

Elsevier required licence: © <2020>. This manuscript version is made available under the CC-BY-NC-ND 4.0 license <http://creativecommons.org/licenses/by-nc-nd/4.0/>
The definitive publisher version is available online at
[<https://www.sciencedirect.com/science/article/pii/S0959652619348231?via%3Dihub>]

Operations scheduling of waste-to-energy plants under uncertainty

Chenlian Hu^{a,b}, Xiao Liu^{a,*}, Jie Lu^b, Chi-Hwa Wang^c

^a Department of Industrial Engineering & Management, Shanghai Jiao Tong University, P.R. China.

^b Centre for Artificial Intelligence, Faculty of Engineering and Information Technology, University of Technology Sydney, Australia.

^c Department of Chemical and Biomolecular Engineering, National University of Singapore, Singapore.

* Corresponding author: Prof. Xiao Liu.

Department of Industrial Engineering & Management, Shanghai Jiao Tong University,
800 Dongchuan Road, Min-Hang District, Shanghai 200240, P.R. China.

E-mail address: x_liu@sjtu.edu.cn

Phone: +65-66016129

Operations scheduling of waste-to-energy plants under uncertainty

Abstract: Waste-to-energy (WTE) technologies provide effective solutions to the compelling challenges of waste management and the energy crisis globally. Many WTE plants utilize the combined heat and power (CHP) operation mode where both electricity and heat can be generated simultaneously. Thus, these WTE CHP plants can supply heat to the local district heating systems and trade power in the electricity markets. As such plants have the responsibilities of treating waste and of fulfilling the allocated district heating demand, necessary operational tasks such as preventive maintenance actions for the production units should be scheduled and performed periodically to ensure their continuous and reliable operations. This paper studies the scheduling of operational tasks in WTE CHP plants that participate in electricity markets and are connected to district heating networks. Firstly, we formulate a two-stage robust optimization model considering the uncertainty of electricity market prices, heat demand, and waste supply. The objective is to derive the robust optimal schedule that maximizes the worst-case operating profit of a WTE CHP plant under uncertainty. Subsequently, we design a constraint generation algorithm for the two-stage robust optimization model. Finally, a case study of scheduling preventive maintenance tasks is conducted for the production units of a WTE CHP plant over a 30-day horizon. The robust schedule thus derived is evaluated by Monte Carlo simulation tests and further compared to the deterministic schedule generated without the consideration of uncertainty. The simulation results show that the robust schedule enables an average profit of 877021.21€ to be attained for the plant over the scheduling horizon. Moreover, it improves the robustness of its deterministic counterpart from 68.4% to 98.8% with an increase of only 0.3% of the operating profit of the plant. In addition, a comprehensive sensitivity analysis is performed to investigate the impacts of different types of uncertainty on the robust schedule for the WTE CHP plant.

Keywords: waste-to-energy, operations scheduling, robust optimization, uncertainty

1 Introduction

Municipal solid waste (MSW) management and the energy crisis have increasingly become two compelling challenges worldwide due to rapid urbanization, growing population, and economic development. Waste-to-energy (WTE) technologies, which can turn waste into various forms of energy, provide elegant and effective solutions to these challenges (Pan et al., 2015; Yi et al., 2018). A variety of WTE options are available, which mainly include thermo-chemical technologies (Shi et al., 2016; Lombardi et al., 2015) and bio-chemical technologies (Pant et al., 2010; Sepehri and Sarrafzadeh, 2018; Sepehri et al., 2019). Incineration with energy recovery is one of the most widely used technologies for MSW treatment, especially in densely populated countries (Kumar and Samadder, 2017). To deal with substantial amounts of MSW, large-scale WTE incineration plants have been established in many countries, such as China, Denmark, the USA, and Japan (Cucchiella et al., 2014).

WTE plants are usually located in proximity to residential or industrial areas for ease of access to MSW. As a result, WTE plants are able to adopt the combined heat and power (CHP) mode to generate both electricity and heat. The electricity generated can be sold on power markets whereas the heat produced can be supplied to local district heating systems or adjacent heat-consuming industrial plants (Ryu and Shin, 2012). In Denmark, a large portion of MSW is treated in WTE incineration plants, of which the majority are CHP producers (Fruergaard et al., 2010). In Sweden, over two million tonnes of MSW are incinerated in WTE CHP plants annually, which provide heat and electricity corresponding respectively to the needs of 810000 and 250000 homes (Avfall Svergie, 2007).

