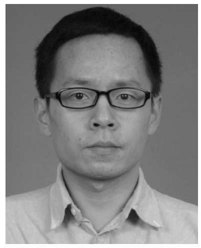

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