# Robust trading strategies for a waste-to-energy combined heat and power plant in a day-ahead electricity market

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**Abstract:** Waste-to-energy (WtE) technologies have been used all over the world as they can solve the dilemma of waste management, energy demand, and global warming. Many modern WtE plants are built and operated in a combined heat and power (CHP) mode due to the high overall energy efficiency. This paper studies robust trading strategies for a WtE CHP plant which sells electricity in a day-ahead electricity market and exports heat to a district heating network. Owing to the requirements of the day-ahead electricity market, plant operators must determine the trading strategy one day before real delivery of electricity. However, many key problem parameters including electricity price, heat demand, and the amount of waste delivered to the plant are uncertain at the day-ahead stage. To derive robust electricity trading strategies for the WtE CHP plant under different types of uncertainty, a two-stage robust optimization model is developed and a solution procedure based on the column-and-constraint generation method is designed. A case study is also performed to illustrate the effectiveness of the robust model and the solution procedure.

*Keywords:* operational strategy, waste-to-energy, robust optimization, uncertainty, electricity market, combined heat and power

## 1. INTRODUCTION

Due to the increasing speed of municipal solid waste (MSW) generation, energy consumption, and greenhouse gas emission, countries worldwide are beset by environmental and energy issues. Waste-to-energy (WtE) technologies have drew the world's attention over the last few decades because they are widely used to solve the dilemma of energy demand, MSW management, and global warming. Unlike traditional power plants, WtE plants are often located close to population centers or industrial parks. Thus, WtE plants can operate in a combined heat and power (CHP) production mode and the residual heat after power production can be used for district heating or exported to nearby heat demanding factories (Ryu and Shin, 2012). Operating WtE plants in a CHP mode can improve the overall thermal efficiency. In Denmark, there are around 30 WtE plants which treat about 3.5 million tonnes of waste annually and the majority of them produce both heat and electricity (Fruergaard et al., 2010). Moreover, these WtE CHP plants export heat to district heating networks and sell electricity on commercial terms (Kirkeby et al., 2014).

Short-term operation of WtE CHP plants is a challenging task. The responsibility of treating waste and the requirement for the simultaneous production of electricity and heat make the operation of WtE CHP plants extremely hard. If WtE CHP plants participate in deregulated electricity markets (e.g., day-ahead markets), the operation task will become even harder. Since electricity is traded one day before real delivery in day-ahead markets, operators of the WtE CHP plants must decide the electricity trading strategy under different types of uncertainty such as electricity price and heat demand. Moreover, the electricity trading strategy to be determined should cover a whole day with hourly granularity which reflects the multi-stage nature of the problem.

Determining the electricity trading strategy for a WtE CHP plant in a day-ahead market is a short-term operation planning problem. Unfortunately, the research related to the short-term operation planning of WtE plants is very limited. Touš et al. (2015) addressed the short-term operation planning of an existing WtE CHP plant in the Czech Republic using a combination of black-box modeling and stochastic simulation. Abaecherli et al. (2017) introduced a very comprehensive mixed-integer linear programming model to tackle the short-term operation planning of an industrial waste incineration plant. However, neither of these two papers considered the operation of WtE plants in deregulated electricity markets. Technically speaking, WtE CHP plants can be seen as CHP systems. Although the study on the operation planning of WtE plants is limited, researchers have developed various stochastic programming models to address the short-term operation planning of CHP systems in deregulated electricity markets under uncertainty. Rolfsman (2004) developed a stochastic model to plan the energy production of a CHP plant in a deregulated electricity market under heat demand and electricity price uncertainty. De Ridder and Claessens (2014) also developed a stochastic programming model to study the optimal power trading strategy for industrial CHP systems in both day-ahead and real-time markets. Dimoulkas and Amelin (2015) proposed a three-stage stochastic model to tackle the unit commitment and energy dispatch problem of a CHP system in a day-ahead electricity market. Kumbartzky et al. (2017) developed a multi-stage stochastic model to optimize the daily operation plan of a CHP plant in multiple electricity markets with electricity price uncertainty. In addition to the stochastic programming approach, researchers started to apply robust optimization (Ben-Tal et al., 2009) to tackle the uncertainty issue in the operation planning of CHP systems. Zugno et al. (2016) proposed a two-stage robust optimization model to address the short-term operation planning problem of a CHP system in both day-ahead and real-time power markets under heat demand and electricity price uncertainty. Although WtE CHP plants can be seen as CHP systems, determining the electricity trading strategy for a WtE CHP plant in a day-ahead market is more complex than a typical CHP system. This is because the amount of waste delivered to a WtE CHP plant is uncertain at the day-ahead stage. This uncertainty may trigger an energy generation shortage in daily operations. Moreover, the primary goal of a WtE plant is to treat waste. Energy production is a useful by-product for cost recovery. Thus, this paper develops a two-stage robust optimization model to generate robust electricity trading strategies for a WtE CHP plant in a day-ahead market under different types of uncertainty.