As MSW is an obnoxious social product generated on a daily basis, the continuous and reliable operation of WTE CHP plants is critical. Hence, necessary operational tasks such as preventive maintenance for their production units should be performed periodically. This paper studies an operational task scheduling problem of WTE CHP plants that participate in liberalized power markets and are connected to district heating networks. The scheduling consists of determining the best timing to perform operational tasks (preventive maintenance actions) for the production units of a WTE CHP plant over a specific horizon. Since the revenue of a WTE CHP plant originates mainly from the sale of energy and from gate fees, plant operators should judiciously schedule the tasks to maximize the operating profit of the plant. Such planning, however, is challenging for two reasons. Firstly, the concomitant responsibilities of treating the delivered MSW and of fulfilling the allocated district heating demand complicate the scheduling. Secondly, as a schedule typically should be determined months in advance of these operational tasks, much information is unknown or only partly predictable during this upstream scheduling, e.g. electricity prices, district heating demand, and MSW supply over the scheduling

horizon. The offshoots of such uncertainty are twofold: if the tasks are scheduled on days with high average electricity prices, the plant may make less profit; and, if they are scheduled on days with high MSW supply or heat demand, the plant may fail to treat all the delivered MSW or to fulfill the allocated heat demand. Thus, the robustness of a task schedule is crucial against the various types of uncertainty. Given the importance of a WTE CHP plant in treating the MSW or satisfying the heat demand, the undesirably low robustness of a task schedule may lead to its occasional failure in those two roles during the scheduling horizon under many possible realizations of the uncertainty. Thus, deriving robust and economical task schedules for the production units of a WTE CHP plant is a complex optimization problem subject to different types of uncertainty. Another key characteristic of the problem is the presence of dynamics. The operation of the production units over the scheduling horizon is linked to the capacity of MSW storage in the waste bunker. This warrants the appropriate modeling of such dynamics to generate effective schedules.

In view of the dynamic feature and the uncertainty in the problem, the framework of two-stage robust optimization (Ben-Tal et al., 2004) is utilized. This framework is a useful mathematical programming method that can effectively model the decision-making process of the problem and appropriately represent the uncertainty. Specifically, this paper builds a two-stage robust optimization model for scheduling operational tasks in WTE CHP plants. Three types of uncertainty are incorporated in the model and described by convex polyhedral sets. The model aims to ascertain the optimal schedule for a WTE CHP plant, which can maximize its worst-case operating profit over a specific scheduling horizon. To solve this model, a constraint generation algorithm based on the framework of Benders' decomposition (Geoffrion, 1972) and the column-and-constraint generation (C&CG) method (Zeng and Zhao, 2013) is designed. The main contributions of this work are summarized as follows:

- 1) A two-stage robust optimization model is formulated for scheduling operational tasks in WTE CHP plants under different types of uncertainty.
- 2) A constraint generation algorithm is designed to solve the two-stage robust optimization model.
- 3) A real-world case study is tested to show the effectiveness of the two-stage robust optimization model and the efficiency of the constraint generation algorithm.

The rest of this work is organized as follows. In Section 2, the research related to operations scheduling of WTE plants is reviewed. Section 3 introduces the two-stage robust optimization model and defines the uncertainty sets. The details of the designed algorithm for the two-stage robust model are shown in Section 4. Section 5 reports the

computational results obtained from a case study with real-world data. A comprehensive sensitivity analysis of the key parameters of the uncertainty sets is also performed. Finally, Section 6 summarizes the paper and provides several future research directions.

2 Related literature

The literature review concerns three aspects: operations scheduling of WTE plants; power system scheduling; and two-stage robust optimization.

2.1 Operations scheduling of WTE plants

Effective operations scheduling in WTE plants is known to improve energy efficiency, ensure continuous operation, and augment economic benefits. However, only a little attention has been paid to such operations scheduling in WTE plants. [Touš et al. \(2015\)](#) studied a short-term operations scheduling problem of a WTE CHP plant in the Czech Republic. They developed a stochastic mathematical model and adopted stochastic simulation to derive effective daily production plans for power and heat under uncertainty. [Liu et al. \(2017\)](#) formulated a mixed-integer linear programming (MILP) model to tackle the mid-term scheduling of preventive maintenance for the production units of a WTE incineration plant and the connected power system devices in the city of Shanghai. [Abaecherli et al. \(2017b\)](#) developed a novel MILP model to optimize short-term schedules for industrial waste incineration to improve both economic and environmental benefits of WTE plants. [Abaecherli et al. \(2017a\)](#) further investigated the integration of planning and scheduling in industrial waste incineration plants. Two MILP models were developed for planning and scheduling of waste incineration. The literature review suggests that, notwithstanding the incipient interest in solving operations scheduling problems, complex operational schemes in WTE plants with the CHP production mode in deregulated power markets and district heating networks are disregarded. In this regard, the uncertainty in electricity prices and in district heating demand represents challenges in operations scheduling of WTE CHP plants.

