A Nonparametric Examination of Market Information:
Application to Technical Trading Rules

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Abstract

This paper develops a nonparametric approach for testing whether an information set is useful for generating greater stock market returns. The approach is model free and thus the test of the information does not depend on the particular assumptions of an asset pricing model. Assuming No Arbitrage, a stochastic discount factor (SDF) is constructed from observed market assets. This SDF can be used as a pricing operator for examining dynamic portfolio returns to indicate the information content in the underlying trading strategy. Trading strategies based on technical trading rules are examined with the developed approach.

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Many traders in stock, commodities, and foreign currency markets buy and sell assets based on rules constructed from past and present price patterns. The Efficient Market Hypothesis (EMH) indicates that such rules, commonly referred to as technical trading rules (TTRs), should have no ability to forecast future movements in returns, or generate excess returns for the investors who use them. Are technical traders mistaken in their confidence in TTRs, or are the trading rules useful despite the lack of economic foundation? There have been a number of recent papers indicating that technical trading portfolio management techniques may lead to returns above normal, including the analysis of Brock, Lakonishok, LeBaron (1992).

This paper presents a model free test procedure for examining whether an information set can be used to generate excess returns. It allows the incorporation of conditioning information, thus addressing whether additional information assists the trader who has access to a basic information set. The procedure makes use of the nonparametric stochastic discount factor presented by Hansen and Jagannathan (1991). Simple technical trading rules are then examined using the procedure.

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Background

Informally, refer to an information set as being useful if it aids the investor in predicting returns, or it helps indicate how a portfolio can be adjusted through time to yield higher returns.

The traditional methods of examining information, or for examining a portfolio based on a dynamic trading strategy, require the use of an asset pricing model. The researcher can employ a *performance* test approach to evaluating a trading strategy. Typically, the researcher applies the trading strategy to a history of asset returns. An asset pricing model, often the CAPM or the APT asset pricing model, is used to evaluate returns generated from the trading strategy. A portfolio that performs better than can be explained by the model indicates that the trading strategy, and thus the information upon which the trading strategy is based, is useful for predicting returns.

Hansen and Singleton (1982) pioneered a second approach, the *orthogonality* test. Researchers generally use it to test the Efficient Market Hypothesis or an asset pricing model. The test examines the null hypothesis that under efficient markets prices reflect all information available to investors. If efficient markets is correct and the asset pricing model correctly reflects the price generating process, then any information that is available to investors should be orthogonal to the errors of the asset pricing model. Parameters of the model are estimated using observed data. The pricing errors of the fitted model are then tested for orthogonality to information possessed by investors.

Relying on an asset pricing model forces a simultaneous test of the information and the asset pricing model employed, including all of the necessary assumptions of that model. In the performance test, a finding of non-zero performance can also be interpreted as a rejection of the asset pricing model rather than an indication of the value of trading strategy. Similarly, in the orthogonality test, the researcher cannot distinguish between a rejection of the EMH and a rejection of the asset pricing model. (For statements concerning the shortcomings of using pricing models as a method of performance evaluation and for proposed nonparametric performance test see Merton (1981), Henriksson and Merton (1981), Kane and Marks (1988), Cumby and Modest (1987), and Chen and Knez (1995) for examples.)

Hansen and Richard (1987) note that estimating an unconditional model when the truth is characterized by a conditional model can lead to false rejection of the null hypothesis through the finding of non-zero performance when examining security returns. Typical examinations in the literature rely on estimating models unconditionally or using difficulty implemented procedures to estimate a conditional model. The procedure developed in this paper easily incorporates conditioning information. As a performance measure, the conditional test serves as an examination of whether a trading rule supplements information that may already be available to investors.

Users of technical trading rules such as the ones examined in this paper and by Brock *et al* (1992) claim that the rules act as a substitute for fundamental information which has not yet become common knowledge, but that a limited population of traders have begun trading on. The claim depends on a less than instantaneous market price adjustment to the new equilibrium, allowing the technical trader to act before the new equilibrium is reached. A finding in which the technical trading rules are useful to the uninformed investor, but not useful to the informed investor, would be consistent with these trader's claims.

The methodology outlined in this paper address unwanted aspect of simultaneous testing of an asset pricing model along with the testing of efficient markets. The developed test is applied to examine the documented success of a number of simple technical trading rules. The returns to a dynamic trading strategy based on the technical trading rules are examined using a conditional as well as unconditional stochastic discount factors.

Section I of this paper contains a description of the return environment that provides a platform for constructing the test. Section II describes how information variables can be tested for information content that would be useful for generating greater returns. In section III, the tests are constructed. Two tests are formulated, based respectively on unconditional and conditional expectations. Section IV provides a description of the data and, in section V, the two tests are applied to a set of TTRs. Section VI contains some concluding statements.

I. The Return Environment

In order to develop the testable implication for evaluating the usefulness of technical trading rule information, it is necessary to discuss the environment under which it is assumed asset payoffs and returns are defined. This section is kept brief. Much of the technical framework necessary to conduct the desired examination of information has be developed and extensively discussed in existing literature. What follows are highlighted aspects that are particularly important to the procedure developed in this paper.

Except where noted, bold type face denotes a vector, bold capitals indicate a matrix. Super- and subscripts other than t indicate information sets. Superscripts indicate a conditioning information set so that $\mathbf{Z}^{\mathrm{I}}$ indicates that $\mathbf{Z}$ is a matrix containing the conditioning information set I. A subscript on m, the stochastic discount factor, indicates that it was constructed using expectations conditional on the information indicated by the subscript.

$\mathbf{x}_t$: Nx1 vector of time t payoffs to assets 1 through N.
$\mathbf{q}_{t-1}$: Nx1 vector of t-1 prices for $\mathbf{x_t}$.
$\mathbf{r}_t$: Nx1 vector of returns to assets 1 through N.
$\mathbf{z}_t$: Kx1 vector of information.
$\mathbf{c}$: Nx1 vector of constants whose elements lie in $\mathbb{R}^N$
θ_t: Nx1 vector of stochastic weights
$\mathbf{R}$: NxT matrix with $\mathbf{r}_t$ as the t^{th} column.
$\mathbf{Z}$: KxT matrix with $\mathbf{z}_t$ as the t^{th} column.
R_t: The (Hilbert) space of time t portfolio returns
$\mathscr{R}$: A construction of R spaces over time
m_t: The stochastic discount factor (SDF)
^{P}: Public information
^{TR}: Technical trading rule information

A. Price and Payoff Process

The general framework for prices and payoffs is as described by Hansen and Richard (1987) and Hansen and Jagannathan (1991) with the price and payoff process $\{\mathbf{q}_t, \mathbf{x}_{t+1}\}$ for t = 1,...,T. Let the function $\pi(\cdot)$ be a pricing operator such that $\mathbf{q}_t = \pi(\mathbf{x}_{t+1})$.

Risk-averse, nonsatiated traders ensure that the intertemporal marginal rate of substitution is strictly positive. This, along with linear pricing, is sufficient to ensure the absence of arbitrage opportunities. No Arbitrage (NA) in turn implies the Law of One Price (LOP), which states that alternative methods of constructing the same payoff must have the same price. The NA condition is a requirement of most standard asset pricing models such as the CAPM and APT models.

B. A Description of the Return Environment

First, consider the space of returns attainable from a constant composition portfolio. This is the return space defined by the returns attainable from all possible linear combinations of N assets.

$$R_t(\mathbf{c}) \equiv \{p_t : p_t = \mathbf{c}'\mathbf{r}_t,\ \mathbf{c}'\mathbf{1} = 1,\ \ \mathbf{r}_t \in \mathrm{L}^2\} \tag{1}$$

where p is the payoff to the portfolio. Further, define the attainable portfolio returns over time from all constant composition portfolio as

$$\mathscr{R}(\mathbf{c}) = \{(r_1 \dots r_T) : r_t \in R_t(\mathbf{c}), \mathbf{c}'\mathbf{1} = 1,\ t = 1, \dots, T\}\ . \tag{2}$$

In the pricing environment described by Hansen and Jagannathan, there is a family of stochastic discount factors, $\mathscr{M}$, able to properly "price" the set of returns such that,

$$1 = \mathrm{E}(r_t m_t) \text{ for all } r_t \text{ in } R_t(\mathbf{c}) \text{ in all } t, \tag{3}$$

where $m_t \in \mathscr{M}$. Of these, there is a unique SDF constructed from a linear span of the N assets.

$$m_t^* = \mathbf{r}_t' E(\mathbf{r}_t \mathbf{r}_t')^{-1} \mathbf{1}. \tag{4}$$

The feature of m_t* which makes it particularly useful for examining information is that it is unable to properly price any asset or portfolio whose return is not in $R_t(\mathbf{c})$, meaning that $\mathrm{E}(r_t m_t^*) \neq 1$ for $r_t \notin R_t(\mathbf{c})$. This is because an asset with a return outside of $R_t(\mathbf{c})$ will have a component that is orthogonal to m_t*, the orthogonal component contributes to the asset's market value, but is missed by m_t*. Other SDFs in $\mathscr{M}$ may be able to price some broader component of the return space not limited to $R_t(\mathbf{c})$. Since it is in not constrained to properly price any return outside $R_t(\mathbf{c})$, m_t* is the least restricted of the set of valid SDFs. The No Arbitrage condition requires that valid SDFs be positive at all times.

