

# Control of Manipulators on Moving Platforms Under Disturbance

by

Jon Woolfrey

A thesis submitted in partial fulfilment of the  
requirements for the degree of Doctor of Philosophy

at the

Centre for Autonomous Systems  
Faculty of Engineering and Information Technology  
**University of Technology Sydney**

February 2020



# Certificate of Original Authorship

I, Jonathan Woolfrey, declare that this thesis is submitted in fulfilment of the requirements for the award of Doctor of Philosophy in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise reference or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

This research is supported by the Australian Government Research Training Program.

Production Note:

Signed: Signature removed prior to publication.

---

Date: 21st February 2020

---



# Control of Manipulators on Moving Platforms Under Disturbance

by

Jon Woolfrey

A thesis submitted in partial fulfilment of the requirements for the  
degree of Doctor of Philosophy

## *Abstract*

Mobile robots, such as underwater vehicles, drones, and rovers, are now being combined with manipulators to perform a variety of work in the field. But current state of the art in control assumes that disturbances from the environment are minimal. However, the effects of wind, waves, and rough terrain may make it difficult for the vehicle to maintain a steady base for the manipulator. Or, in some cases, the vehicle may lack the control authority to negate disturbances in all directions. In this thesis, predictions of the base motion are used to formulate control strategies that enable a manipulator to proactively counter, and even make use of, these disturbances.

Time series and Fourier series are commonly applied to many predictive control methods in literature. However, there are contradictory results in performance for different applications. To clarify these discrepancies, an objective comparison in prediction performance is made between time series, Fourier series, and Gaussian Process Regression (GPR) using motion data from underwater robots in waves. Analysis of the forecast errors and uncertainties show that GPR can produce better short-term. Furthermore, time series was found to be overconfident in the prediction, whereas Fourier series had the largest uncertainty.

A predictive control method is then presented that enables a manipulator on a free-moving platform to maintain a steady end-effector pose. By using forecasts of the base motion,

the manipulator can anticipate and negate this disturbance. Simulations and experiments are conducted, and it is shown that the proposed predictive control can reduce tracking error by 60% compared to a PID feedback controller. Moreover, kinematic constraints can be satisfied whilst simultaneously minimizing task error.

A control strategy is also developed that allows a redundant manipulator to use the inertial forces produced by base disturbance to reduce joint torque. Further improvements are made by predicting changes in gravitational acceleration with respect to the manipulator. It is shown that joint torques can be reduced by 25% compared to a local minimization of the weighted torque norm.

Lastly, a torque minimization method is presented for redundant manipulators handling large external forces. Most literature only addresses the internal dynamics. This thesis presents a method to minimize torque from both an external loading and the internal dynamics. This method can be applied to manipulators on moving platforms, and further enhanced by incorporating the base motion predictions.

# *Acknowledgements*

- Dr. Andrew To For his generous help with data collection and conducting experiments for my research.
- Clyde Webster For his assistance with my research experiments.
- Prof. Dikai Liu For offering me the opportunity to complete my PhD, and unwavering support.
- Dr. Gavin Paul For giving me the opportunity to teach and share my knowledge.
- Dr. Khoa Le For his generosity in helping me with experiments.
- Dr. Marc Carmichael For the years of discussions, advice, and teaching opportunities.
- Sheila Sutjipto For her endless patience helping me with experiments and listening to me prattle on about my research.
- Dr. Stephen Woodcock For his guidance with time series prediction.
- Dr. Teresa Vidal-Calleja For her help with implenting Gaussian Process Regression.
- Dr. Wenjie Lu For all the knowledge, help, guidance, and support. I would not have made it this far otherwise.
- My cat Kismet For her emotional support, and constant reminders not to take life too seriously.
- My parents Christopher & Brigitte Woolfrey, for their endless years of care and support, without whom this would not have been possible.