2.2 Power system scheduling

Generally, WTE plants are power producers since most of them are MSW incinerators with power recovery. Although the literature on operations scheduling of WTE plants is limited, researchers have addressed various scheduling problems in power systems. [Yamin \(2004\)](#) presented a comprehensive review of methods for power generation scheduling in centralized and decentralized power systems. Different optimization methods including deterministic, heuristic, and hybrid approaches have been proposed to solve unit

commitment problems (Bhardwaj et al., 2012) and economic dispatch problems (Mahor et al., 2009) in generation scheduling of power systems. In addition to power generation scheduling, researchers have studied maintenance scheduling in power systems over the last several decades (Kralj and Petrović, 1988; Khalid and Ioannis, 2012). Froger et al. (2016) reviewed various maintenance scheduling problems including generator maintenance scheduling and transmission maintenance scheduling in regulated and deregulated power markets, alongside key features such as network structures, fuel constraints, and uncertainty management.

Despite the elucidation of scheduling problems in power systems, most research has focused on traditional thermal power plants. Unlike these thermal counterparts, WTE plants base their energy production on waste, which generally cannot be purchased externally and whose sourcing could be uncertain. Moreover, WTE plants house waste bunkers with fixed storage capacities for MSW for future energy production. A parallel may thus be drawn between operations scheduling in WTE plants and that in hydropower systems, since hydropower plants likewise entails a resource (water) in their energy production and have storage facilities (reservoirs). Researchers have studied scheduling problems in hydropower systems over the last decade. Hongling et al. (2008) provided a comprehensive survey on hydropower scheduling in deregulated electricity markets with the consideration of the uncertainty in electricity prices and water inflow. Nazari-Heris et al. (2017) reviewed solution methods for short-term scheduling of hydro-based power systems. Only a few papers have tackled maintenance scheduling problems in hydropower systems. Guedes et al. (2015) proposed a differential evolution algorithm to solve a combined problem for scheduling power generation and preventive maintenance for a cascaded hydropower system. Helseth et al. (2018) adopted the stochastic dynamic programming approach to address a hydropower maintenance scheduling problem in a deregulated market context considering the uncertainty in water inflow and energy prices. Of note, however, operations scheduling in WTE CHP plants is distinctive from hydropower systems in several manners. Firstly, the main goal of WTE CHP plants is the treatment of waste. Secondly, the plants are usually connected to district heating networks; the responsibility of fulfilling the heat demand critically determines ideal schedules. Finally, different types of uncertainty including MSW supply, district heating demand, and electricity prices warrant consideration in the operations scheduling of WTE CHP plants. To the best of our knowledge, our work represents the pioneering research into the elucidation of the scheduling of operational tasks in WTE CHP plants that participate in deregulated power markets under uncertainty.

2.3 Two-stage robust optimization

Two-stage robust optimization is an optimization framework for two-stage or multi-stage decision-making problems with uncertain data (Ben-Tal et al., 2004). It is also called adaptable or adjustable robust optimization (Zeng and Zhao, 2013), which is an important extension of the classic robust optimization approach (Ben-Tal and Nemirovski, 1998, 2000). In two-stage robust optimization, the uncertain data is assumed to take values from a predefined uncertainty set. Herein, decisions are separated into two stages. The first-stage (*here-and-now*) decisions need to be determined before the uncertain data is observed. Given the first-stage decisions, the second-stage (*wait-and-see*) decisions can be adjusted and determined after the uncertain data is observed. Due to its modeling capacity for various classes of optimization problems and computational tractability for different types of uncertainty sets, two-stage robust optimization finds utility in many real-life applications (Yanikoğlu et al., 2019).

However, it is very challenging to solve two-stage robust optimization models. As discussed in Ben-Tal et al. (2004), even trivial two-stage robust optimization models can be computationally intractable. Several solution strategies have been proposed to address the computational tractability issue. The most popular one is to use affine decision rules, which assume second-stage decision variables to be affinely dependent on the uncertain data (Ben-Tal et al., 2004). Using this strategy, two-stage robust models are able to be transformed to linear programming models. However, only sub-optimal solutions can be obtained. Later, Thiele et al. (2009) developed a cutting-plane method to generate the optimal solutions for two-stage robust optimization problems based on Kelly’s algorithm (Kelley, 1960). Zeng and Zhao (2013) designed a C&CG procedure which aims to obtain the optimal solutions of two-stage robust optimization problems with faster speed. However, these two algorithms may not be able to address large-size problems. Recently, Zhen et al. (2018) proposed a Fourier-Motzkin elimination procedure to tackle two-stage robust optimization problems with fixed recourse. This procedure can obtain the optimal solutions for small-size problems and generate good-quality feasible solutions for large-size instances.