The return space can be broadened not only by including more assets, but also by considering the returns attainable from a dynamic portfolio of the original N assets. Let $\mathscr{F}_{t\text{-}1}$ indicate the conditional information set available to the trader at time *t*-1. Further, let $\theta_t = \theta(\mathscr{F}_{t\text{-}1})$ be an Nx1 vector of weights based on the information revealed in $\mathscr{F}_{t\text{-}1}$. The information set $\mathscr{F}_{t\text{-}1}$ represents the most recently observe data that the trader can use in constructing the portfolio that yields returns at time *t*. The return space $\mathscr{R}(\theta(\mathscr{F}_{t\text{-}1}))$ consists of all returns attainable from dynamic portfolios based on the $\mathscr{F}_{t\text{-}1}$ information. (Chen and Knez (1995) provide a more detailed explanation of the dynamic portfolio return space.)

Let $\mathscr{M}(\mathscr{F}_{t\text{-}1})$ indicate the more restricted subset of $\mathscr{M}$ able to properly price the conditional returns in $\mathscr{R}(\theta(\mathscr{F}_{t\text{-}1}))$. By iterated expectations, any SDF able to price the conditional returns is able to price the unconditional returns as well (under the conditions of the EMH where prices properly reflect the $\mathscr{F}_{t\text{-}1}$ information). As with the unconstrained case, there is a unique SDF $m_{i,t}$* which is constructed as a linear span of the returns in $\mathscr{R}(\theta(\mathscr{F}_{t\text{-}1}))$. Of the SDFs in $\mathscr{M}(\mathscr{F}_{t\text{-}1})$, $m_{i,t}$* is the least restricted.

Asset pricing models, such as the CAPM, the APT, or Lucas's Consumption CAPM, suggest restrictions on returns such that the SDF implied by the model properly prices all returns in the market. Restrictions to the price and return process in addition to those described above are required to create the necessary environment for the model to work. These additional restrictions are avoided by using the nonparametric SDF.

II. Testable Implications

The testable implications arise from the distinction between what can be priced by the nonparametric SDF and what cannot. The unconditional test relies on m_t* and the fact that it properly prices any return in $\mathscr{R}(\mathbf{c})$, but incorrectly price any return outside of this space. Likewise, $m_{i,t}$* prices returns in $\mathscr{R}(\theta(\mathscr{F}_{t-1}))$, but not the those outside of this expanded conditional return space. Under the null hypothesis that the TTR information is useless, a portfolio adjusted through time according to a TTR is priced by the SDF as though it remains inside the return space. The alternative hypothesis that the TTRs are useful would be supported by a finding that the TTR generated returns are outside of the return space used to construct m_t* or $m_{i,t}$*.

To differentiate between returns that can be price by a SDF and those which cannot, consider a Kx1 vector with elements $\mathbf{z}_t$ from the information set $\mathscr{F}_{t-1}$. Normalize the series such that $E[\mathbf{z}_t] = \mathbf{1}$.

A. Unconditional Testing

By construction, the SDF m_t* successfully prices the returns to an asset *if* that asset can be replicated by a constant composition portfolio of the base assets used to construct m_t*. This is an *if and only if* statement with one exception. A dynamic portfolio of the base assets can be successfully priced by m_t* if the information upon which the dynamic strategy is based is orthogonal to the returns.

Since there exists a SDF such that

$$E[\mathbf{1} - (\mathbf{r}_t \cdot m_{i,t}) | \mathscr{F}_{t-1}] = \mathbf{0}, \tag{5}$$

it is also true that

$$\mathrm{E}[\mathbf{1} - (\mathbf{r}_t \cdot m_{i,t}) | \mathscr{F}_{t-1}] \otimes \mathbf{z}_t = \mathbf{0}. \tag{6}$$

Since $\mathscr{F}_{t-1}$ contains $\mathbf{z}_t$, it can be moved inside the expectations operator. By iterated expectations

$$\mathrm{E}[\mathbf{1} \otimes \mathbf{z}_t - (\mathbf{r}_t \otimes \mathbf{z}_t \cdot m_{i,t})] = \mathbf{0}. \tag{7}$$

$$\mathbf{1} - \mathrm{E}[(\mathbf{r}_t \otimes \mathbf{z}_t \cdot m_{i,t})] = \mathbf{0}. \tag{7'}$$

Thus, $m_{i,t}$ is able to price $\mathbf{r}_t z_{k,t}$ for k = 1, ..., K as well as being able to price the original returns, $\mathbf{r}_t$. The constructed series $\mathbf{r}_t z_{k,t}$ can be thought of as an asset that the conditional $m_{i,t}$ must be able to price in addition to the original returns $\mathbf{r}_t$. Consider $\mathbf{r}_t z_{k,t}$ to be the return to artificial assets. The return series $\{r_{n,t} z_{k,t},\ t = 1,2,\ldots\}$ is the return to purchasing and selling asset n according to the information source $z_{k,t}$. This represents a dynamic portfolio since the value of z_k is time dependent. Mathematically, this return series can be treated as if it is the returns to a new asset, independent of the original N assets in $\mathbf{r}_t$. In general, the artificial return is outside the return space $\mathscr{R}(\mathbf{c})$ since any constant composition portfolio of the original N assets cannot replicate it. Equations (5) through (7') demonstrate that the SDF $m_{i,t}$ that prices the original set of returns using conditional expectations also prices unconditionally the expanded set of returns that includes the returns to a portfolio constructed using a dynamic trading strategy based on information in the conditioning information set.

Construct $m_{i,t}^*$ as a linear combination of $\mathbf{r}_t$ and $\mathbf{r}_t z_{k,t}$, satisfying Equation (7). In addition to pricing the returns to a constant composition portfolio of the original assets, it also prices the returns to dynamic portfolios of the original assets where the dynamic strategy is limited to linear functions of the information variables $z_{k,t}$ for k = 1, ..., K. It cannot accurately price any return not included in this linear space.

The unconditional test derives from the fact that m_t* is less constrained than $m_{i,t}$*. Since the random variable $\mathbf{r}_t z_{k,t}$ is not a linear combination of the $\mathbf{r}_t$ returns, it is outside the space of $\mathscr{R}(\mathbf{c})$, and thus is improperly priced by m_t*. In general,

$$E[\mathbf{1}\otimes\mathbf{z}_t - (\mathbf{r}_t\otimes\mathbf{z}_t\cdot m_t^*)] \neq \mathbf{0}. \tag{8}$$

However, the special case in which $z_{k,t}$ is orthogonal to the returns, and thus orthogonal to m_t* as well, results in

$$E[\mathbf{1}z_{k,} - (\mathbf{r}_t z_{k,t}\cdot m_t^*)] = E[\mathbf{1} - (\mathbf{r}_t\cdot m_t^*)]E[z_{k,t}] = \mathbf{0}. \tag{9}$$

B. Conditional Testing

By construction, the SDF $m_{i',t}$* successfully prices the returns to an asset *if* that asset can be replicated by a constant composition portfolio of the base assets used to construct $m_{i',t}$*, or by a dynamic portfolio of the base assets in which the dynamic trading strategy is a linear function of the information in $\mathscr{F}_{t-1}$'. A dynamic portfolio based on a broader set of information, $\mathscr{F}_{t-1}$, cannot be priced by $m_{i',t}$* with one exception. The exception is if the addition information is orthogonal to the returns generated from $\mathscr{F}_{t-1}$'.