The rest of the paper is organized as follows. In Section 2, we first describe the problem, then introduce the two-stage robust optimization model and the uncertainty sets. The solution procedure for the developed model is discussed in Section 3. In Section 4, results from a case study are presented. Finally, we conclude the paper in Section 5.

## 2. PROBLEM DESCRIPTION AND MODEL FORMULATION

### 2.1 Problem description

We consider a WtE CHP plant that sells electricity in a day-ahead electricity market and exports heat to a district heating network. The plant has two kinds of CHP production units which are back-pressure and extraction units. A diagram which roughly shows the operation process of the WtE CHP plant is depicted in Fig. 1.

In Fig. 1, waste is first delivered to the plant and temporarily stored in a waste bunker. Then, waste is combusted to release the chemical energy which is transferred to CHP units to generate heat and electricity. Since the electricity trading process happens one day before real delivery in the day-ahead market, the plant operators have to determine the electricity trading strategy under different types of uncertainty. To tackle the problem, we develop a two-stage robust model that considers three types of uncertainty including district heating demand, day-ahead electricity price, and the amount of waste delivered to the plant. The first-stage decisions are about the hourly amount of electricity to be sold in the day-ahead market (trading strategy) for a particular day. The second-stage (recourse) decisions are related to waste incineration, heat production, and electricity delivery on the same day. The model aims to derive the robust optimal electricity trading strategy that can minimize the worst-case daily expense of the WtE CHP plant under uncertainty.

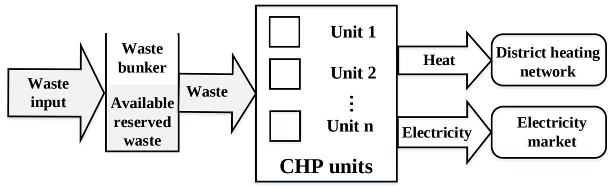


Fig. 1. The operation process of the WtE CHP plant

### 2.2 Model formulation

*Sets:*

| | |
|---|---|
| $T$ | set of time periods, $T = \{1, \cdots, \|T\|\}$ |
| $I$ | set of production units, $I = \{1, \cdots, \|I\|\}$ |
| $I_{bp}$ | set of back-pressure units, $I_{bp} \subseteq I$ |
| $I_{ex}$ | set of extraction units, $I_{ex} \subseteq I$ |

*Parameters:*

| | |
|---|---|
| $H_i^{max}$ | maximum heat output of unit $i$ |
| $H_i^{min}$ | minimum heat output of unit $i$ |
| $P_i^{max}$ | maximum power output of unit $i$ |
| $P_i^{min}$ | minimum power output of unit $i$ |
| $R_i^{HD}$ | heat ramp-down limit of unit $i$ |
| $R_i^{HU}$ | heat ramp-up limit of unit $i$ |
| $R_i^{PD}$ | power ramp-down limit of unit $i$ |
| $R_i^{PU}$ | power ramp-up limit of unit $i$ |
| $M_i^{max}$ | maximum waste consumption for unit $i$ |
| $M_i^{min}$ | minimum waste consumption for unit $i$ |
| $W_{max}$ | maximum amount of waste in the waste bunker |
| $W_{min}$ | minimum amount of waste in the waste bunker |
| $c_i$ | marginal operating cost for waste treatment of unit $i$ |
| $r_i$ | heat-to-power ratio of unit $i$ |
| $f_i^p$ | marginal waste consumption for power production of unit $i$ |
| $f_i^h$ | marginal waste consumption for heat production of unit $i$ |
| $\lambda_t$ | day-ahead electricity price in period $t$ |
| $d_t$ | heat demand in period $t$ |
| $q_t$ | amount of waste delivered to the plant in period $t$ |
| $w_0$ | amount of waste stored in the waste bunker at the beginning of the time horizon |
| $p_{i0}$ | power output of unit $i$ at the beginning of the time horizon |
| $h_{i0}$ | heat output of unit $i$ at the beginning of the time horizon |

*Decision variables:*

| | |
|---|---|
| $x_t$ | amount of electricity to be sold in the day-ahead market in period $t$ |
| $p_{it}$ | power output of unit $i$ in period $t$ |
| $h_{it}$ | heat output of unit $i$ in period $t$ |
| $w_t$ | amount of waste stored in the waste bunker in period $t$ |