3 Model

3.1 Problem description

Consider a WTE plant that is owned and operated by the private sector in a specific region. The WTE plant adopts the CHP mode to generate both power and heat. As back-pressure and extraction units are two typical types of production units in CHP systems, the WTE CHP plant is assumed to house both types of CHP units. The operational

scheme of such a plant is shown in Fig. 1. The MSW generated by the local households is transported daily by trucks to the plant, where it is then deposited in the waste bunker for energy production. Thereafter, the MSW is first incinerated to liberate its chemical energy, which converts water into high-pressure steam through heating in boilers. The steam is then conveyed to the CHP units to produce electricity and heat which are subsequently used by the local households. In a back-pressure unit, the steam turbine utilizes only part of the high-pressure steam to generate power and leaves an output of high-temperature steam for district heating. Thus, a fixed relationship is present between the generation of power and that of heat in back-pressure units. Conversely, in an extraction unit, a flexible amount of steam can be extracted before traversing the steam turbine. The extracted steam can be used by the local district heating system and the rest is used for power generation. Thus, the power and heat generation in extraction units is more flexible. The power generated by the WTE CHP plant is considered to be sold in the regional electricity market. The heat produced is assumed to be exported and sold to the local district heating system with a fixed price set by the local regulator.

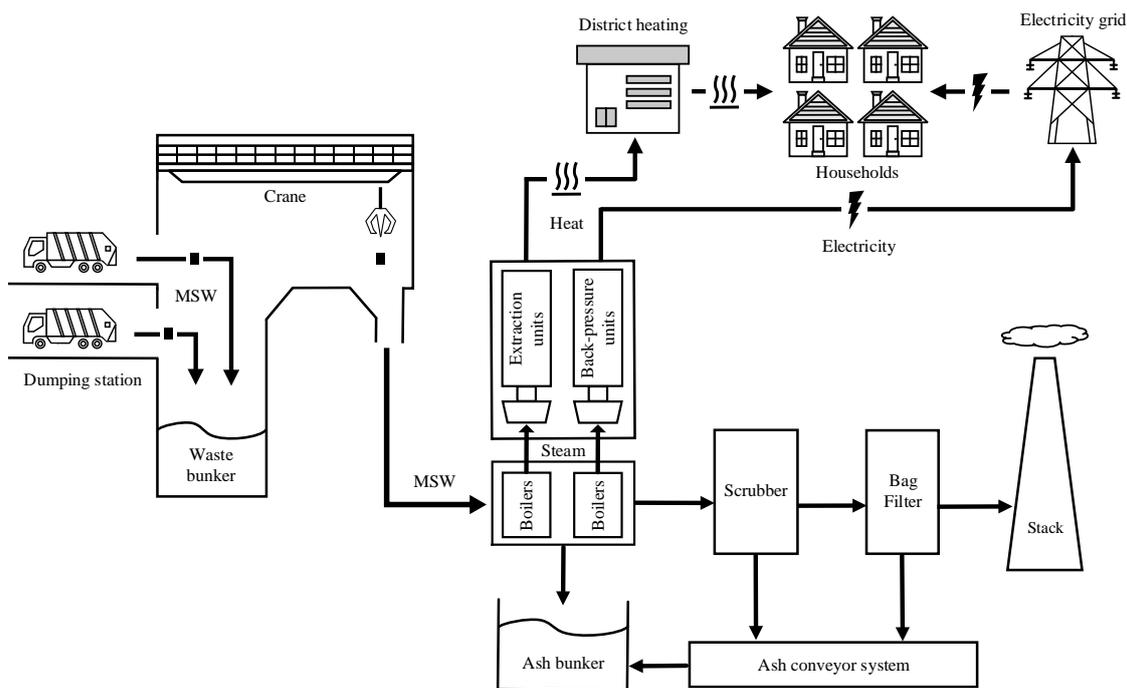


Fig. 1. Operational scheme of the WTE CHP plant.

As the WTE CHP plant has the twofold responsibilities of treating MSW and of fulfilling part of the district heating demand, its continuous and reliable operation is critical. To prevent unexpected unit failures and ensure a continuous high-efficiency operation, necessary operational tasks such as preventive maintenance for the production units of the plant should be scheduled and performed periodically. It is noteworthy that

WTE plants typically operate in a round-the-clock manner (24 hours a day, seven days a week), except for scheduled downtime for maintenance of major system components such as incineration and production units (Rogoff and Screve, 2019). As the revenue of the WTE CHP plant originates mainly from selling energy and receiving gate fees, plant operators need to schedule the best timing to perform these necessary operational tasks for the CHP units over a specific horizon. The scheduling horizon is typically considered to be one or several months subdivided into daily intervals. However, many types of key information are uncertain when the operators plan effective schedules. For example, the daily average electricity prices and MSW supply over the scheduling horizon are difficult to know and hard to predict for the operators. Thus, we formulate a two-stage robust optimization model that aims to derive the optimal schedule for the operational tasks for the CHP production units under uncertainty. The operational tasks considered in the problem concern the shutting down of each CHP unit for fixed time periods to conduct preventive maintenance actions such as inspection and lubrication of steam turbines. Three types of uncertainty are considered in the model: daily average electricity prices, daily total heat demand, and daily total MSW supply over the scheduling horizon. The decision-making process of the developed model is shown in Fig. 2. It is clear that the decision-making process is divided into two stages. In the first stage (Day 0), operators of the plant determine the optimal time periods among the scheduling horizon (Day 1-Day T) to perform the operational task for each CHP unit (task schedule). It should be noted that the schedule is determined without knowing the exact values of the daily average electricity prices, daily total MSW supply, and heat demand over the scheduling horizon. In the second stage (Day 1-Day T), the uncertain daily average electricity prices, daily total MSW supply, and heat demand become known. The operators determine the optimal heat and power generation and the MSW storage level in each time period of the scheduling horizon to maximize the total operating profit of the plant based on the task schedule established in the first stage. Herein, the stages correspond to steps in the decision-making process and do not necessarily refer to the time periods. The two-stage model aims to generate the optimal robust schedule for the operational tasks for the production units determined in the first stage, which can then maximize the worst-case profit of the plant over the scheduling horizon in the second stage.