The argument follows the same line of reasoning applied to the unconditional SDF. Consider $\{z_{k,t}, k = 1, \ldots, K-1\}$ to be elements of an element of $\mathscr{F}_{t-1}$' but not $z_{K,t}$, which is an element of $\mathscr{F}_{t-1}$. Assume $\mathscr{F}_{t-1}' \subset \mathscr{F}_{t-1}$. Construct a stochastic discount factor $m_{i',t}$* such that

$$E[\mathbf{1} - (\mathbf{r}_t m^*_{i',t})|\mathscr{F}_{t-1}'] = \mathbf{0}. \tag{10}$$

The SDF $m_{i',t}$* is unable to price $\mathbf{r}_t z_{K,t}$, yielding

$$\mathrm{E}[\mathbf{1}z_{K,} - (\mathbf{r}_t z_{K,t} \cdot m_{i',t}^*)] \neq \mathbf{0}. \tag{11}$$

The exception is if $\mathbf{z}_K$ is orthogonal to the base returns and to the dynamic returns, $\mathbf{r}_t z_{k,t}$.

III. Construction of the Tests

The empirical evaluation of a dynamic strategy in general, and of TTRs within the context of this paper, depends on a measure of the extent to which a SDF, unconditional or conditional, is unable to price the returns to the dynamic trading strategy in question. The measure of this pricing error, λ, has a value of zero under the null hypothesis that the trading strategy is useless. The test can be characterized as either an orthogonality test between the series of pricing errors and the TTR information set, or as a performance measure resulting from pricing a dynamic portfolio return based on the TTR using the a nonparametric SDF.

A. Derivation of the Test

For any SDF m, define

$$\varepsilon_t(\mathrm{m}) = [\mathbf{r}_t m_t - \mathbf{1}] \tag{12}$$

as the Nx1 vector of pricing errors. The realization of $(\mathbf{r}_t m_t - \mathbf{1})$ is an orthogonal series to the information used to construct the discount factor. In the unconditional case, the error is orthogonal to the returns. In the conditional case, the error is orthogonal to the returns and to linear combinations of the returns and the conditioning information. Consider an information set consisting of technical trading rules, $\mathscr{F}^{TR}$. Under the null hypothesis that the TTR information is useless for generating excess returns, the pricing error series is orthogonal to the technical trading rule information. The test will be whether a vector of trading rule information, $\mathbf{Z}^{\mathrm{TR}} \in \mathscr{F}^{\mathrm{TR}}$, is orthogonal to the pricing error.

The first test determines whether technical trading rules contain any useful information. This requires construction of the unconditional SDF,

$$m_t^* = \mathbf{r}_t{}' E(\mathbf{r}_t \mathbf{r}_t{}')^{-1} \mathbf{1}_{\mathrm{N}}, \tag{13}$$

from the observed returns. Define $\boldsymbol{\lambda}(m^*,\mathbf{Z}^{\mathrm{TR}})$ as E[ε(m*)⊗$\mathbf{Z}^{\mathrm{TR}}$] or

$$\boldsymbol{\lambda}(m^*,\mathbf{Z}^{\mathrm{TR}}) = E[(\mathbf{r}_t \otimes \mathbf{z}_t^{\mathrm{TR}})m_t^* - \mathbf{1} \otimes \mathbf{z}_t^{\mathrm{TR}}]. \tag{14}$$

Define $\hat{\boldsymbol{\lambda}}(m^*,\mathbf{Z}^{TR})$ as the sample analogue of $\boldsymbol{\lambda}(m^*,\mathbf{Z}^{\mathrm{TR}})$,

$$\hat{\boldsymbol{\lambda}}(m^*,\mathbf{Z}^{\mathrm{TR}}) = \frac{1}{T}\sum_{t=1}^{T}[(\mathbf{r}_t \otimes \mathbf{z}_t^{\mathrm{TR}})m_t^* - \mathbf{1}] \tag{15}$$

where in sample, m_t* is estimated by

$$\hat{m}_t^* = \mathbf{r}_t{}' T\left[\sum(\mathbf{r}_t \mathbf{r}_t{}')\right]^{-1} \mathbf{1}_{\mathrm{N}}. \tag{16}$$

Define an H-statistic as

$$H_T = T \cdot \hat{\boldsymbol{\lambda}}(m^*,\mathbf{Z}^{\mathrm{TR}})' W_T \cdot \hat{\boldsymbol{\lambda}}(m^*,\mathbf{Z}^{\mathrm{TR}}) \tag{17}$$

which indicates whether $\hat{\boldsymbol{\lambda}}(m^*,\mathbf{Z}^{\mathrm{TR}})$ is statistically different from zero. The weighting matrix W_T is symmetric and positive definite and can be chosen optimally as the inverse of the consistent estimate of the covariance matrix of $\hat{\boldsymbol{\lambda}}(m^*,\mathbf{Z}^{\mathrm{TR}})$. Under the null hypothesis that $\boldsymbol{\lambda}(m^*,\mathbf{Z}^{\mathrm{TR}})$ is zero and that $\hat{\boldsymbol{\lambda}}(m^*,\mathbf{Z}^{\mathrm{TR}})$ is asymptotically normally distributed with finite variance, H_T is asymptotically χ^2-distributed with N·k degrees of freedom.[1] The H-statistic is a non-negative measure of the value of the technical trading rule information.

Because the information in $\mathbf{Z}^{TR}$ is actually a trading strategy, the statistic $\boldsymbol{\lambda}(m^*,\mathbf{Z}^{TR})$ is a dual measure of both orthogonality and performance. It is a test of orthogonality between the two series $\varepsilon(m^*)$ and $\mathbf{Z}^{TR}$. In this sense, the test can be though of as a GMM test with the number of coefficients being estimated equal to zero. On the other side of the duality, $(\mathbf{r}\otimes \mathbf{Z}^{TR})$ are the actual returns to portfolios based on the dynamic trading strategies of the technical trading rules. As such, $[\mathbf{1}\otimes\mathbf{z}_t^{TR} - (\mathbf{r}_t\otimes\mathbf{z}_t^{TR})m_t^*]$ is a measure of the performance of the portfolio as determined by the SDF m_t^*. Positive (negative) values indicate positive (negative) performance for the portfolio. For this analysis, the interest is in the "performance" of the TTR information, and not the particular trading strategy. Any deviation of $\boldsymbol{\lambda}(m^*,\mathbf{Z}^{TR})$ away from zero, positive or negative, indicates that the TTRs are correlated with returns and thus the information can be used by an investor to increase returns. Thus, the H-statistic is the final measure of interest since it measures whether the performance of the portfolio is statistically different from zero.

The conditional test is to determine whether the technical trading rules contain useful information beyond that which is common knowledge about market conditions. Consider $\mathscr{F}_{t-1}^{P}$, the set of publicly available information. Let $z_t^{P} \in \mathscr{F}_{t-1}^{P}$ be a $(k_p+1)\times 1$ vector with k_p

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[1] Many studies have indicated that returns have fat tail distributions. If so, convergence of $\hat{\lambda}(m^*, Z^{TR})$ to a normal distribution may be slow or it may never converge. In such a case H_T may not be approximated by a χ^2 distribution at the sample size used in this paper. Thus, the critical value for rejection of the null may be higher than that of a χ^2 distribution.

conditioning variables and a vector of 1s as the (k+1) row. Construct a conditional stochastic discount factor $m_{p,t}^{*}/\pi(m_{p,t}^{*}) \in R_t(\theta(\mathscr{F}_{t-1}^{\mathrm{P}}))$ such that the Kronecker product of the pricing error of observed returns with the conditioning public information is zero. Find the Tx1 vector $\mathbf{m}_p^{*}$ such that

$$\mathbf{O} = E[(\mathbf{r}_t \otimes \mathbf{z}_t^{\mathrm{P}}) \cdot m_{p,t}^{*} - \mathbf{1} \otimes \mathbf{z}_t^{\mathrm{P}}]. \tag{18}$$