# Contents

<b>Declaration of Authorship</b>	<b>iii</b>
<b>Abstract</b>	<b>v</b>
<b>Acknowledgements</b>	<b>vii</b>
<b>List of Figures</b>	<b>xi</b>
<b>List of Tables</b>	<b>xiii</b>
<b>Nomenclature</b>	<b>xvii</b>
<b>Glossary of Terms</b>	<b>xix</b>
<b>1 Introduction</b>	<b>1</b>
In Scope: . . . . .	11
Out of Scope: . . . . .	13
1.1 Thesis Outline . . . . .	14
1.2 Publications . . . . .	15
<b>2 Review of Related Work</b>	<b>17</b>
2.1 Intervention Tasks in Field Robotics . . . . .	17
2.2 Control of Manipulators Mounted on Mobile Platforms . . . . .	19
2.3 Forecasting Methods for Predictive Control . . . . .	21
2.3.1 Analysis of Motion Data . . . . .	21
2.4 Forecast Accuracy . . . . .	25
2.4.1 Root Mean Squared Error (RMSE) . . . . .	26
2.4.2 Akaike Information Criterion (AIC) . . . . .	26
2.4.3 Bayes Information Criterion (BIC): . . . . .	27
2.4.4 Theil's Inequality Coefficient . . . . .	27
2.4.5 Standard Error of Predictions for Assessing Forecast Uncertainty . . . . .	28
2.5 Predictive Control . . . . .	29
2.6 Other Control Methods for Disturbance Compensation . . . . .	31
2.7 Torque Minimization of Redundant Manipulators on Fixed Bases . . . . .	33

	Torque Minimization in the Null Space: . . . . .	35
	Minimum Kinetic Energy Method: . . . . .	35
	Null Space Damping Method: . . . . .	35
	Simultaneous Velocity & Torque Minimization: . . . . .	35
	Other Relevant Literature: . . . . .	36
2.8	Conclusions . . . . .	37
<b>3</b>	<b>Comparison of Forecasting Methods</b>	<b>39</b>
3.1	Problem Formulation . . . . .	39
	3.1.1 Case Study using Autonomous Underwater Vehicle (AUV) Motion Disturbance . . . . .	40
	Data Collection Process: . . . . .	41
3.2	Time Series . . . . .	42
	3.2.1 Structure of a Time Series Model . . . . .	42
	3.2.2 Determination of Model Parameters . . . . .	42
	3.2.3 Analysis of a Sample of AUV Motion Data . . . . .	44
	3.2.4 Producing Forecasts with Time Series . . . . .	44
3.3	Fourier Series . . . . .	46
	3.3.1 Superposition of Sinusoids . . . . .	46
	3.3.2 Extended Kalman Filter (EKF) . . . . .	47
	3.3.3 Forecasts from the Fourier Series with EKF . . . . .	48
3.4	Gaussian Process Regression . . . . .	49
	3.4.1 Kernel Functions . . . . .	50
	Matérn: . . . . .	50
	Rational Quadratic: . . . . .	50
	Periodic Rational Quadratic: . . . . .	50
	3.4.2 Mean Functions . . . . .	51
3.5	Comparison of Forecast Models . . . . .	51
	3.5.1 Model Parameters . . . . .	51
	Time Series: . . . . .	51
	Fourier Series: . . . . .	51
	Gaussian Process Regression: . . . . .	52
	3.5.2 Antarctic AUV . . . . .	52
	Mean Forecast Accuracy: . . . . .	53
	Forecast Uncertainty: . . . . .	53
	3.5.3 Submerged Pile Inspection Robot (SPIR) . . . . .	58
	3.5.3.1 Mean Forecast Accuracy . . . . .	58
	3.5.3.2 Forecast Uncertainty . . . . .	59
3.6	Discussion . . . . .	64
	3.6.1 Limitations . . . . .	64
	3.6.2 Time series vs Fourier Series . . . . .	64
	3.6.3 Model Parameters . . . . .	65
	3.6.4 Forecast Uncertainty . . . . .	65
	3.6.5 Vehicle Morphology & Stationarity . . . . .	66