The two-stage robust model can be formulated as follows:

$$\min_{\boldsymbol{x}} \max_{\boldsymbol{d}\in\boldsymbol{D},\boldsymbol{\lambda}\in\boldsymbol{\Lambda},\boldsymbol{q}\in\boldsymbol{Q}} R(\boldsymbol{x},\boldsymbol{d},\boldsymbol{\lambda},\boldsymbol{q}) \quad (1)$$

$$\text{s.t.} \quad \sum_{i\in I} P_i^{min} \le x_t \le \sum_{i\in I} P_i^{max}, \quad \forall t\in T \quad (2)$$

$$x_t \ge 0, \quad \forall t\in T \quad (3)$$

where $R(\boldsymbol{x},\boldsymbol{d},\boldsymbol{\lambda},\boldsymbol{q})$ represents the expense of the WtE plant given electricity trading strategy $x_t$, heat demand $d_t$, day-ahead electricity price $\lambda_t$, and the amount of waste delivered to the plant $q_t$. Note that notation $\boldsymbol{x}$ subsumes $x_t$. Notations $\boldsymbol{d}$, $\boldsymbol{\lambda}$, and $\boldsymbol{q}$ subsume uncertain parameters $d_t$, $\lambda_t$, and $q_t$, respectively. Moreover, $\boldsymbol{d}$, $\boldsymbol{\lambda}$, and $\boldsymbol{q}$ respectively take values from the predefined uncertainty sets $\boldsymbol{D}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{Q}$. These uncertainty sets will be discussed in detail in the next subsection. $R(\boldsymbol{x},\boldsymbol{d},\boldsymbol{\lambda},\boldsymbol{q})$ equals the optimal objective value of the following problem

$$R(\boldsymbol{x},\boldsymbol{d},\boldsymbol{\lambda},\boldsymbol{q}) = \min \sum_{i\in I} c_i \sum_{t\in T}(f_i^p p_{it} + f_i^h h_{it}) - \sum_{t\in T} x_t\lambda_t \quad (4)$$

$$\text{s.t.}\ p_{it} = r_i h_{it}, \quad \forall i\in I_{bp}, t\in T \quad (5)$$

$$p_{it} \ge r_i h_{it}, \quad \forall i\in I_{ex}, t\in T \quad (6)$$

$$f_i^p p_{it} + f_i^h h_{it} \ge (f_i^p + f_i^h/r_i)P_i^{min}, \forall i\in I_{ex}, t\in T \quad (7)$$

$$f_i^p p_{it} + f_i^h h_{it} \le f_i^p P_i^{max}, \quad \forall i\in I_{ex}, t\in T \quad (8)$$

$$H_i^{min} \le h_{it} \le H_i^{max}, \quad \forall i\in I, t\in T \quad (9)$$

$$M_i^{min} \le f_i^p p_{it} + f_i^h h_{it} \le M_i^{max}, \quad \forall i\in I, t\in T \quad (10)$$

$$p_{it} - p_{i(t-1)} \le R_i^{PU}, \quad \forall i\in I, t\in T \quad (11)$$

$$p_{i(t-1)} - p_{it} \le R_i^{PD}, \quad \forall i\in I, t\in T \quad (12)$$

$$h_{it} - h_{i(t-1)} \le R_i^{HU}, \quad \forall i\in I, t\in T \quad (13)$$

$$h_{i(t-1)} - h_{it} \le R_i^{HD}, \quad \forall i\in I, t\in T \quad (14)$$

$$\sum_{i\in I} h_{it} >= d_t, \quad \forall t\in T \quad (15)$$

$$\sum_{i\in I} p_{it} = x_t, \quad \forall t\in T \quad (16)$$

$$w_t = w_{t-1} + q_t - \sum_{i\in I}(f_i^p p_{it} + f_i^h h_{it}), \quad \forall t\in T \quad (17)$$

$$W_{min} \le w_t \le W_{max}, \quad \forall t\in T \quad (18)$$

$$w_{|T|} <= w_0 \quad (19)$$

$$h_{it}, p_{it}, w_t \ge 0, \quad \forall i\in I, t\in T. \quad (20)$$

Objective function (1) minimizes the worst-case daily expense of the WtE plant. Constraints (2) ensure that the amount of electricity to be sold in the day-ahead market should be more than the plant's minimum power output and less than its maximum output in period $t$. Objective function (4) minimizes the daily expense of the WtE plant, expressed as the difference between the operating cost and the revenue from selling electricity. Note that we do not consider the revenue from heat sales in objective (4). Because heat price is assumed to be unchanged and this revenue does not affect the optimal solution. Constraints (5) and (6) reflect the relationship of the power and heat generation in back-pressure and extraction units, respectively. Constraints (7)-(9) define the feasible regions for the power and heat production of back-pressure and extraction units. Constraints (10) impose the upper and lower bounds for waste consumption of unit $i$ in period $t$. Constraints (11) and (12) respectively impose the upward and downward power ramping limits for unit $i$. Constraints (13) and (14) respectively impose the upward and downward heat ramping limits for unit $i$. Constraints (15) ensure that the heat produced by all units can meet the demand of the district heating network in period $t$. Note that surplus heat may be produced since the primary goal of the WtE plant is to treat waste. Constraints (16) guarantee that the power produced by all units in period $t$ should be equal to the amount of electricity sold in the day-ahead market. Constraints (17) calculate the amount of waste stored in the waste bunker in period $t$. Constraints (18) ensure that the amount of waste stored in the waste bunker does not exceed the upper and lower limits. Constraint (19) forces that the amount of waste in the waste bunker at the end of the optimization horizon to be less than that at the beginning. This condition corresponds to the WtE plant's primary goal of treating waste. Constraints (3) and (20) deal with the nature of the variables. In this paper, we refer to formulation (1)-(3) as the first-stage problem and formulation (4)-(20) as the second-stage problem.

