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Isogeometric topology optimization for auxetic metamaterials and structures

by JIE GAO

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Certificate of Original Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Abbreviations

SIMP	Solid Isotropic Material with Penalization
ESO	Evolutionary Structural Optimization
BESO	Bi-directional Evolutionary Structural Optimization
LSM	Level Set Method
PLSM	Parameterized Level Set Method
CSRBF	Compactly Supported Radial Basis Function
HJ-PDE	Hamilton-Jacobian Partial Derivative Equation
OC	Optimality Criteria
MMA	Method of Moving Asymptotes
IGA	Isogeometric Analysis
ITO	Isogeometric Topology Optimization
M-ITO	Multi-material Isogeometric Topology Optimization
DDF	Density Distribution Function
TVF	Field of Topology Variable
DVF	Field of Design Variable
NPR	Negative Poisson's Ratio
NURBS	Non-Uniform Rational B-splines

Abstract

It is known that topology optimization is located at the conceptual design phase, which can effectively determine the numbers, connectivity and existence of holes in the structural design domain and evolve design elements to improve the concerned performance. General speaking, topology optimization works as an important tool to seek for the optimal material distribution, which has been identified as one of the most promising sub-field of structural optimization due to its superior features occurring in the conceptual design stage without prior knowledge of the design domain. In the current work, the main intention is to propose a novel numerical method for the topology optimization with more effectiveness and efficiency for the single-material structures and structures with multiple materials. Meanwhile, the proposed topology optimization method is also applied to implement the rational design of auxetic metamaterials and auxetic composites. In Chapter 1, we provide a brief description for the main intention of the current work. In Chapter 2, the comprehensive review about the developments of topology optimization, isogeometric topology optimization and the rational design of auxetic materials is provided.

In Chapter 3, a more effective and efficient topology optimization method using isogeometric analysis is proposed for continuum structures using an enhanced density distribution function (DDF). The construction of the DDF mainly involves two steps: (1) the smoothness of nodal densities is improved by the Shepard function; (2) the higher-order NURBS basis functions are combined with the smoothed nodal densities to construct the DDF with the continuity. A topology optimization formulation to minimize the structural mean compliance is developed

using the DDF and isogeometric analysis (IGA) is applied to solve structural responses. An integration of the geometry parametrization and numerical analysis offer several benefits for the optimization.

The Chapter 4 intends to develop a Multi-material Isogeometric Topology Optimization (M-ITO) method. Firstly, a new Multi-material Interpolation model is established with the use of NURBS (Non-uniform Rational B-splines), termed by the "N-MMI" model, which mainly involves three components: (1) Multiple Fields of Design Variables (DVFs); (2) Multiple Fields of Topology Variables (TVFs); (3) Multi-material interpolation. Two different M-ITO formulations are developed using the N-MMI model to address the problems with multiple volume constraints and the total mass constraint, respectively. The decoupled expression and serial evolving of the DVFs and TVFs can effectively eliminate numerical difficulties in the multi-material problems.

In Chapter 5, the proposed ITO method is applied for the systematic design of both 2D and 3D auxetic metamaterials. An energy-based homogenization method (EBHM) to evaluate the macroscopic effective properties is numerically implemented by IGA, with the imposing of periodic boundary formulation on material microstructure. An ITO formulation for 2D and 3D auxetic metamaterials is developed using the DDF, where the objective function is defined as a combination of the homogenized elastic tensor. A relaxed optimality criteria (OC) method is used to update the design variables, due to the non-monotonic property of the problem.

In Chapter 6, the proposed M-ITO method is applied for the systematic design of both 2D and 3D auxetic composites. The homogenization, that evaluates macroscopic effective properties of auxetic composites, is numerically implemented by IGA, with the imposing of the periodic boundary formulation on composite microstructures. The developed N-MMI model is applied to describe the multi-material topology and evaluate the multi-material properties. A topology optimization formulation for the design of both two-dimensional (2D) and three-dimensional (3D) auxetic composites is developed. Finite element simulations of auxetic composites are discussed using the ANSYS to show different deformation mechanisms.

Finally, conclusions and prospects are given in Chapter 7.

Key words: Topology optimization; Isogeometric analysis, Auxetic metamaterials; Auxetic composites; Homogenization.

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