The formula for computing the conditional stochastic discount factor $m_{p,t}^{*}$ is

$$m_{p,t}^{*} = \mathbf{r}_t \otimes \mathbf{z}_t^{\mathrm{P}}\mathbf{'}\, E[(\mathbf{r}_t \otimes \mathbf{z}_t^{\mathrm{P}})(\mathbf{r}_t \otimes \mathbf{z}_t^{\mathrm{P}})']^{-1}(\mathbf{1}_{\mathrm{N}} \otimes \mathbf{z}_t^{\mathrm{P}}). \tag{19}$$

In sample, this is estimated by

$$\hat{m}_{p,t}^{*} = \mathbf{r}_t \otimes \mathbf{z}_t^{\mathrm{P}}\mathbf{'}\, T\left[\sum(\mathbf{r}_t \otimes \mathbf{z}_t^{\mathrm{P}})(\mathbf{r}_t \otimes \mathbf{z}_t^{\mathrm{P}})\mathbf{'}\right]^{-1}(\mathbf{1}_{\mathrm{N}} \otimes \mathbf{z}_t^{\mathrm{P}}). \tag{20}$$

The proposed test is to examine whether the SDF constructed using the public information is able to price returns constructed using the technical trading rules. If so, this indicates that technical trading rules are orthogonal to returns attainable through a trading strategy based on the public information, and thus cannot be used to expand the space of attainable returns for an informed investor. If the conditional SDF is unable to price the $r_t z_{t,k}^{\mathrm{TR}}$ returns, then the trading rules remain conditionally correlated with pricing error and can be used to generate a larger set of attainable returns. The H-statistic for the conditional test is computed using the technical trading rule information

$$H_T = T \cdot \hat{\boldsymbol{\lambda}}(m_p^{*}, \mathbf{Z}^{\mathrm{TR}})' \cdot W_T \cdot \hat{\boldsymbol{\lambda}}(m_p^{*}, \mathbf{Z}^{\mathrm{TR}}) \tag{21}$$

where

$$\boldsymbol{\lambda}(m_p^{*}, \mathbf{Z}^{\mathrm{TR}}) = E[(\mathbf{r}_t \otimes \mathbf{z}_t^{\mathrm{TR}}) \cdot m_{p,t}^{*} - \mathbf{1} \otimes \mathbf{z}_t^{\mathrm{TR}}] \tag{22}$$

and $\hat{\boldsymbol{\lambda}}(m_p^{*}, \mathbf{Z}^{\mathrm{TR}})$ is the sample analogue of $\boldsymbol{\lambda}(m_p^{*}, \mathbf{Z}^{\mathrm{TR}})$.

The H-statistic is asymptotically χ^2-distributed with $N\cdot(k_{TR})$ degrees of freedom under the null that the pricing error is zero. Comparing the conditional to the unconditional pricing error, the "artificial asset" being priced, $(\mathbf{r}_t \otimes \mathbf{z}_t^{\mathrm{TR}})$, is the same in both cases. The value of the returns, determined by the different SDF, is what has changed.

B. The Empirical Test

In the empirical analysis that follows, it is convenient to execute a linear transformation of what was described in the previous section. Rather than examining returns and determining whether an artificial asset is properly priced by the SDF, excess returns are examined. The SDF **m*** is able to price any linear function of the returns used to construct it. Let $\mathbf{r}_1$ be the return to a risk free bond. Excess returns such as $\mathbf{r}_2 - \mathbf{r}_1$ represent a zero cost portfolio.

The unconditional test is of the form

$$\mathbf{O} = E[((\mathbf{r}_t - r_{1,t}) \otimes \mathbf{z}_t^{\mathrm{TR}}) \cdot m_t^*] \tag{23}$$

which derives from the fact that under the null,

$$E[(r_{n,t} \otimes \mathbf{z}_t^{\mathrm{TR}}) \cdot m_t^*] = 1 = E[(r_{1,t} \otimes \mathbf{z}_t^{\mathrm{TR}}) \cdot m_t^*],\ n = 2,\ldots,\mathrm{N}. \tag{24}$$

Thus, the new pricing error is

$$\lambda(m^*, \mathbf{Z}^{\mathrm{TR}}) = E[((\mathbf{r}_t - r_{1,t}) \otimes \mathbf{z}_t^{\mathrm{TR}}) \cdot m_t^*]. \tag{25}$$

Although this test is weaker than pricing each asset separately, in practice, the H-statistic is only minimally affected (decreased by less than 0.01, much less in most cases).

The series $(\mathbf{r}_t - r_{1,t}) \otimes \mathbf{z}_t^{\mathrm{TR}}$ represent the return to a dynamic zero cost portfolio. A buy indicator instructs the trader to short the bond in order to purchase a risky asset. The sell indicator has the trader short the risky asset to purchase the bond. The hold signal indicates that neither position should be taken. If λ is positive, it indicates that the trading rule on average tends to signal "buy" when the excess returns are realized positive and "sell" when excess returns are realized negative (a weighted average, actually, as determined by the discount factor).

The conditional test uses the same procedure as the unconditional, substituting $m_{p,t}$* in place of m_t* in Equation (25).

C. Interpreting λ

At the most basic level, λ is an indicator variable. In the unconditional case, a zero value indicates that the asset being examined is in $\mathscr{R}(\mathbf{c})$, whereas a non-zero value indicates the asset is outside of $\mathscr{R}(\mathbf{c})$ and cannot be replicated by a linear combination of the original assets used to construct the SDF.

It is tempting to assign an economic interpretation to a non-zero value of λ. Unfortunately, the aspect that makes the procedure particularly useful for examining information also strips λ of economic meaning. As an illustration, consider three assets with returns r_1, r_2, and r_3 (where E(**rr**'), **r** = $\{r_1, r_2, r_3\}$, is non-singular). The first two are used the construct the SDF m_t*. These two assets then define the return space $\mathscr{R}(\mathbf{c})$. Using m_t* to price the returns to the third asset will result in a non-zero λ value. Let $\hat{r}_3 = a + b_1 r_1 + b_2 r_2$ be the projection of r_3 onto $\mathscr{R}(\mathbf{c})$ so that $r_3 = \hat{r}_3 + e$, where e is orthogonal to r_1 and r_2 with E(e) = 0. Thus, looking at excess returns

$$\pi(r_{3,t} - r_{1,t}) = E[(r_{3,t} - r_{1,t})m_t^*] = E[(\hat{r}_{3,t} - r_{1,t})m_t^*] = \frac{a}{r_f} + b_1 + b_2 - 1. \tag{26}$$

The outcome results from the facts that e and m_t* are uncorrelated, m_t* prices r_1 and r_2, and 1/E(m_t*) = r_f, the riskfree rate of return. A $\lambda > 0$ value indicates that, though an asset paying $r_3 - r_1$ has a zero cost, the portfolio of r_1 and r_2 that replicates $\hat{r}_3$ implies ($r_{3,t}$ - $r_{1,t}$) costs $\lambda \neq 0$ units to construct. Interpretation of λ is limited because r_3 does not exist in the space of r_1 and r_2. Thus, declaring a price for r_3 based on the value of $\hat{r}_3$ in $\mathscr{R}(\mathbf{c})$ space can be misleading. The price is not constructed within the context of a model. Only after imposing a model by say, claiming that m_t* is an estimate of the intertemporal marginal rate of substitution, can one say that λ is then the market value of the excess return, even though the actual price is zero.

The inability to assign an economic interpretation to λ limits the issues that this analysis can be used to address. Under the Efficient Markets Hypothesis, if the third asset exists in market equilibrium, then r_3 must have a price of one. Trivially, a SDF able to price all three returns is the m_t* constructed from all three assets. It is only in special cases in which theory indicates the third asset should be properly priced by a SDF constructed from the first two assets. This includes the situation explored in this paper where r_3 is the return to a dynamic trading strategy of the first two assets, with a null hypothesis that the trading strategy is not useful for generating excess returns. For this reason, the test procedure outlined in this paper is only useful for assessing information and examining the efficient markets hypothesis on the basis that prices should already reflect that information.

Using a given SDF, portfolios are rankable according to λ. Expanding the set of returns used to construct the SDF (expanding $\mathscr{R}(\mathbf{c})$) will change the λ value of a portfolio and possibly the relative rankings. For each portfolio, the direction of the change in λ depends on the direction of the correlation between the newly captured component of the portfolio's return and the expanded SDF. As long as the expanded $\mathscr{R}(\mathbf{c})$ does not fully contain the portfolio being priced, the direction of the correlation is unconstrained.