### *2.3 Uncertainty set definition*

Defining uncertainty sets is important for an effective representation of different types of uncertainty in the problem. Based on the uncertainty sets introduced in Bertsimas and Sim (2004), we define three uncertainty sets $\boldsymbol{D}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{Q}$ which respectively model the uncertain parameters $d_t$, $\lambda_t$, and $q_t$. The details of these uncertainty sets are shown in the following equations (21a)-(21c).

$$\boldsymbol{D} = \left\{ \boldsymbol{d} : d_t = \overline{d}_t + z_t^d \hat{d}_t, |z_t^d| \le 1, \sum_{t\in T} |z_t^d| \le \Gamma_d, t\in T \right\} \quad (21a)$$

$$\boldsymbol{\Lambda} = \left\{ \boldsymbol{\lambda} : \lambda_t = \overline{\lambda}_t + z_t^\lambda \hat{\lambda}_t, |z_t^\lambda| \le 1, \sum_{t\in T} |z_t^\lambda| \le \Gamma_\lambda, t\in T \right\} \quad (21b)$$

$$\boldsymbol{Q} = \left\{ \boldsymbol{q} : q_t = \overline{q}_t + z_t^q \hat{q}_t, |z_t^q| \le 1, \sum_{t\in T} |z_t^q| \le \Gamma_q, t\in T \right\} \quad (21c)$$

In uncertainty set $\boldsymbol{D}$, $\overline{d}_t$ denotes the nominal value of the uncertain heat demand $d_t$. $\hat{d}_t$ denotes the maximum possible deviation of $d_t$ from its nominal value. $z_t^d$ denotes the auxiliary variable and $\Gamma_d$ is the uncertainty budget which limits the variation of $d_t$. From (21a), it is clear that $d_t$ takes values from the interval $[\overline{d}_t - \hat{d}_t, \overline{d}_t + \hat{d}_t]$ for all $t\in T$. However, it is controlled by $\Gamma_d$. If $\Gamma_d = 0, d_t = \overline{d}_t$. If $\Gamma_d = |T|$, $d_t$ can take any value in the interval $[\overline{d}_t - \hat{d}_t, \overline{d}_t + \hat{d}_t]$. The parameters of uncertainty sets $\boldsymbol{\Lambda}$ and $\boldsymbol{Q}$ have similar meanings as those introduced in set $\boldsymbol{D}$.

## 3. SOLUTION PROCEDURE

To solve the above two-stage robust model, we design a solution procedure based on the column-and-constraint generation (C&CG) method proposed in Zeng and Zhao (2013). The solution procedure relies on an iterative process where a master problem, a feasibility check problem, and a subproblem are solved in each iteration. To derive the solution procedure with simplicity and clarity, we use a general formulation to represent the proposed robust model as shown in (1)-(20). The corresponding general formulation can be expressed as follows:

$$\min_{\boldsymbol{x}} \quad \max_{\boldsymbol{d}\in \boldsymbol{D},\boldsymbol{\lambda}\in\boldsymbol{\Lambda},\boldsymbol{q}\in\boldsymbol{Q}} R(\boldsymbol{x},\boldsymbol{d},\boldsymbol{\lambda},\boldsymbol{q}) \tag{22a}$$

$$\text{s.t.} \quad \boldsymbol{Ax} \le \boldsymbol{b} \tag{22b}$$

$$\boldsymbol{x} \ge \boldsymbol{0} \tag{22c}$$

where $R(\boldsymbol{x},\boldsymbol{d},\boldsymbol{\lambda},\boldsymbol{q})$ is given as

$$R(\boldsymbol{x},\boldsymbol{d},\boldsymbol{\lambda},\boldsymbol{q}) = \min_{\boldsymbol{y}} \boldsymbol{c}_{\boldsymbol{y}}^{\top}\boldsymbol{y} - \boldsymbol{\lambda}^{\top}\boldsymbol{x} \tag{23a}$$