The assumptions underlying the two-stage robust optimization model are as follows:

- The WTE CHP plant needs to fulfill part of the district heating demand in all time periods of the scheduling horizon.
- The power generated by the plant is sold in the regional electricity market and the heat produced is sold to the local district heating system with a fixed price.
- Each type of uncertainty is captured by a convex polyhedral set.

- The first-stage decisions are made before the scheduling horizon begins and the uncertain parameters are not observed.
- The second-stage decisions are made after the uncertain parameters are observed in all time periods of the scheduling horizon.

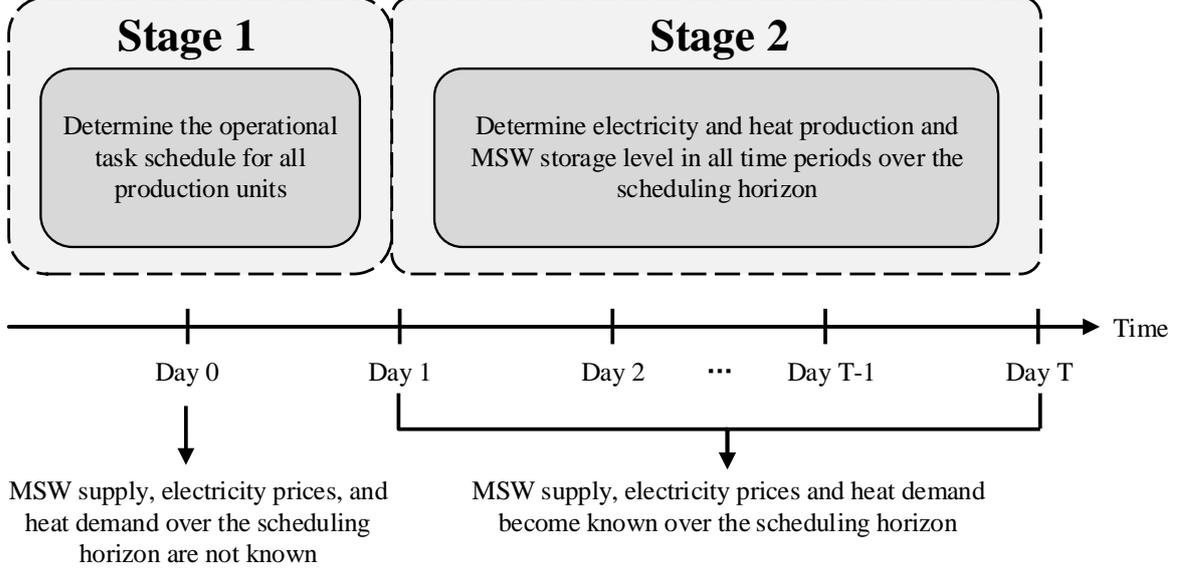


Fig. 2. Overview of the two-stage decision-making process.

3.2 Two-stage robust optimization model

Sets:

\mathcal{T}	set of scheduling time periods (days), $\mathcal{T} = \{1, \dots, \mathcal{T} \}$
\mathcal{I}	set of all CHP units, $\mathcal{I} = \{1, \dots, \mathcal{I} \}$
\mathcal{I}_{bp}	set of back-pressure units, $\mathcal{I}_{bp} \subseteq \mathcal{I}$
\mathcal{I}_{ex}	set of extraction units, $\mathcal{I}_{ex} \subseteq \mathcal{I}$

Parameters:

HD_i^{max}	daily maximum heat production capacity of unit i
HD_i^{min}	daily minimum heat production capacity of unit i
PD_i^{max}	daily maximum power production capacity of unit i
PD_i^{min}	daily minimum power production capacity of unit i
MD_i^{max}	daily maximum MSW consumption of unit i
MD_i^{min}	daily minimum MSW consumption of unit i
ET_i	earliest start time of the operational task for unit i
LT_i	latest start time of the operational task for unit i
DT_i	duration of the operational task for unit i