The conditional test is simply a special case of this type of expansion where the additional assets are dynamic portfolios of the original assets. As such, a portfolio with a non-zero $\boldsymbol{\lambda}(m^*,\mathbf{Z}^{\mathrm{TR}})$ but a zero-valued $\boldsymbol{\lambda}(m_p^*,\mathbf{Z}^{\mathrm{TR}})$ is a portfolio that cannot be replicated from the constant composition portfolios that define $\mathscr{R}(\mathbf{c})$, but is replicated by a linear combination of the original returns and the returns to dynamic portfolios based on the conditioning information set.

IV. Data

A. Returns and Trading Rules

Daily returns to holding US Treasury Bills are available from the Federal Reserve. The Daily returns to a value-weighted market portfolio of all the securities traded at the NYSE and AMEX are available from the Center for Research in Securities Prices (CRSP). These two return series are the basis of the return space, where $\mathbf{r}_1$ represents the T-Bill return, and

$\mathbf{r}_2$ represents the CRSP market return. Both series begin on July 3, 1962, and the last trading day in the series is Dec. 29, 1989.

The technical trading rules are typically buy-sell indicators that are based on recent price patterns of the security being considered. In practice, one of the simple and commonly watched trading rules involves comparing a short period moving average of the price of the security to a long period moving average. When the short period moving average, say the average price of the security over the last five trading days, rises above the long period moving average, say the average of the price over the last 200 trading days, this is a buy indicator. When the short period moving average drops below the long period moving average, this is a sell indicator. Short periods tested are one day (present price) and five day averages. Long periods tested are 50 and 200 day averages.

Placing bands around the long run average is another common practice. A one percent band means that the short period moving average must move above 101% percent of the long period moving average to be a buy indicator and must move below 99% to be a sell indicator. If the short period is within this range, the hold signal tells the trader not to take a position. The notation (s,l,b) indicates the short-period (s), long-period (l), and band width (b) rule used for a particular trading rule.

The risky asset under consideration is the CRSP value-weighed market portfolio, thus the technical trading rule series are constructed using the price series of this portfolio. The TTR series, z_t^{TR}, consists of 1, 0, and -1 values, with 1 indicating a buy period and -1 indicating a sell period. When the one percent band is used, 0 indicates that the short period moving average is within the band.

B. Public Information

There are two types of conditioning information used in this analysis. The first group consists of four market indicators. These variables are available at the daily frequency. All four are variables that indicate fundamental conditions of the market and are also often found in the existing literature testing the forecastability of returns (see for examples Chen, Roll and Ross (1985), Ferson and Korajczyk (1992), Connor and Korajczyk (1992), and Farnsworth *et al* (1995)). The first two variables are term structure variables, which indicate the difference between the returns on long term and short term government bonds. The variable *Term1* is the difference between 10-year and 3-month government bonds. *Term2* is the difference between 1-year and 3-month bonds. The third variable, *Qual*, is the difference in the rate of return on low risk and high risk bonds. *Qual* is computed as the difference in the rate of return on Baa and AAA rated bonds. The fourth variable, *Divyld*, is the dividend yield ratio, which is the dividend of the CRSP market portfolio divided by its price. *Term1*, *Term2*, and *Qual* are all obtained from the Federal Reserve. The *Divyld* variable is computed from information in CRSP.

The second set of conditioning information variables consists of indicators of the macro economy. *Ip* measures industrial production and is available at the monthly frequency. *M1* is the money stock and is available at the weekly frequency. Both variables are from the Federal Reserve. To include these variables with the daily frequency observations, each day in a week or month is assigned a value based on what is most recently available to the investor.

Two difficulties arise from the attempt to replicate the information actually available to investors. The first is that the values used have been revised by the Federal Reserve, and thus may not represent the actual values that were available to investors at the time. The second is that I do not have the time that the observations were revealed (i.e. the official publication date of the macroeconomic variables).

The empirical analysis found neither of the macroeconomic variables to be correlated with the daily returns, nor did they have an interesting impact on the pricing of the TTR based returns (neither alone nor in conjunction with other variables). The results from the conditional tests based on the macroeconomic variables are thus not included as part of the results section.

V. Results

Results are divided into four categories. Subsections *A* and *B* discuss the basic unconditional and conditional testing of the TTR returns. Subsections *C* and *D* address some of the issues raised concerning the basic tests. Subsection C is an analysis based on a SDF constructed using 24 assets. Subsection *D* replaces the market portfolio with IBM stock returns as the second asset in order to make the TTR more true to a real world application.

A. Results of the Unconditional Test

Table 1 reports the results of the both the unconditional and conditional tests of the TTRs. The first column identifies the trading rules by number. The second column gives the description of the trading rule. The third column reports the results of pricing the dynamic portfolio based on TTRs using the unconditional SDF. The top number in each cell of the table is the sample pricing error, $\hat{\boldsymbol{\lambda}}(m^*, \mathbf{Z}^{TR})$. The middle number of the cell, in italics, is the

H-statistic associated with the λ estimate. The χ^2 critical values for the individual tests are 2.71 for 90% confidence and 3.84 for 95% confidence. The bottom number, in parenthesis, is the p-value measuring the probability of obtaining the given λ estimate if the null hypothesis is true. Towards the bottom of the column is the H-statistic of the joint test that use of all eight of the trading rules contribute to expanded returns. The χ^2 critical values for the joint test are 13.36 for 90% confidence and 15.51 for 95% confidence.

The pricing errors reported in column 3 are all positive indicating that the average value of excess returns of the TTR dynamic portfolios are positive. The performance interpretation is that the TTRs' returns offer higher valued returns when the trader has no other information and is considering a buy and hold trading strategy. The joint test that the trading rules provide useful information is significant at the 95% level of confidence. Individually, the (5, 50, 0) and the (5, 50, 0.01) trading rules provide the trader with least useful information, with λ estimates that are the closest to zero. The (1, 200, 0) and (1, 200, 0.01) rules provide the trader with the greatest value, according to the unconditional SDF. In addition to the (1, 200) rules, the zero band (1, 50, 0) and (5, 200, 0) are also priced statistically significantly different from zero with 90% confidence. Overall, the results indicate that a number of the trading rules provide useful information to the trader who has no other source of information.
(Table 1 about here)

Brock *et al* (1992) found that for the rules with bands, a buy signal was associated with higher average returns and a sell signal was associated with more negative average returns as compared to the corresponding zero band signal. With frictionless trading, use of the banded rules might appear to be the preferred trading rule, but the result is misleading.

When implemented, the banded rules issue hold signals during periods that have lower expected profits, but profitable periods nonetheless, that the zero band rules take advantage of. The comparative success of the zero band trading rules may reflect the lost opportunities.[2]

The market conditioning information variables, unfortunately, do not cover the full sample period of the return series. For comparison purposes, the unconditional test is performed on each of the sub samples for which the conditioning information is available. The subsets are designated Subset A, for the sample for which the term structure variables are available; Subset B, for the sample for which the *Qual* variable is available; and Subset C, which is the intersection of Subsets A and B. Subset A contains 6628 observations, all but 86 observations of the full data set. Subsets B and C contain respectively 1010 and 999 observations from the end of the data set.

The unconditional tests using Subset A, reported in column 4 of Table 1, provide essentially the same results as the full data set. The unconditional performance tests for Subsets B and C, reported in columns 8 and 10 of Table 1, offer very different evaluations of the usefulness of the TTR than is obtained from the full data set. The λ estimates differ, with the (5, 50, 0) and (5, 50, 0.01) performing much better than most of the other trading rules. Further, none of the λ estimates are statistically significant from zero. Breaking the full data set up into subsets of 1000 observations each reveals that estimates of λ differ greatly over the different periods, possibly indicating nonstationarity in the system. Different market

[2] Alternatively, in a market where trading costs are imposed the banded trading rules may be preferred. Trading in order to earn low expected returns may not cover for the cost of the transaction.

conditions may be more or less accommodating to the use of these Moving Average TTRs, possibly depending on the availability of information and on trader behavior. The last 1000 observations appear to occur at a time that the trading rules are of little use to the otherwise uninformed investor.