$$\text{s.t.}\ \boldsymbol{By} \ge \boldsymbol{Ed} + \boldsymbol{Fq} + \boldsymbol{Gx} + \boldsymbol{g} \tag{23b}$$

$$\boldsymbol{y} \ge \boldsymbol{0}. \tag{23c}$$

In formulation (22), $\boldsymbol{x}$ denotes the first-stage decision variables. Constraints (22b) and (22c) correspond to constraints (2) and (3), respectively. In formulation (23), $\boldsymbol{y}$ denotes the second-stage decision variables that subsume variables $h_{it}$,$p_{it}$,$w_t$. $\boldsymbol{c}_{\boldsymbol{y}}$ represents the corresponding coefficients in the objective function (4). Constraints (23b) subsume constraints (5)-(19), where matrices $\boldsymbol{B}$, $\boldsymbol{E}$, $\boldsymbol{F}$, and $\boldsymbol{G}$ respectively denote the corresponding coefficients of variables $\boldsymbol{y}$, $\boldsymbol{d}$, $\boldsymbol{q}$, and $\boldsymbol{x}$. Vector $\boldsymbol{g}$ represents the corresponding constants.

Next, we present the subproblem and the feasibility check problem which need to be solved in the solution procedure. The subproblem is shown in the following formulation (24):

$$R(\boldsymbol{x}) = \max_{\boldsymbol{d}\in \boldsymbol{D},\boldsymbol{\lambda}\in\boldsymbol{\Lambda},\boldsymbol{q}\in\boldsymbol{Q}} R(\boldsymbol{x},\boldsymbol{d},\boldsymbol{\lambda},\boldsymbol{q}) \tag{24}$$

where $R(\boldsymbol{x})$ denotes the worst-case second-stage objective value over the uncertainty sets given the first-stage decisions $\boldsymbol{x}$. As shown in formulation (24), the subproblem has a max-min optimization structure, which makes it difficult to solve. Since the inner second-stage minimization problem $R(\boldsymbol{x},\boldsymbol{d},\boldsymbol{\lambda},\boldsymbol{q})$ in formulation (23) is a linear program, we can dualize it and rewrite the subproblem with a new formulation (25). Note that the term $\boldsymbol{\lambda}^{\top}\boldsymbol{x}$ is a fixed constant given $\boldsymbol{x}$ and $\boldsymbol{\lambda}$ in the objective function of the second-stage problem (23). Thus, $\boldsymbol{\lambda}^{\top}\boldsymbol{x}$ keeps unchanged when performing the dual transformation.

$$R(\boldsymbol{x}) = \max_{\boldsymbol{d},\boldsymbol{\lambda},\boldsymbol{q},\boldsymbol{\mu}} (\boldsymbol{Ed} + \boldsymbol{Fq} + \boldsymbol{Gx} + \boldsymbol{g})^{\top}\boldsymbol{\mu} - \boldsymbol{\lambda}^{\top}\boldsymbol{x} \tag{25a}$$

$$\text{s.t.}\ \boldsymbol{B}^{\top}\boldsymbol{\mu} \le \boldsymbol{c}_{\boldsymbol{y}} \tag{25b}$$

$$\boldsymbol{\mu} \ge \boldsymbol{0}, \boldsymbol{d}\in\boldsymbol{D}, \boldsymbol{\lambda}\in\boldsymbol{\Lambda}, \boldsymbol{q}\in\boldsymbol{Q}. \tag{25c}$$

In formulation (25), $\boldsymbol{\mu}$ denotes the dual variables corresponding to constraints (23b). However, the bilinear terms $\boldsymbol{d}^{\top}\boldsymbol{\mu}$ and $\boldsymbol{q}^{\top}\boldsymbol{\mu}$ in (25a) still make the subproblem (25) difficult to handle. Thus, we use the linearization method discussed in Thiele et al. (2009) to tackle the bilinear terms and the subproblem (25) can be solved efficiently after linearization. Note that the inner second-stage problem (23) of the subproblem (24) can be infeasible given some first-stage decisions $\boldsymbol{x}$ and realizations of the uncertain parameters. Thus, we need to cut the first-stage decisions which may make the second-stage problem (23) infeasible. Effective C&CG feasibility cuts can be generated by solving the following feasibility check problem (26):

$$F(\boldsymbol{x}) = \max_{\boldsymbol{d}\in\boldsymbol{D},\boldsymbol{q}\in\boldsymbol{Q}} \min_{\boldsymbol{y},\boldsymbol{\xi}} \boldsymbol{1}^{\top}\boldsymbol{\xi} \tag{26a}$$

$$\text{s.t.}\ \boldsymbol{By} + \boldsymbol{\xi} \ge \boldsymbol{Ed} + \boldsymbol{Fq} + \boldsymbol{Gx} + \boldsymbol{g} \tag{26b}$$

