

AN EXPLORATION OF THE INTERPLAY BETWEEN  
ASSESSMENT AND MOBILE PEDAGOGIES,  
IN SECONDARY SCHOOL MATHEMATICS

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## CERTIFICATE OF ORIGINAL AUTHORSHIP

I, Pauline Wong Wing Man Kohlhoff, declare that this thesis is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the Faculty of Arts and Social Sciences at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

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Pauline Kohlhoff

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# Prologue

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I once had the opportunity to participate in a truly great mathematics lesson. Aimed at Stage 3 students (Years 5 and 6), the lesson began with the premise that the students were to run a stall at a school fair, which would offer passers-by a turn at a game of chance.

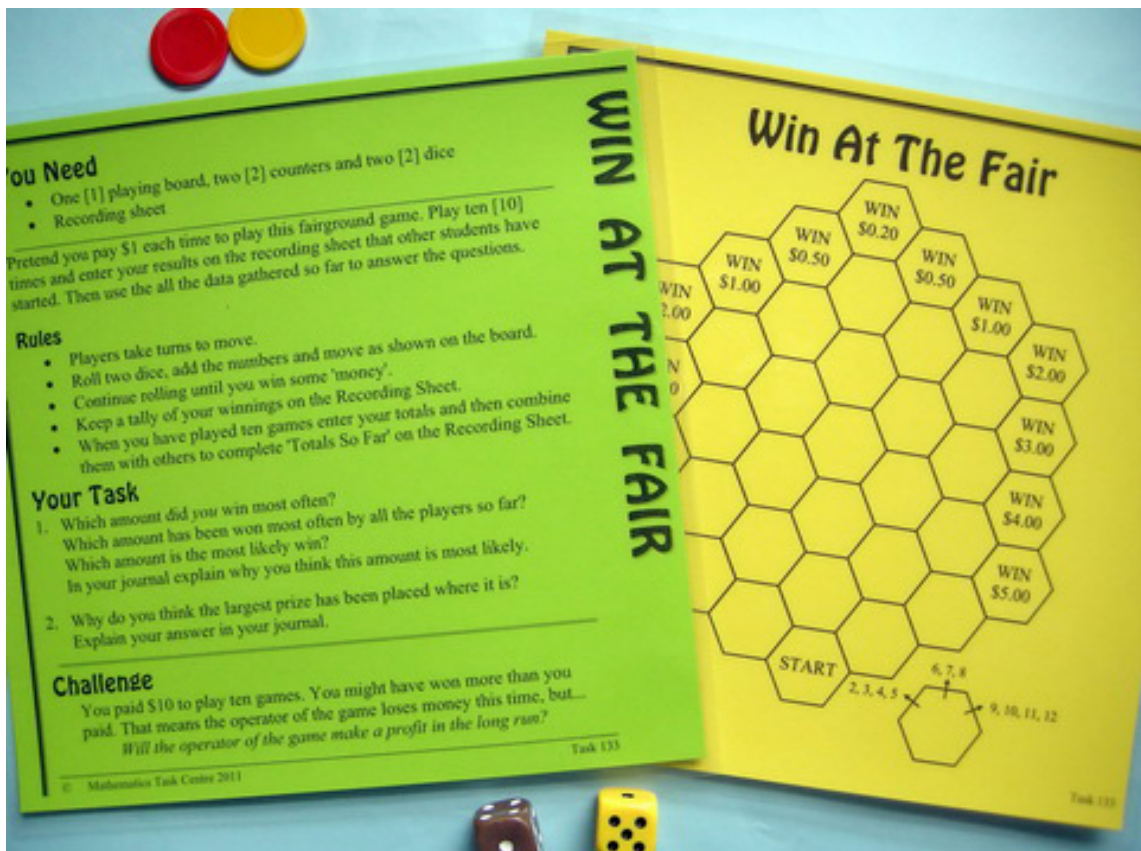
Customers would pay one dollar to be allowed to play this game, at the conclusion of which the game would “pay out” different sums, depending on the position on the board that was reached by the customer.