τ_i	heat-to-power ratio of unit i
CV_i	variable operating cost of unit i
CM_i	daily cost to perform the operational task for unit i
L^{max}	maximum allowable amount of MSW in the waste bunker
L^{min}	minimum required amount of MSW in the waste bunker
l_0	amount of MSW in the waste bunker at the beginning of the scheduling horizon
l_{end}	minimum amount of MSW in the waste bunker at the end of the scheduling horizon
π_i^p	MSW consumption per unit of power production for unit i
π_i^h	MSW consumption per unit of heat production for unit i
GF	marginal gate fee for MSW treatment
N	number of units can be shut down simultaneously
ξ_t	average electricity price in time period t
d_t	total heat demand in time period t
w_t	total amount of MSW supplied to the WTE plant in time period t

Decision variables:

x_{it}	binary variable, "1" if unit i is operating in time period t ; "0" otherwise
z_{it}	binary variable, "1" if the operational task is being performed for unit i in time period t ; "0" otherwise
h_{it}	heat production from unit i in time period t
p_{it}	power production from unit i in time period t
l_t	MSW storage level in the waste bunker in time period t

We formulate the two-stage robust optimization model in the following equations (1)-(20):

$$\max_{\mathbf{x}, \mathbf{z}} - \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} CM_i z_{it} + \min_{\xi \in \Xi, \mathbf{d} \in \mathcal{D}, \mathbf{w} \in \mathcal{W}} R(\mathbf{x}, \mathbf{z}, \xi, \mathbf{d}, \mathbf{w}) \quad (1)$$

$$\text{s.t. } x_{it} + z_{it} = 1, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \quad (2)$$

$$\sum_{t \in \mathcal{T}} z_{it} = DT_i, \quad \forall i \in \mathcal{I} \quad (3)$$

$$z_{it} - z_{i(t-1)} \leq z_{i(t+DT_i-1)}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \quad (4)$$

$$\sum_{t=1}^{ET_i-1} z_{it} = 0, \quad \forall i \in \mathcal{I} \quad (5)$$

$$\sum_{t=LT_i+DT_i}^{|\mathcal{T}|} z_{it} = 0, \quad \forall i \in \mathcal{I} \quad (6)$$

$$\sum_{i \in \mathcal{I}} z_{it} \leq N, \quad \forall t \in \mathcal{T} \quad (7)$$

$$x_{it}, z_{it} \in \{0, 1\}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \quad (8)$$

where \mathbf{x} and \mathbf{z} denote the first-stage decisions which respectively subsume x_{it} and z_{it} for

all $i \in \mathcal{I}$ and $t \in \mathcal{T}$. Vectors $\boldsymbol{\xi}, \mathbf{d}, \mathbf{w}$ respectively contain uncertain vectors $\xi_t, d_t,$ and w_t for all $t \in \mathcal{T}$. Note that uncertain parameters $\boldsymbol{\xi}, \mathbf{d},$ and \mathbf{w} respectively assume values in the uncertainty sets $\Xi, \mathcal{D},$ and \mathcal{W} which are defined in the subsection 3.3. In the first-stage problem (1)-(8), the objective function (1) maximizes the worst-case profit of the WTE CHP plant over the scheduling horizon: it equals the worst-case operating profit minus the total cost to perform all operational tasks. Constraints (2) ensure that each CHP unit cannot operate if the operational task is in the midst of execution. This coincides with the fact that most WTE plants operate in a round-the-clock manner except for scheduled downtime for maintenance of major system components. Constraints (3) guarantee that the operational task for each CHP unit should be performed for the required time periods. Constraints (4) ensure that, upon embarking, the operational task for each CHP unit has to be completed. Constraints (5) and (6) respectively determine the earliest and latest times to perform the task for each CHP unit. Constraints (7) limit the maximum number of CHP units that can be shut down synchronously in each time period. Constraints (8) ensure that all first-stage decision variables are binary. Function $R(\mathbf{x}, \mathbf{z}, \boldsymbol{\xi}, \mathbf{d}, \mathbf{w})$ in objective function (1) denotes the operating profit of the plant over the scheduling horizon, given the determined schedule (\mathbf{x}, \mathbf{z}) , electricity prices $\boldsymbol{\xi}$, heat demand \mathbf{d} , and MSW supply \mathbf{w} . $R(\mathbf{x}, \mathbf{z}, \boldsymbol{\xi}, \mathbf{d}, \mathbf{w})$ can be calculated by solving the following second-stage problem:

$$R(\mathbf{x}, \mathbf{z}, \boldsymbol{\xi}, \mathbf{d}, \mathbf{w}) = \max \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} p_{it} \xi_t + GF \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} (\pi_i^p p_{it} + \pi_i^h h_{it}) - \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} CV_i (\pi_i^p p_{it} + \pi_i^h h_{it}) \quad (9)$$

$$\text{s.t. } p_{it} = \tau_i h_{it}, \quad \forall i \in \mathcal{I}_{bp}, t \in \mathcal{T} \quad (10)$$