B. Conditioning on Market Information

Columns 5 through 7, 9, and 11 through 13 of Table 1 contain the results from the conditional tests of the TTRs, based on the market information. Recall the conditional measure of λ,

$$\lambda(m_p^*, \mathbf{Z}^{\mathrm{TR}}) = E[((r_{2,t} - r_{1,t}) \otimes \mathbf{z}_t^{\mathrm{TR}}) \cdot m_{p,t}^*]. \tag{27}$$

Only *Term1*, *Term2*, and *Qual* are included in this table. Including dividend yield as a conditioning public information variable proved to have negligible effect on the λ and H-statistic estimates. This was true both when *Divyld* was examined independently and when examined in conjunction with the other information variables. One explanation is that *Divyld* is uncorrelated with one day ahead returns. Further examination indicated that this seems to be the case.

In subset A, the unconditional test results are very close to those of the full data set. The λ and H-statistics remain relatively unchanged when *Term1* and *Term2* are each separately included as conditioning information variables. When both the term structure variables are included, the λ estimates on all of the trading rules decrease and all of the H-statistics become insignificantly different from zero. The return series attainable through the use of the TTRs, when evaluated with the knowledge of both term structure variables, loses value to the extent that they no longer provide the trader with statistically significant excess returns. The duality of the test allows two interpretations of these results. The more financial oriented interpretation is that the value to the trader provided by the TTR based portfolio over a constant composition portfolio is largely encompassed by a trading strategy based on the two term structure variables. A statistical interpretations is simply that the trading rules are less correlated with the pricing error of the conditional SDF as compared to the unconditional SDF. When the trader prices the returns using the knowledge of the term structure, the returns offered by the TTRs appear to be insignificantly correlated with the pricing errors (though still positive correlated).

In both subsets B and C the unconditional individual tests have insignificantly small H-statistics. The conditional tests for subsets B and C have pricing errors that tend to be larger than those of the unconditional tests. All of the conditional pricing errors decrease when compared to the unconditional pricing errors (except for trading rule 8 in subset B). In a number of cases, the conditional pricing error is farther from zero, with large magnitude negative pricing errors. The results indicate the TTR trading strategies which offer mostly small advantages when evaluated with no conditioning information, are determined to offer mostly negative value when the trader has the term structure and quality information variables.

Recall though, that the test is for informational content, and thus the distance from zero is the measure of how much information is contained in the trading rule. By enlarge, the pricing errors are insignificantly different from zero. In the last column, only the (1, 50, 0) trading rule has a negative pricing error with a significant H-statistic (the (1, 50, 0.01) trading rule has a p-value of 0.109). However, the negative λ estimate for each trading rule indicates that to an informed investor, during the period covered by this sub sample, a contrarian trading rule would earn positive profits.

C. Using Additional Assets to Construct ***m****

The unconditional tests implemented in section *A* above may not yield results all that different from a standard CAPM analysis. The CAPM model's pricing mechanism is based on the same returns used in the construction of the unconditional SDF, i.e., the returns to a market portfolio and to a risk free asset.

The pricing condition required by the standard unconditional CAPM can be converted into the SDF framework. The CAPM SDF is

$$m_t^{\text{CAPM}} = a - b(r_m - r_f)$$

where

$$b = \frac{E(r_m - r_f)}{\sigma_m^2 r_f} \text{ and } a = \frac{1}{r_f} + bE(r_m).$$

The CAPM SDF applied to the TTR returns yields the λ estimates reported in Table 2. As a comparison to the nonparametric SDF, the CAPM pricing errors are similar, though with smaller pricing errors on all but TTRs (5,200,0) and (5,200,0.01).

(Table 2 about here)

Another concern with using the nonparametric SDF based on such a small set of assets is that the dynamic "asset" $\mathbf{r}_t z_{k,t}$ replicates an asset that already exists in the market, but is not included in the base set of N asset. In this case, the test procedure assigns $\mathbf{r}_t z_{k,t}$ a non-zero λ, giving the appearance that the information is useful to the investor. The reality may be that, for the purpose of forming a portfolio using the two base assets, $\mathbf{r}_t z_{k,t}$ replicates an existing traded asset already available in the market. As a performance measure, we may wish to give $\mathbf{r}_t z_{k,t}$ a zero performance rating in this special case.

To address this second concern, ideally, the SDF would be constructed from all of the tradable assets of the market. Thus, a market SDF $\mathbf{m}_m$*, which is an element of $\mathscr{M}$, would be used to price the dynamic portfolio returns of the TTRs. A dynamic trading strategy that could not be properly priced by $\mathbf{m}_m$* would truly represent returns that could not be replicated by any constant composition portfolio of existing marketed assets. Unfortunately, calculating the market SDF using all of the existing traded assets is computationally infeasible. As a substitute, a SDF is constructed using a total of twenty-four aggregated portfolios. In addition to the original two assets, the market portfolio and T-bill returns, are the returns to ten portfolios constructed by grouping assets according to the capitalization size of the firm and the returns to twelve portfolios constructed by grouping firms by their SIC codes.

Using the same procedures outlined in section IV, the stochastic discount factors are created, but in this case, the non-negativity constraint on the SDF is binding. Thus, the procedure for estimating a non-negative SDF as outlined in Hansen and Jagannathan (1991) must be used for this section. Refer to the non-negatively constrained SDFs as $\mathbf{m}_{24}^{+}$ and $\mathbf{m}_{24,p}^{+}$.

The results from using $\mathbf{m}_{24}^{+}$ and $\mathbf{m}_{24,p}^{+}$ to price the same $(r_{2,t} - r_{1,t})\mathbf{z}_t^{\mathrm{TR}}$ returns are contained in Table 3. For the full data set, the use of $\mathbf{m}_{24}^{+}$ generally lowers the estimates of the pricing errors, but does not particularly change the conclusions drawn from Table 1.

However, the conditional tests using $\mathbf{m}_{24,p}^{+}$ as the SDF differ greatly from the original results. In Subset A, the trading rules are useful to the trader regardless of the term structure information. This indicates that in the return space constructed from the twenty-four assets, making use of the term structure information does not contribute to the investor's attempt to replicate the returns derived from the TTRs. For Subsets B and C, the TTR returns are priced with λ estimates that are all statistically insignificantly difference from zero, but unlike the results in Table 1, the pricing errors are primarily positive.

(Table 3 here)

Imposing non-negativity on the SDF significantly changes the outcome of the test. Though not reported, the $\mathbf{m}_{24}$* and $\mathbf{m}_{24,p}$* SDFs without the non-negativity imposed provide results similar to those reported in Table 1. The (1, 50, 0), (1, 200, 0), (1, 50, 0.01) and (1, 200, 0.01) trading rules have λ estimates significantly different from zero in the full sample unconditional test, but all become insignificant when conditioning on *Term1* and *Term2*.

D. Trading IBM Stock

Traders use TTRs to trade actual tradable marketed assets. The CRSP market portfolio is not among these assets. Furthermore, individual assets are more likely to suffer from localized information dispersion that the TTRs are believed to take advantage of. For this reason, the performance of the TTRs used to trade a dynamic portfolio in IBM stock are examined (daily prices and returns from CRSP). The same eight moving average TTRs are computed using the price of IBM stock. The results of pricing the dynamic portfolio returns are reported in Table 4. For the full data set, and for subset A, all of the H-statistics indicate that the unconditional λ estimates are significantly different from zero at the 10% level. Many are significant at the 5% level. As with the market portfolio, the λ estimates are all positive for these data sets. As with the market portfolio, conditioning on both *Term1* and *Term2* reduces the λ estimates, but unlike the market portfolio, the conditional λ estimates remain significantly different from zero. The term structure variables may provide good information about the state of the overall market, but not on the state of IBM stock in particular. The usefulness of the trading rules is more likely to be affected by information specific to IBM fundamentals.

Another difference between using IBM stock and the market portfolio is that the conditional estimates of λ in subset C remain near zero, rather than turning negative. Finally, the Joint tests of the λ estimates are each insignificantly different from zero.

(Table 4 here)

VI. Concluding Remarks

A test method is developed for determining whether a set of information is useful for predicting asset returns which is model free and can incorporate conditioning information. Examining some simple technical trading rule strategies finds estimates for the performance measure, λ, that are positive and significantly different from zero for many of the trading rules. Using basic fundamental conditioning information that is composed of two term structure variables, the λ and H-statistics remain positive, but are smaller and insignificantly different from zero. These results indicate that the conditioning information is important for evaluating performance. The investor who uses a dynamic trading strategy based on the term structure variables may not find the TTRs useful, as does the uninformed investor.