---

3.7	Conclusion . . . . .	67
<b>4</b>	<b>Predictive Control of Manipulators on Moving Platforms</b>	<b>71</b>
4.1	Problem Scenario . . . . .	71
4.2	Problem Formulation . . . . .	74
4.2.1	Manipulator Kinematics . . . . .	74
4.2.2	Predictive End-Effector Control (PEEC) . . . . .	75
	Constrained solution: . . . . .	79
	Unconstrained solution: . . . . .	80
4.3	Verification of the Predictive End-Effector Control Method . . . . .	81
4.3.1	Simulation . . . . .	81
	Outline: . . . . .	81
	Results for the Unconstrained Case: . . . . .	83
	Results for the Constrained Case: . . . . .	85
4.3.2	Experimental Results . . . . .	87
	Outline for the Relative Motion Test: . . . . .	87
	Results for Relative Motion Test: . . . . .	89
	Outline for the Moving Base Test: . . . . .	90
	Results for the Moving Base Test: . . . . .	91
4.4	Discussion . . . . .	93
4.4.1	Orientation Error . . . . .	93
4.4.2	Dynamic Forces . . . . .	93
4.4.3	Base Motion Disturbance . . . . .	94
4.4.4	Feedback Control & Future Development . . . . .	94
4.5	Conclusion . . . . .	95
<b>5</b>	<b>Joint Torque Minimization of Manipulators on Moving Platforms</b>	<b>97</b>
5.1	Problem Scenario . . . . .	97
5.2	Modelling of the Manipulator and Platform . . . . .	99
5.3	Manipulator Control . . . . .	101
5.3.1	Weighted Least Norm (WLN) of the Joint Torques . . . . .	101
5.3.2	Predictive Joint Torque Minimization (PJTM) . . . . .	102
	Joint Control Resolution: . . . . .	102
	Predictions of gravitational forces: . . . . .	103
5.3.3	End-Effector Control . . . . .	105
5.4	Verification of the Predictive Joint Torque Minimization . . . . .	105
5.4.1	Implementation . . . . .	105
5.4.2	Results . . . . .	106
5.5	Discussion . . . . .	110
5.5.1	Choice of Forecast Horizon . . . . .	110
5.5.2	Null Space Damping & Energy Enervation . . . . .	111
5.5.3	Limitations . . . . .	111
5.6	Conclusion . . . . .	112
<b>6</b>	<b>Joint Torque Minimization with Large External Forces</b>	<b>113</b>

6.1	Problem Scenario . . . . .	113
6.2	Problem Formulation . . . . .	114
6.3	Joint Torque Minimization With External Forces . . . . .	115
6.3.1	Weighted Least Norm Solution . . . . .	115
6.3.2	External Joint Torque Minimization (EJTM) Method . . . . .	116
6.3.3	Incorporation of Physical Constraints . . . . .	118
6.4	Case Studies on Fixed Bases . . . . .	119
6.4.1	Torque Minimization for Heavy Lifting . . . . .	120
	Results: . . . . .	121
6.4.2	Torque Minimization for High-Pressure Blasting . . . . .	125
	Outline . . . . .	125
	Results: . . . . .	128
6.5	Case Study With A Moving Base . . . . .	131
	Outline: . . . . .	131
	Results: . . . . .	132
6.6	Discussion . . . . .	135
6.6.1	Choice of Control Parameters . . . . .	135
6.6.2	Considerations and Limitations . . . . .	135
6.6.3	Relationship to Cartesian Stiffness Control . . . . .	136
6.7	Conclusion . . . . .	138
<b>7</b>	<b>Conclusions</b> . . . . .	<b>139</b>
7.1	Summary of Contributions . . . . .	140
7.1.1	Comparison of Forecasting Models . . . . .	140
7.1.2	Predictive Control of Manipulators on Moving Platforms . . . . .	141
7.1.3	Joint Torque Minimization on Moving Platforms . . . . .	141
7.1.4	Joint Torque Minimization with External Forces . . . . .	142
7.2	Limitations . . . . .	142
7.2.1	Base Motion . . . . .	142
7.2.2	Base Inertia and Dynamics . . . . .	143
7.3	Future Work . . . . .	144
7.3.1	Prediction Methods for Nonstationary Data . . . . .	144
7.3.2	Disturbance Predictions . . . . .	144
7.3.3	Extension to the Predictive Control Method . . . . .	144
7.3.4	Simultaneous Vehicle-Manipulator Control . . . . .	145
7.3.5	Applications and Validation with Physical Experiments . . . . .	145
	<b>Appendices</b> . . . . .	<b>147</b>
	<b>Bibliography</b> . . . . .	<b>159</b>