$$p_{it} \geq \tau_i h_{it}, \quad \forall i \in \mathcal{I}_{ex}, t \in \mathcal{T} \quad (11)$$

$$x_{it} HD_i^{min} \leq h_{it} \leq x_{it} HD_i^{max}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \quad (12)$$

$$\pi_i^p p_{it} + \pi_i^h h_{it} \geq (\pi_i^p + \pi_i^h / \tau_i) PD_i^{min} x_{it}, \quad \forall i \in \mathcal{I}_{ex}, t \in \mathcal{T} \quad (13)$$

$$\pi_i^p p_{it} + \pi_i^h h_{it} \leq \pi_i^p PD_i^{max} x_{it}, \quad \forall i \in \mathcal{I}_{ex}, t \in \mathcal{T} \quad (14)$$

$$x_{it} MD_i^{min} \leq \pi_i^p p_{it} + \pi_i^h h_{it} \leq x_{it} MD_i^{max}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \quad (15)$$

$$\sum_{i \in \mathcal{I}} h_{it} \geq d_t, \quad \forall t \in \mathcal{T} \quad (16)$$

$$l_t = l_{t-1} + w_t - \sum_{i \in \mathcal{I}} (\pi_i^p p_{it} + \pi_i^h h_{it}), \quad \forall t \in \mathcal{T} \quad (17)$$

$$L^{min} \leq l_t \leq L^{max}, \quad \forall t \in \mathcal{T} \quad (18)$$

$$l_{|\mathcal{T}|} \geq l_{end} \quad (19)$$

$$h_{it}, p_{it}, l_t \geq 0, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}. \quad (20)$$

In the second-stage problem (9)-(20), the objective function (9) maximizes the operating profit of the plant, given a determined schedule and the realized uncertain parameters. The operating profit equals the revenues from selling power and receiving gate fees minus the total operating cost over the planning horizon. Note that the revenue from supplying heat to the local district heating network is omitted in the objective function (9) since the heat price is assumed to be fixed and the revenue from supplying heat does not affect the optimal solution of the second-stage problem. Constraints (10) and (11) respectively show the relationship between power and heat production in the back-pressure units and in the extraction units. Constraints (12) ensure the heat production of each CHP unit is within its capacity range. Constraints (13) and (14) determine the feasible energy production zone of each extraction unit. Constraints (15) impose the maximum and minimum limits of MSW consumption of each CHP unit in each time period. Constraints (16) guarantee that the total heat generated by all CHP units can fulfill the allocated heat demand from the local district heating network in all time periods. Constraints (17) determine the MSW storage level in the bunker in each time period. Constraints (18) guarantee that the stored MSW stays within the lower and upper limits of the waste bunker in any time period. Constraint (19) guarantees that the amount of MSW should exceed a predefined threshold in the last time period of the horizon for future operations of the plant. Constraints (20) ensure that all second-stage decision variables are non-negative.

Next, a further discussion of the two-stage robust model (1)-(20) is provided. As shown in constraints (3), it is necessary for the operational task (preventive maintenance) to be performed for each CHP unit for fixed time periods. Thus, the heat and power production of the WTE CHP plant will be affected during the downtime for such maintenance of each CHP unit. Moreover, the profit of the plant over the scheduling horizon will decline compared to normal continuous operation. However, the main goal of the two-stage robust model (1)-(20) is to generate the optimal task schedule under the various types of uncertainty in the first stage. In the second stage, the WTE CHP plant can then optimize the energy production and MSW incineration to maximize its worst-case operating profit over the scheduling horizon through the optimal schedule thus determined. Generally, the optimal task schedule depends on many key parameters of the proposed two-stage model, especially the uncertain parameters including the daily average electricity prices, daily total MSW supply, and heat demand. For example, when the MSW supply and the MSW storage is low, it might be an opportunity to perform the maintenance tasks. However, if the heat demand is also high during these time periods, the scheduled downtime may need to be shifted to other feasible time periods due to the need to fulfill the allocated heat demand from the local district heating network. In addition, if the maintenance tasks are scheduled during time periods with low average electricity prices, the plant may make more profit because electricity can be generated

and sold on other time periods with high average prices. Conversely, the plant may make less profit if the tasks are scheduled during time periods with high electricity prices.