Including the quality conditioning measure required that the sample size be reduced from 6714 to 1010. In this sub sample, the H-statistics are insignificant for many of the trading rules, rendering the results inconclusive. Nevertheless, the differences between the unconditional and conditional test are interesting. The trading rules tend to have unconditional λ estimates that are positive or close to zero, but the conditional estimates are uniformly negative, indicating a negative performance measure for the trading rules to an informed investor. Though statistically insignificant, the indication is that the TTRs are information that may be useful for making predictions during this time period.

Expanding the basic set of assets used in constructing the SDF indicates whether the information remains useful to an investor who has the ability to trade assets other than just the basic T-bill and the market portfolio. The most notable difference generated from using the expanded return space is that the TTRs remain useful, even when the two term structure variables are conditioned upon. The results indicate that the TTR-based portfolios do not replicate other existing assets in the market.

Finally, using IBM stock returns rather than the market portfolio finds the TTRs to be effective, with the term structure conditioning information decreasing the performance measure, but many of the TTR performances remain significantly different from zero.

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Table 1
Test of the pricing rule $E[(r_{2,t} - r_{1,t})\mathbf{z}_t^{TR} m_t^*] = 0$
Asset 1: 3 month T-bill; Asset 2: CRSP value weighted market portfolio
Term1 = 10 year bond rate - 3 month bond rate
Term2 = 1 year bond rate - 3 month bond rate
Qual = Baa - AAA returns

			Subset A				Subset B		Subset C			
Trading Rule #	(s,l,b)	Full Data Set	Unconditional	Conditional Term1	Term2	Term1&2	Unconditional	Conditional Qual	Unconditonal	Conditional T1 & Q	T2 & Q	T1,T2&Q
1	1,50,0	1.816	1.864	1.767	1.924	1.213	-0.824	-2.920	-0.309	-7.150	-6.581	-7.140
		2.768	*2.898*	*2.192*	*3.000*	*1.011*	*0.035*	*0.646*	*0.005*	*4.413*	*3.754*	*4.403*
		(0.096)	(0.089)	(0.139)	(0.083)	(0.315)	(0.852)	(0.422)	(0.944)	(0.036)	(0.053)	(0.036)
2	5,50,0	0.841	0.928	0.901	1.002	0.333	2.729	0.572	3.058	-4.024	-3.494	-4.016
		0.593	*0.717*	*0.569*	*0.814*	*0.076*	*0.382*	*0.025*	*0.474*	*1.393*	*1.055*	*1.389*
		(0.441)	(0.397)	(0.451)	(0.367)	(0.783)	(0.537)	(0.874)	(0.491)	(0.238)	(0.304)	(0.239)
3	1,200,0	2.478	2.505	2.499	2.601	1.806	3.832	2.387	4.076	-3.028	-2.710	-3.031
		5.159	*5.234*	*4.382*	*5.486*	*2.242*	*0.754*	*0.432*	*0.843*	*0.788*	*0.635*	*0.791*
		(0.023)	(0.022)	(0.036)	(0.019)	(0.134)	(0.385)	(0.511)	(0.359)	(0.375)	(0.426)	(0.374)
4	5,200,0	1.944	1.954	1.471	1.962	0.736	0.406	1.475	0.660	-0.955	-1.759	-0.992
		3.172	*3.184*	*1.518*	*3.121*	*0.372*	*0.008*	*0.165*	*0.022*	*0.078*	*0.267*	*0.085*
		(0.075)	(0.074)	(0.218)	(0.077)	(0.542)	(0.929)	(0.685)	(0.882)	(0.780)	(0.605)	(0.771)
5	1,50,.01	1.582	1.684	1.619	1.747	1.081	1.164	-0.932	1.639	-5.142	-4.678	-5.137
		2.387	*2.685*	*2.040*	*2.798*	*0.888*	*0.074*	*0.073*	*0.146*	*2.568*	*2.137*	*2.564*
		(0.122)	(0.101)	(0.153)	(0.094)	(0.346)	(0.786)	(0.787)	(0.702)	(0.109)	(0.144)	(0.109)
6	5,50,.01	0.877	0.921	0.891	0.998	0.301	2.168	0.063	2.603	-4.329	-3.848	-4.323
		0.743	*0.814*	*0.628*	*0.926*	*0.070*	*0.262*	*0.000*	*0.373*	*1.855*	*1.470*	*1.851*
		(0.389)	(0.367)	(0.428)	(0.336)	(0.791)	(0.609)	(1.000)	(0.541)	(0.173)	(0.225)	(0.174)
7	1,200,.01	2.261	2.243	2.022	2.296	1.336	0.844	0.755	1.046	-3.604	-3.661	-3.620
		5.457	*5.334*	*4.281*	*5.580*	*1.867*	*0.069*	*0.055*	*0.104*	*1.169*	*1.267*	*1.182*
		(0.019)	(0.021)	(0.039)	(0.018)	(0.172)	(0.793)	(0.815)	(0.747)	(0.280)	(0.260)	(0.277)
8	5,200,.01	1.378	1.341	0.922	1.359	0.200	-1.227	-0.008	-1.026	-2.824	-3.441	-2.856
		1.692	*1.591*	*0.624*	*1.585*	*0.029*	*0.080*	*0.000*	*0.055*	*0.767*	*1.111*	*0.785*
		(0.193)	(0.207)	(0.430)	(0.208)	(0.865)	(0.777)	(1.000)	(0.815)	(0.381)	(0.292)	(0.376)
Joint test		17.33	18.01	15.86	17.96	14.27	11.23	10.93	10.89	10.65	11.06	10.67
No. of Obs.		6714	6628				1010		999			

(s,l,b) represents the MA trading rule; s = short period, l = long period, and b = band width. Each cell reports λ estimates and individual H-statistics (*in italics*, p-values in parenthesis) for unconditional and conditional test. The individual H-statistics are χ^2 distributed with 1 degree of freedom under the null. Critical values are 2.71 for 90% confidence and 3.84 for 95% confidence. The joint H-statistics are distributed χ^2 with 8 degrees of freedom under the null. Critical values are 13.36 for 90% confidence and 15.51 for 95% confidence.

Table 2
Test of the pricing rule $E[(r_{2,t} - r_{1,t})\mathbf{z}_t^{TR} m_t] = 0$
Asset 1: 3 month T-bill; Asset 2: CRSP value weighted market portfolio
SDF: *m*(CAPM)

Trading Rule #	(s,l,b)	Full Data Set
1	1,50,0	1.501 *2.116* (0.146)
2	5,50,0	0.517 *0.251* (0.616)
3	1,200,0	2.295 *4.951* (0.026)
4	5,200,0	2.231 *4.676* (0.031)
5	1,50,.01	1.227 *1.633* (0.201)
6	5,50,.01	0.565 *0.351* (0.554)
7	1,200,.01	2.243 *5.391* (0.020)
8	5,200,.01	1.598 *2.589* (0.108)
Joint test		17.57
No. of Obs.		6714

(*s*,*l*,*b*) represents the MA trading rule; *s* = short period, *l* = long period, and *b* = band width. Each cell reports λ estimates and individual H-statistics (*in italics*, p-values in parenthesis) for unconditional and conditional test. The individual H-statistics are χ^2 distributed with 1 degree of freedom under the null. Critical values are 2.71 for 90% confidence and 3.84 for 95% confidence. The joint H-statistics are distributed χ^2 with 8 degrees of freedom under the null. Critical values are 13.36 for 90% confidence and 15.51 for 95% confidence.