$$\boldsymbol{y},\boldsymbol{\xi} \ge \boldsymbol{0} \tag{26c}$$

where $\boldsymbol{\xi}$ represents slack variables of constraints (26b). Note that the subproblem (24) is unbounded if and only if $F(\boldsymbol{x}) > 0$ at optimality given the first-stage decisions $\boldsymbol{x}$. Feasibility check problem (26) also can be solved by dualizing its inner minimization problem and it can be transformed to the following formulation (27):

$$F(\boldsymbol{x}) = \max_{\boldsymbol{d},\boldsymbol{q},\boldsymbol{\beta}} (\boldsymbol{Ed} + \boldsymbol{Fq} + \boldsymbol{Gx} + \boldsymbol{g})^{\top}\boldsymbol{\beta} \tag{27a}$$

$$\text{s.t.}\ \boldsymbol{B}^{\top}\boldsymbol{\beta} \le \boldsymbol{0} \tag{27b}$$

$$\boldsymbol{\beta} \le \boldsymbol{1} \tag{27c}$$

$$\boldsymbol{\beta} \ge \boldsymbol{0}, \boldsymbol{d}\in\boldsymbol{D}, \boldsymbol{q}\in\boldsymbol{Q} \tag{27d}$$

where $\boldsymbol{\beta}$ denotes the dual variables corresponding to constraints (26b). After introducing the subproblem (25) and feasibility check problem (27), we present the detailed solution procedure for the proposed two-stage robust model.

**Solution procedure based on the C&CG method**

1. Set lower bound $LB = -\infty$, upper bound $UB = +\infty$, counter $k = 0$, and set $\boldsymbol{S} = \emptyset$.
2. Solve the following master problem (28):

$$\min_{\boldsymbol{x},\theta} \quad \theta \tag{28a}$$

$$\text{s.t.} \quad \boldsymbol{Ax} \le \boldsymbol{b} \tag{28b}$$

$$\theta \ge \boldsymbol{c}_{\boldsymbol{y}}^{\top}\boldsymbol{y}^{l} - \boldsymbol{\lambda}_{l}^{*\top}\boldsymbol{x}, \forall l \in \boldsymbol{S} \tag{28c}$$

$$\boldsymbol{By}^{l} \ge \boldsymbol{Ed}_{l}^{*} + \boldsymbol{Fq}_{l}^{*} + \boldsymbol{Gx} + \boldsymbol{g}, \forall l \le k \tag{28d}$$

$$\boldsymbol{x} \ge \boldsymbol{0}, \boldsymbol{y}^{l} \ge \boldsymbol{0}, \forall l \le k. \tag{28e}$$

   Derive the optimal solution $(\boldsymbol{x}_{k+1}^{*}, \theta_{k+1}^{*}, \boldsymbol{y}^{1*}, \cdots, \boldsymbol{y}^{k*})$ and update $LB = \theta_{k+1}^{*}$.
3. Solve the feasibility check problem (27) with $\boldsymbol{x}_{k+1}^{*}$. If $F(\boldsymbol{x}_{k+1}^{*}) > 0$, create variables $\boldsymbol{y}^{k+1}$ and add the following constraints:

$$\boldsymbol{By}^{k+1} \ge \boldsymbol{Ed}_{k+1}^{*} + \boldsymbol{Fq}_{k+1}^{*} + \boldsymbol{Gx} + \boldsymbol{g} \tag{29}$$

   to the master problem (28). $\boldsymbol{d}_{k+1}^{*}$ and $\boldsymbol{q}_{k+1}^{*}$ are the optimal outcomes of the uncertain parameters solving $F(\boldsymbol{x}_{k+1}^{*})$. Update $k = k + 1$ and go to Step 2. Otherwise, go to Step 4.
4. Solve the subproblem (25) with $\boldsymbol{x}_{k+1}^{*}$ and update $UB = \min\{UB, R(\boldsymbol{x}_{k+1}^{*})\}$. If $|UB - LB|/LB < \epsilon$, return $\boldsymbol{x}_{k+1}^{*}$ as the optimal solution and terminate. Otherwise, create variables $\boldsymbol{y}^{k+1}$ and add the following constraints:

$$\theta \ge \boldsymbol{c}_{\boldsymbol{y}}^{\top}\boldsymbol{y}^{k+1} - \boldsymbol{\lambda}_{k+1}^{*\top}\boldsymbol{x} \tag{30a}$$

$$\boldsymbol{By}^{k+1} \ge \boldsymbol{Ed}_{k+1}^{*} + \boldsymbol{Fq}_{k+1}^{*} + \boldsymbol{Gx} + \boldsymbol{g} \tag{30b}$$

   to the master problem (28) where $\boldsymbol{d}_{k+1}^{*}$, $\boldsymbol{\lambda}_{k+1}^{*}$, and $\boldsymbol{q}_{k+1}^{*}$ are the optimal outcomes solving $R(\boldsymbol{x}_{k+1}^{*})$. Update $k = k+1, \boldsymbol{S} = \boldsymbol{S}\cup\{k+1\}$ and go to Step 2.