The teacher gathered the students around and selected one student to pretend to be a customer. The student proceeded to roll the dice and play the game to its conclusion, apparently “winning” twenty cents in the process. If this sequence of events had occurred in an actual game, then the customer would have had to pay a dollar to play in the first place, and so they would in fact have lost eighty cents; this was a point that was impressed upon the students.

The teacher then sent the students off in pairs to play five rounds of this game. Payout amounts were recorded on the whiteboard at the front of the room, and within ten minutes the whiteboard was covered in tally marks indicating how the students’ games had progressed.

At this point, the teacher stopped the class and asked them if they had noticed anything about the game. The class then proceeded to add up the takings from the game (\$87) and then the amount the game had paid out (\$124). The students were horrified - the game lost money for them! This couldn't be allowed - the principal would be furious. Something had to be done.

The first question at this point was - well, what was a reasonable amount to pay out, on average, for every customer? The students agreed that fifty cents would be about right. The game would be profitable, but it would not be obviously taking advantage of the customers; there would still be customers who would come away happy, and who would spread the word about how great the game was. Fifty cents was considered to be a good compromise.



Game board for “Win at the Fair” (mathematicscentre.com, 2017)

The teacher then explained that the game was available online. The students could change the parameters of the game and then ask the computer to run a thousand simulated games, to see what the effect of the change might be, on the amount that was paid back out to the customers.

And so, as a class, they played around with the parameters of the game. The students changed the effect of rolling 5, so that it would cause a movement upwards instead of to the left. They changed one of the pay-out amounts down from \$4 to thirty cents. With each change, they ran the simulation and checked if the average pay-out was getting any closer to fifty cents.

This lesson was run as a demonstration class at a teachers' conference in Sydney. The teacher, in this case, was Charles Lovitt; the students were all either primary school teachers or secondary mathematics teachers. At the conclusion of the activity, Lovitt explained that the purpose of the lesson was to help students understand how gambling games worked, so that, should they be faced with the prospect of playing the "pokies" at their local club, they would have an appreciation of why the game was guaranteed to lose them money. It was not, as advertised, a game of chance. Rather, it was a game of maths. Games were designed by people who understood that, in the long run, the game must have the advantage. The hope was that, through identifying with the game vendor, the students would learn that these games were rigged.

Following this, I spoke with a number of teachers who had attended that session. They - unanimously - loved it. They understood why this teaching was important, and they could appreciate the cleverness of the lesson design. They could see how all students could be engaged with this activity - after all, it only really required them to roll dice in order to be able to fully participate - as well as the scope it offered for extending the high achievers. The lesson ticked so many boxes - it was fun, it allowed the students to work

collaboratively and to take charge of their learning, and the mathematics it taught was so critically important.

It also, very interestingly, used technology in a completely authentic way. The technology was used to do one of the things that technology does best: it freed the students from having to do something repetitive and time-consuming, so that they could step back and see the big picture. It was central to the lesson, but at the same time, unobtrusive.

By the end of that lesson, I had learned a lot. So, I am sure, had many of the other teachers in the room. But what I found particularly interesting about that lesson was that, even though I knew that it taught some highly relevant mathematics, and taught it very well, I could only think of one way in which we could reasonably assess the depth of that learning. To my mind, the learning from that lesson could only be assessed twenty years later, after the students had likely long forgotten a goodly proportion of their school maths - and the assessment would simply be, do they play the pokies? If they did, then the learning had apparently not occurred to the extent that was intended; and if they did not, then what were the reasons for their abstinence?

While twenty years is clearly unreasonable for the purposes of school assessment, it is my belief that an immediate short-term assessment of this learning would not have told us anything meaningful at all. A similar effect might be had from the teacher simply telling the students, “Gambling is bad” - a lesson that would take five seconds, instead of fifty minutes. How would we know? For that matter, how can we ever know how embedded the learning has become? And, as far as assessment is concerned, do we ever have the opportunity to really value such intangible characteristics of students’ mathematical thinking?