# List of Figures

1.1	Two examples of field robots designed for observation tasks in harsh environments. . . . .	2
1.2	Examples of non-inertial mobile-manipulator systems. . . . .	3
1.3	The Submerged Pile Inspection Robot (SPIR) is designed to operate in shallow bathymetry to clean infrastructure. . . . .	4
1.4	Kinematic effects of base motion must be accounted for through the joint control. . . . .	6
1.5	A manipulator in a gravity field is equivalent to a manipulator accelerating upward. . . . .	9
1.6	The gravity vector with respect to a mobile manipulator. . . . .	9
1.7	The configuration of the robot arm affects its ability to withstand large external forces. . . . .	11
1.8	The scope of topic areas in this thesis. . . . .	13
1.9	The flow of ideas in this thesis. . . . .	14
2.1	Relative motion between the world frame and manipulator results in task error. . . . .	20
2.2	Two different AUVs were used to test motion data. . . . .	22
2.3	A sample of the Inertial Measurement Unit (IMU) data collected from the SPIR. . . . .	22
2.4	The SPIR motion data decomposed in to a sum of 4 sinusoids. . . . .	23
2.5	The motion data exhibits strong correlation across time. . . . .	24
2.6	Both linear regression and conditional probability can be used to predict the data. . . . .	25
2.7	An example of a predicted observation falling outside the 95% confidence interval of the forecast. . . . .	28
3.1	Correlograms for a sample of pitch motion (rad/s) shows statistically significant partial-autocorrelation values for up to 50 lagged observations. . . .	44
3.2	A 30s sample of IMU data recorded for the Antarctic AUV under wave excitation. . . . .	52
3.3	Forecasts of linear motion (red) for the Antarctic AUV prototype, with 95% confidence interval (grey) . . . . .	54
3.4	Forecasts of angular motion (red) for the Antarctic AUV prototype, with 95% confidence interval (grey). . . . .	55
3.5	A sample of the IMU data captured for the SPIR. . . . .	58

3.6	Correlograms for the heave motion of the SPIR v 3.0. The over-inflated partial-autocorrelation values are due to the nonstationary nature of the data. . . . .	59
3.7	Forecasts of linear motion for the SPIR (red), with 95% confidence interval (grey). . . . .	60
3.8	Forecasts of angular motion for the SPIR (red) with 95% confidence interval (grey). . . . .	61
3.9	A small net force is generated on the SPIR due to its size relative to the passing wave. . . . .	67
4.1	Relative frames in a mobile manipulation scenario. . . . .	71
4.2	A fixed target with respect to a moving frame can be conceptualized as a sequence of moving targets from the perspective of the moving frame. . . . .	72
4.3	The objective of the predictive control is to find a sequence of control actions to minimize the error across the prediction horizon. . . . .	76
4.4	Increasing the penalty term on the sequential control actions can be used to smooth the velocities, but increases tracking error. . . . .	79
4.5	Base motion for the predictive control simulation was generated from motion data of an AUV in waves. . . . .	82
4.6	The predictive control method reduced tracking error when the base is under disturbance. . . . .	83
4.7	Screenshots for the trajectory tracking simulation. . . . .	84
4.8	Using constraints on the kinematic feasibility, the predictive control method can satisfy joint constraints and reduce tracking error from disturbance. . . . .	85
4.9	Maximum joint speeds can be satisfied whilst reducing tracking error with the proposed method. . . . .	86
4.10	Photos taken during the relative motion test for the predictive control with the UR3. . . . .	88
4.11	The predictive control method can better track a moving target relative to the manipulator base frame. . . . .	89
4.12	Set up for the moving base test. . . . .	90
4.13	Images from the video recording of the moving base test. . . . .	90
4.14	The predictive control method reduce position and orientation error for the free-moving base experiment. . . . .	91
4.15	Plot of the position error of the end-effector in Cartesian space for the moving base test. . . . .	92
5.1	Relative change in gravity from the perspective of the manipulator. . . . .	98
5.2	Taking the arithmetic mean of the change in gravity across the prediction horizon reveals more information about the trend than a simple difference. . . . .	104
5.3	Base motion of the manipulator used in the simulation was generated from motion data captured from an AUV under wave excitation. . . . .	106
5.4	Norm of joint torques for torque minimization under base disturbance. By forecasting the change in base orientation, the shift in gravity can be utilized to reduce joint torque over the long term. . . . .	107