In each iteration of the above solution procedure, the master problem (28) is initially solved to generate the first-stage decisions $\boldsymbol{x}^{*}$. A lower bound $LB$ is also calculated based on the optimal objective value of the master problem. Next, the feasibility check problem (27) is solved.

If $F(\boldsymbol{x}^*) > 0$, constraints (29) which serve as feasibility cuts are added to the master problem. If $F(\boldsymbol{x}^*) = 0$, the subproblem (25) is solved. Constraints (30a)-(30b) which serve as optimality cuts are added to the master problem. An upper bound $UB$ is also calculated based on the optimal value of function $R(\boldsymbol{x}^*)$. The solution procedure stops when $|UB - LB|/LB$ is less than a predefined tolerance $\epsilon$.

## 4. CASE STUDY

The results of a case study are presented in this section. In this study, we consider a WtE CHP plant that sells electricity in a day-ahead electricity market and exports heat to a district heating network. The WtE plant consists of an extraction unit and a back-pressure unit. The technical parameters related to the CHP units of the WtE plant are shown in Table 1. These parameters are mainly derived from the production units of the existing WtE CHP plants as reported in Force Technology (2018) and Energinet.dk (2012). The maximum and minimum allowable amount of waste $W_{max}$ and $W_{min}$ in the waste bunker are 8000 and 2000 tonnes, respectively. The initial amount of waste $w_0$ in the waste bunker is set to 3000 tonnes.

Table 1. Parameters for the CHP units

| Parameters | Unit | CHP units | |
|---|---|---|---|
| | | Extraction | Back-pressure |
| $H_i^{max}$ | MWh | 14 | 30 |
| $H_i^{min}$ | MWh | 0 | 4 |
| $P_i^{max}$ | MWh | 12 | 12 |
| $P_i^{min}$ | MWh | 4 | 1.6 |
| $R_i^{HD}$ | MWh/h | 6 | 3 |
| $R_i^{HU}$ | MWh/h | 6 | 3 |
| $R_i^{PD}$ | MWh/h | 6 | 4 |
| $R_i^{PU}$ | MWh/h | 6 | 4 |
| $M_i^{max}$ | tonne | 12 | 18 |
| $M_i^{min}$ | tonne | 4 | 2.4 |
| $c_i$ | €/tonne | 53 | 50 |
| $r_i$ | - | 0.65 | 0.40 |
| $f_i^p$ | tonne/MWh | 1 | 1 |
| $f_i^h$ | tonne/MWh | 0.19 | 0.20 |
| $p_{i0}$ | MWh | 6 | 10.8 |
| $h_{i0}$ | MWh | 3 | 27 |

We consider a 24-hour operating time horizon and assume each time period to be one hour. Since uncertain parameters $\lambda_t$, $d_t$, and $q_t$ take values from the predefined uncertainty sets, the data used to construct the uncertainty sets is described as follows. The day-ahead electricity price uncertainty set $\boldsymbol{\Lambda}$ is built with data from the El-spot electricity market (Energinet.dk, 2018). The district heating demand uncertainty set $\boldsymbol{D}$ is built with the heat consumption data in the west Copenhagen area, Denmark (Madsen, 2018). However, the data is modified to fit the case study. The uncertainty set $\boldsymbol{Q}$ which models the amount of waste delivered to the plant is built with the simulated data based on the capacity of the WtE plant since no real-world data is available for this parameter. The nominal values of the electricity price $\overline{\lambda}_t$ and the heat demand $\overline{d}_t$ in each time period are shown in Fig. 2. The nominal amount of waste delivered to the plant $\overline{q}_t$ in each time period is shown in Fig. 3. We assume that each uncertain parameter can deviate by up to 10% of its nominal value. We also assume that uncertainty budgets $\Gamma_d = \Gamma_\lambda = \Gamma_q = 18$ and set the tolerance of the optimality gap to 0.01%. The robust model is solved by CPLEX 12.3 on a laptop with an Intel Core i7 2.9GHz CPU and 16GB memory.