Table 3

Test of the pricing rule $E[(r_{2,t} - r_{1,t})\mathbf{z}_t^{TR} m_t] = 0$

Asset 1: 3 month T-bill; Asset 2: CRSP value weighted market portfolio

SDF: m+(24), constructed from 24 assets, positivity imposed

Term1 = 10 year bond rate - 3 month bond rate

Term2 = 1 year bond rate - 3 month bond rate

Qual = Baa returns - AAA returns

			Subset A				Subset B		Subset C			
Trading Rule #	(s,l,b)	Full Data Set	Unconditional	Conditional Term1	Term2	Term1&2	Unconditional	Conditional Qual	Unconditonal	Conditional T1 & Q	T2 & Q	T1,T2&Q
1	1,50,0	1.461	1.505	1.745	1.490	1.844	-1.740	-1.922	0.196	0.716	-0.174	-1.463
		1.996	*2.100*	*2.844*	*2.043*	*3.121*	*0.225*	*0.297*	*0.003*	*0.035*	*0.002*	*0.170*
		(0.158)	(0.147)	(0.092)	(0.153)	(0.077)	(0.635)	(0.586)	(0.960)	(0.851)	(0.961)	(0.680)
2	5,50,0	0.485	0.575	0.812	0.578	0.876	1.860	1.524	3.632	3.606	2.446	1.809
		0.219	*0.307*	*0.615*	*0.307*	*0.705*	*0.258*	*0.187*	*0.867*	*0.898*	*0.480*	*0.260*
		(0.639)	(0.580)	(0.433)	(0.579)	(0.401)	(0.612)	(0.666)	(0.352)	(0.343)	(0.488)	(0.610)
3	1,200,0	2.214	2.239	2.476	2.162	2.444	3.010	1.486	3.954	3.751	3.616	3.053
		4.587	*4.646*	*5.726*	*4.302*	*5.485*	*0.675*	*0.177*	*1.028*	*0.972*	*1.050*	*0.741*
		(0.032)	(0.031)	(0.017)	(0.038)	(0.019)	(0.411)	(0.674)	(0.311)	(0.324)	(0.305)	(0.389)
4	5,200,0	2.130	2.129	2.351	2.014	2.241	2.540	1.604	2.249	2.441	3.544	4.687
		4.244	*4.204*	*5.164*	*3.733*	*4.613*	*0.480*	*0.207*	*0.332*	*0.411*	*1.009*	*1.747*
		(0.039)	(0.040)	(0.023)	(0.053)	(0.032)	(0.488)	(0.649)	(0.564)	(0.521)	(0.315)	(0.186)
5	1,50,.01	1.186	1.287	1.539	1.277	1.629	0.085	-0.225	1.870	2.059	0.754	0.337
		1.518	*1.772*	*2.558*	*1.728*	*2.808*	*0.001*	*0.005*	*0.252*	*0.325*	*0.052*	*0.010*
		(0.218)	(0.183)	(0.110)	(0.189)	(0.094)	(0.981)	(0.946)	(0.616)	(0.569)	(0.819)	(0.920)
6	5,50,.01	0.524	0.568	0.821	0.553	0.880	1.218	0.813	3.014	3.358	2.156	1.308
		0.300	*0.349*	*0.738*	*0.328*	*0.830*	*0.124*	*0.060*	*0.662*	*0.872*	*0.422*	*0.155*
		(0.584)	(0.555)	(0.390)	(0.567)	(0.362)	(0.725)	(0.806)	(0.416)	(0.350)	(0.516)	(0.694)
7	1,200,.01	2.159	2.134	2.382	2.048	2.309	1.170	0.003	1.646	1.773	2.312	1.957
		4.995	*4.850*	*6.088*	*4.460*	*5.683*	*0.140*	*0.000*	*0.280*	*0.302*	*0.492*	*0.334*
		(0.025)	(0.028)	(0.014)	(0.035)	(0.017)	(0.708)	(0.999)	(0.597)	(0.582)	(0.483)	(0.563)
8	5,200,.01	1.493	1.444	1.686	1.342	1.584	0.552	-0.347	0.258	0.642	1.836	2.451
		2.248	*2.088*	*2.857*	*1.787*	*2.473*	*0.025*	*0.011*	*0.005*	*0.030*	*0.285*	*0.556*
		(0.134)	(0.148)	(0.091)	(0.181)	(0.116)	(0.875)	(0.918)	(0.945)	(0.863)	(0.593)	(0.456)
Joint test		17.37	17.99	18.84	17.55	18.66	11.16	10.57	11.50	10.46	10.15	10.70
No. of Obs.		6714	6628				1010		999			

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Table 4
Test of the pricing rule $E[(r_{2,t} - r_{1,t})\mathbf{z}_t^{TR} m_t^*] = 0$
Asset 1: 3 month T-bill; Asset 2: IBM stock.
Term1 = 10 year bond rate - 3 month bond rate
Term2 = 1 year bond rate - 3 month bond rate
Qual = Baa - AAA returns

			Subset A				Subset B		Subset C			
Trading Rule #	(s,l,b)	Full Data Set	Unconditional	Conditional Term1	Term2	Term1&2	Unconditional	Conditional Qual	Unconditonal	Conditional T1 & Q	T2 & Q	T1,T2&Q
1	1,50,0	4.075	4.035	3.337	3.692	3.345	1.792	0.412	1.964	1.487	1.037	1.934
		5.860	*5.722*	*3.509*	*4.487*	*3.527*	*0.144*	*0.008*	*0.171*	*0.104*	*0.051*	*0.170*
		(0.015)	(0.017)	(0.061)	(0.034)	(0.060)	(0.704)	(0.929)	(0.679)	(0.747)	(0.821)	(0.680)
2	5,50,0	4.725	4.544	3.886	4.248	3.896	-1.381	-2.788	-1.715	-2.272	-2.698	-1.831
		7.881	*7.257*	*4.760*	*5.940*	*4.785*	*0.086*	*0.374*	*0.130*	*0.244*	*0.348*	*0.153*
		(0.005)	(0.007)	(0.029)	(0.015)	(0.029)	(0.769)	(0.541)	(0.718)	(0.621)	(0.555)	(0.696)
3	1,200,0	3.540	3.137	2.293	2.758	2.306	0.339	-1.020	-0.412	-0.930	-1.332	-0.664
		4.422	*3.458*	*1.656*	*2.502*	*1.675*	*0.005*	*0.050*	*0.007*	*0.041*	*0.085*	*0.020*
		(0.035)	(0.063)	(0.198)	(0.114)	(0.196)	(0.944)	(0.823)	(0.933)	(0.840)	(0.771)	(0.888)
4	5,200,0	3.525	3.119	2.280	2.722	2.292	0.161	-1.222	-0.134	-0.649	-1.065	-0.274
		4.384	*3.418*	*1.638*	*2.438*	*1.654*	*0.001*	*0.072*	*0.001*	*0.020*	*0.054*	*0.003*
		(0.036)	(0.064)	(0.201)	(0.118)	(0.198)	(0.975)	(0.788)	(0.975)	(0.888)	(0.816)	(0.956)
5	1,50,.01	3.683	3.747	3.064	3.424	3.073	0.477	-1.068	0.876	0.164	-0.252	0.543
		5.355	*5.500*	*3.262*	*4.275*	*3.282*	*0.011*	*0.061*	*0.037*	*0.001*	*0.003*	*0.015*
		(0.021)	(0.019)	(0.071)	(0.039)	(0.070)	(0.916)	(0.805)	(0.847)	(0.975)	(0.956)	(0.903)
6	5,50,.01	3.562	3.473	2.862	3.219	2.873	-0.872	-2.335	-0.959	-1.619	-2.024	-1.246
		5.080	*4.791*	*2.878*	*3.821*	*2.900*	*0.039*	*0.302*	*0.046*	*0.140*	*0.222*	*0.079*
		(0.024)	(0.029)	(0.090)	(0.051)	(0.089)	(0.843)	(0.583)	(0.830)	(0.708)	(0.638)	(0.779)
7	1,200,.01	3.270	2.884	2.031	2.496	2.044	-1.662	-3.007	-2.160	-2.648	-3.056	-2.326
		3.944	*3.056*	*1.353*	*2.136*	*1.370*	*0.130*	*0.455*	*0.215*	*0.346*	*0.467*	*0.257*
		(0.047)	(0.080)	(0.245)	(0.144)	(0.242)	(0.718)	(0.500)	(0.643)	(0.556)	(0.494)	(0.612)
8	5,200,.01	3.338	2.932	2.116	2.544	2.127	-0.596	-2.013	-1.069	-1.751	-2.124	-1.434
		4.097	*3.148*	*1.464*	*2.215*	*1.480*	*0.017*	*0.206*	*0.053*	*0.152*	*0.227*	*0.098*
		(0.043)	(0.076)	(0.226)	(0.137)	(0.224)	(0.896)	(0.650)	(0.818)	(0.697)	(0.634)	(0.754)
Joint test		10.75	9.68	7.19	8.24	7.20	8.20	8.63	8.40	8.83	8.77	9.07
No. of Obs.		6714	6628				1010		999			

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