5.5	Individual joint torques for trajectory tracking with a moving base. The proposed control method shows reduced torque for joints 3, 4, 5 and 6. . . .	108
5.6	Joint positions over time for trajectory tracking under disturbance using torque minimization. . . . .	108
5.7	Screenshots from the simulation of joint torque minimization with base motion disturbance. . . . .	109
5.8	Average change in base orientation for different prediction lengths. . . . .	110
6.1	The manipulator must overcome gravitational forces when lifting and lowering the object, but must also account for the object's momentum. . . .	120
6.2	The proposed torque minimization method can reduce the torque required for heavy lifting by reconfiguring the redundant portion of the manipulator.	121
6.3	Screenshots of the heavy lifting simulation. Notice that the EJTM method brings joint 3 under the arm to provide support. . . . .	122
6.4	Joint angles for the heavy lifting scenario. The WLN method violates joint constraints. The EJTM method satisfies constraints and minimizes total joint torque. . . . .	123
6.5	Joint torques for the heavy lifting scenario. . . . .	123
6.6	A 3kg load on the end-effector overloads the joint torques in a sub-optimal joint configuration . . . . .	124
6.7	The manipulator autonomously reconfigures itself to a stiff posture, and is now able to support the 3kg weight. . . . .	124
6.8	A virtual end-effector with a virtual ball joint can be appended to the model to permit control over the blasting angle, and add redundancy to the system.	125
6.9	Nozzle reaction forces are referenced in frame {6} for blasting. . . . .	126
6.10	Using the proposed control method, the total joint torques required are reduce for high-pressure blasting. . . . .	128
6.11	Screenshots of the high-pressure blasting simulation. . . . .	129
6.12	The proposed control method is able to keep all joint torques within limits, even with large payload forces. The Weighted Least Norm (WLN) method violated the torque limit for joint 4. . . . .	130
6.13	The proposed method is able to satisfy joint constraints, even for the virtual joints in the model. . . . .	130
6.14	Base motion for the torque minimization with added end-effector forces, generated from IMU data of an AUV in waves. . . . .	132
6.15	Torque minimization with large payload forces can still be gauranteed even with base motion disturbance. . . . .	132
6.16	Screenshots for the weight lifting with moving base simulation. . . . .	133
6.17	Individual joint torques for the heavy lifting scenario with moving base. . .	134
6.18	Individual joint angles with large payload forces and moving base disturbances.	134





# List of Tables

3.1	Theil's Coefficient for the forecasts of the Antarctic AUV data. . . . .	56
3.2	Standard Error for the forecasts of the Antarctic AUV data. . . . .	57
3.3	Theil's Coefficient for forecasts of the SPIR motion data. . . . .	62
3.4	Standard Error for the forecasts of the SPIR motion data. . . . .	63
4.1	Feedback control gains and weighting matrices in the predictive control for the simulation. . . . .	82
4.2	Summary of performance results for the simulation. . . . .	84
4.3	Summary of results with joint velocities capped at 10 RPM. . . . .	86
4.4	Feedback control gains and predictive control weightings for the experiments on the UR3. . . . .	87
4.5	Summary of performance for the relative motion test. . . . .	89
4.6	Performance results for the moving base test. . . . .	92
5.1	Control parameters for torque minimization with base disturbance. . . . .	106
5.2	Mean Sum of Absolute Joint Torques . . . . .	107
6.1	Physical properties of the Sawyer manipulator . . . . .	121
6.2	Control parameters in the heavy-lifting scenario. . . . .	121
6.3	Physical properties of the igus Robolink manipulator . . . . .	127
6.4	Control parameters for the high-pressure blasting scenario . . . . .	127
6.5	Control parameters for torque minimization with external forces and moving base. . . . .	131



# Acronyms & Abbreviations

<b>2D</b>	2-Dimensional
<b>ACFR</b>	Australian Center for Field Robotics
<b>ADCP</b>	Acoustic Doppler Current Profiler
<b>AGV</b>	Autonomous Ground Vehicle
<b>AIC</b>	Akaike Information Criterion
<b>AR</b>	Autoregression
<b>ARMA</b>	Autoregression Moving-Average
<b>AUV</b>	Autonomous Underwater Vehicle
<b>BIC</b>	Bayes Information Criterion
<b>CAS</b>	Centre for Autonomous Systems
<b>DOF</b>	Degree(s)-of-Freedom
<b>EJTM</b>	External Joint Torque Minimization
<b>EKF</b>	Extended Kalman Filter
<b>DLS</b>	Damped Least Squares
<b>GP</b>	Gaussian Process
<b>GPR</b>	Gaussian Process Regression
<b>IMU</b>	Inertial Measurement Unit