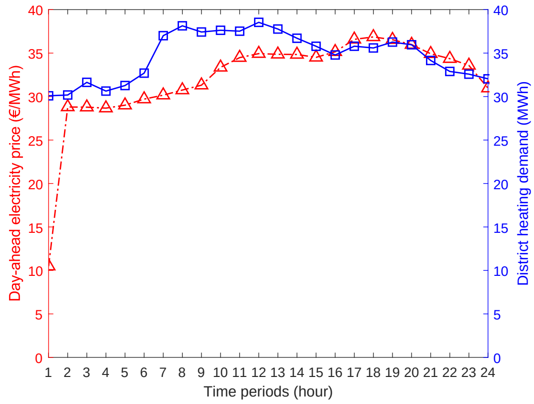


Fig. 2. Nominal day-ahead electricity price and district heating demand

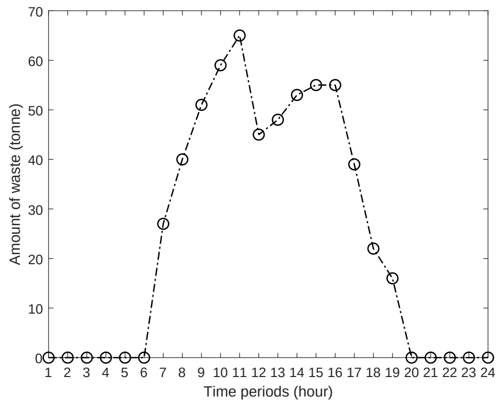


Fig. 3. Nominal amount of waste delivered to the plant

The robust electricity trading strategy for the WtE CHP plant in the case study is shown in Fig. 4. Note that we also generate the deterministic trading strategy by running the proposed model without considering uncertainty. The deterministic strategy is depicted in Fig. 4 as well. Let us first focus on the robust trading strategy. It is clear that a relatively large amount of electricity is sold between periods 12 and 21 where the day-ahead electricity prices are relatively high. Next, we compare the robust trading strategy with the deterministic strategy. By comparing these two strategies, we observe that the robust strategy sells more electricity in most of the time periods. This behavior is mainly caused by the uncertainty of the amount of waste delivered to the plant and the district heating demand. In the worst-case scenario, more waste is delivered to the plant and needs to be burned because the primary goal of the WtE plant is to treat waste. Moreover, higher heat demand needs to be satisfied by burning extra waste. Thus, the robust trading strategy determines to sell more electricity to reduce the plant's expense in the worst-case scenario.

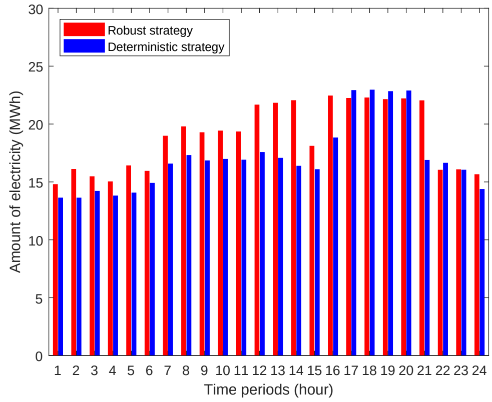


Fig. 4. Electricity trading strategies in the case study

Next, we solve the two-stage robust model with different values of the uncertainty budgets $\Gamma_d$, $\Gamma_\lambda$, and $\Gamma_q$, and report the worst-case expense of the WtE CHP plant and the corresponding CPU time in Table 2. Note that we still assume $\Gamma_d = \Gamma_\lambda = \Gamma_q$ and each uncertain parameter can deviate by up to 10% of its nominal value.

Table 2. Worst-case expense with different values of the uncertainty budgets

| Uncertainty Budgets | 6 | 12 | 18 | 24 |
|---|---|---|---|---|
| Expense (€) | 17167.7 | 18163.2 | 18674.9 | 19012.6 |
| CPU Time (s) | 710.5 | 3514.1 | 28.5 | 3.0 |

From Table 2, we find that the worst-case expense of the WtE plant increases as the uncertainty budgets increase. This is because more scenarios can be considered in the uncertainty sets when the uncertainty budgets become larger. Thus, the worst-case expense increases as the solution becomes more robust and conservative. In addition, the robust optimal solutions for different values of the uncertainty budgets can be obtained within one hour. This shows the effectiveness of the solution procedure.

## 5. CONCLUSION

In this paper, a two-stage robust optimization model is developed to help a WtE CHP plant derive robust electricity trading strategies in a day-ahead electricity market. The proposed model incorporates three types of uncertainty including day-ahead electricity price, heat demand, and the amount of waste delivered to the plant. The uncertain parameters are described by polyhedral uncertainty sets. To solve the model, a solution procedure based on the C&CG method is designed. Results from a case study show that the proposed two-stage model and solution procedure are effective to generate robust electricity trading strategies for the WtE CHP plant under three types of uncertainty. Moreover, trading strategies with different levels of robustness can be generated by adjusting the values of the uncertainty budgets in the uncertainty sets.

## ACKNOWLEDGEMENTS

This research is funded by the National Research Foundation (NRF), Prime Minister's Office, Singapore under its Campus for Research Excellence and Technological Enterprise (CREATE) program.

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