<b>KF</b>	Kalman Filter
<b>MA</b>	Moving-Average
<b>MKE</b>	Minimum Kinetic Energy
<b>MPC</b>	Model Predictive Control
<b>P</b>	Proportional
<b>PD</b>	Proportional-Derivative
<b>PEEC</b>	Predictive End-Effector Control
<b>PID</b>	Proportional-Integral-Derivative
<b>PJTM</b>	Predictive Joint Torque Minimization
<b>QP</b>	Quadratic Programming
<b>RMSE</b>	Root Mean Squared Error
<b>ROS</b>	Robot Operating System
<b>ROV</b>	Remote Operated Vehicle
<b>SAUVIM</b>	Semi-Autonomous Underwater Vehicle for Intervention Missions
<b>SPIR</b>	Submerged Pile Inspection Robot
<b>UAV</b>	Unmanned Aerial Vehicle
<b>UAVMS</b>	Unmanned Aerial Vehicle-Manipulator System
<b>UTS</b>	University of Technology Sydney
<b>UVMS</b>	Underwater Vehicle-Manipulator System
<b>WLN</b>	Weighted Least Norm

# Nomenclature

## General Formatting Style

$x \in \mathbb{R}$	A scalar
$\mathbf{x} \in \mathbb{R}^m$	A vector
$\hat{\mathbf{x}} \in \mathbb{R}^m$	A unit vector
$\mathbf{X} \in \mathbb{R}^{m \times n}$	A matrix

## Operators

$\ \cdot\ $	Euclidean norm
$*$	Quaternion multiplication
$f(\cdot) \in \mathbb{R}$	A scalar function
$\mathbf{f}(\cdot) \in \mathbb{R}^m$	A vector function
$\mathbf{F}(\cdot) \in \mathbb{R}^{m \times n}$	A matrix function
$S(\cdot) \in \mathbb{R}^{m \times m}$	Skew-symmetric matrix operator

## Accents

$\dot{x}$	First time-derivative $dx/dt$
$\ddot{x}$	Second time-derivative $d^2x/dt^2$
$\hat{x}$	State estimate of $x$

## Specific Symbols

$\{F\}$	A frame of reference in Cartesian space
$\mathbf{J}^\dagger \in \mathbb{R}^{n \times m}$	Pseudoinverse of a non-square matrix $\mathbf{J} \in \mathbb{R}^{m \times n}$ , where $m \neq n$
$\mathbf{J}_W^\dagger \in \mathbb{R}^{n \times m}$	Weighted pseudoinverse of a non-square matrix $\mathbf{J} \in \mathbb{R}^{m \times n}$

---

$\mathbf{N}_W \in \mathbb{R}^{n \times n}$	Weighted null space projection matrix for a redundant manipulator.
$\mathbf{p} \in \mathbb{R}^3$	A position or translation vector in Cartesian space
${}^A\mathbf{p} \in \mathbb{R}^3$	A position or translation vector specified in frame $\{A\}$
$\mathbf{p}_A^B \in \mathbb{R}^3$	A translation vector from frame $\{A\}$ to frame $\{B\}$ , w.r.t. $\{A\}$
$\mathbf{q} \in \mathbb{R}^n$	Joint position vector for a manipulator
$Q \in \mathbb{H}$	A quaternion
$\mathbf{R}_A^B \in \mathbb{SO}(3)$	A rotation matrix from frame $\{A\}$ to frame $\{B\}$
$\sigma_x^2 \in \mathbb{R}$	Variance of a single random variable $x$ .
$\Sigma_x \in \mathbb{R}^{n \times n}$	Variance-covariance matrix of a set of random variables $\mathbf{x} \in \mathbb{R}^n$
$\mathbf{T}_A^B \in \mathbb{SE}(3)$	A homogeneous transformation matrix from frame $\{A\}$ to frame $\{B\}$
$\mathbf{w} \in \mathbb{R}^m$	A wrench of forces and torques.
$\hat{y}(t+i t+j)$	A state estimate of $y$ at time $t+i$ from time $t+j$ .

# Glossary of Terms

Autonomous	Without human intervention.
Bathymetry	The measurement of depth of water in oceans, seas, or lakes.
End-effector	The extremity of a robot arm designed for interacting with the external environment.
Heave	Vertical motion (z-direction).
Manipulator	A robotic arm design to handle physical objects.
Pitch	Rotation about the y-axis.
Roll	Rotation about the x-axis .
Surge	Forward motion (x-direction).
Sway	Sideways motion (y-direction).
Yaw	Rotation about the z-axis .