3.3 Uncertainty set definition

In the two-stage robust optimization model (1)-(20), uncertain electricity prices $\boldsymbol{\xi}$, heat demand \mathbf{d} , and MSW supply \mathbf{w} respectively assume values in the corresponding uncertainty sets Ξ , \mathcal{D} , and \mathcal{W} . Since certain types of uncertainty sets can make two-stage robust optimization models computationally attractive, an appropriate and practical definition of the uncertainty sets is critical. Following the concept of the budget-constrained uncertainty set proposed in [Bertsimas and Sim \(2004\)](#), the corresponding uncertainty sets Ξ , \mathcal{D} , and \mathcal{W} can be defined in the following equations (21a)-(21c).

$$\Xi = \left\{ \boldsymbol{\xi} : \xi_t = \bar{\xi}_t + \eta_t^\xi \hat{\xi}_t, |\eta_t^\xi| \leq 1, \sum_{t \in \mathcal{T}} |\eta_t^\xi| \leq \Gamma_\xi, t \in \mathcal{T} \right\} \quad (21a)$$

$$\mathcal{D} = \left\{ \mathbf{d} : d_t = \bar{d}_t + \eta_t^d \hat{d}_t, |\eta_t^d| \leq 1, \sum_{t \in \mathcal{T}} |\eta_t^d| \leq \Gamma_d, t \in \mathcal{T} \right\} \quad (21b)$$

$$\mathcal{W} = \left\{ \mathbf{w} : w_t = \bar{w}_t + \eta_t^w \hat{w}_t, |\eta_t^w| \leq 1, \sum_{t \in \mathcal{T}} |\eta_t^w| \leq \Gamma_w, t \in \mathcal{T} \right\} \quad (21c)$$

In the uncertainty set Ξ , the uncertain electricity price ξ_t in time period t is expressed as the sum of $\bar{\xi}_t$ and $\eta_t^\xi \hat{\xi}_t$. $\bar{\xi}_t$ is the nominal value of ξ_t . $\bar{\xi}_t$ can be obtained by any effective forecast technique. $\hat{\xi}_t$ represents the maximum deviation of ξ_t from $\bar{\xi}_t$. $\hat{\xi}_t$ can be inferred from the historical data or set based on the decision-makers' experience. η_t^ξ is an auxiliary variable which belongs to the interval $[-1, 1]$ for all $t \in \mathcal{T}$. Moreover, the sum of the absolute value of all η_t^ξ is bounded by the uncertainty budget Γ_ξ . Generally, Γ_ξ controls the level of uncertainty in set Ξ . If $\Gamma_\xi = 0$, $\xi_t = \bar{\xi}_t$ for all $t \in \mathcal{T}$. No uncertainty is considered in set Ξ . If $\Gamma_\xi = |\mathcal{T}|$, ξ_t can take any value in the interval $[\bar{\xi}_t - \hat{\xi}_t, \bar{\xi}_t + \hat{\xi}_t]$ for all $t \in \mathcal{T}$ and set Ξ has the highest level of uncertainty. The parameters in the uncertainty set \mathcal{D} for heat demand and the uncertainty set \mathcal{W} for MSW supply are similar to those discussed in set Ξ .

4 Solution algorithm

As discussed in [Ben-Tal et al. \(2004\)](#), a classic two-stage robust optimization model can be hard to tackle due to the multi-level optimization structure. In this study, we design a constraint generation algorithm for the developed robust model (1)-(20) with the help of the C&CG method ([Zeng and Zhao, 2013](#)) and the Benders' decomposition

framework (Geoffrion, 1972). For the purpose of introducing the algorithm with clarity and simplicity, we first rewrite the developed model (1)-(20) in the following generic matrix formulation (22)-(23):

$$\max_{\mathbf{x}, \mathbf{z}} \quad -\mathbf{c}_z^\top \mathbf{z} + \min_{\xi \in \Xi, \mathbf{d} \in \mathcal{D}, \mathbf{w} \in \mathcal{W}} R(\mathbf{x}, \mathbf{z}, \xi, \mathbf{d}, \mathbf{w}) \quad (22a)$$

$$\text{s.t.} \quad \mathbf{A}_x \mathbf{x} + \mathbf{A}_z \mathbf{z} \leq \mathbf{b} \quad (22b)$$

$$\mathbf{x}, \mathbf{z} \in \{0, 1\}^{|\mathcal{I}| \times |\mathcal{T}|} \quad (22c)$$

where $R(\mathbf{x}, \mathbf{z}, \xi, \mathbf{d}, \mathbf{w})$ is expressed as

$$R(\mathbf{x}, \mathbf{z}, \xi, \mathbf{d}, \mathbf{w}) = \max_{\mathbf{h}, \mathbf{p}, \mathbf{l}} \xi^\top \mathbf{p} + \mathbf{c}_p^\top \mathbf{p} + \mathbf{c}_h^\top \mathbf{h} \quad (23a)$$

$$\text{s.t.} \quad \mathbf{E}_p \mathbf{p} + \mathbf{E}_h \mathbf{h} + \mathbf{E}_l \mathbf{l} \geq \mathbf{D}_x \mathbf{x} + \mathbf{D}_z \mathbf{z} + \mathbf{q} \quad (23b)$$

$$\mathbf{G}_h \mathbf{h} \geq \mathbf{d} \quad (23c)$$

$$\mathbf{F}_l \mathbf{l} + \mathbf{F}_p \mathbf{p} + \mathbf{F}_h \mathbf{h} = \mathbf{w} \quad (23d)$$

$$\mathbf{h}, \mathbf{p}, \mathbf{l} \geq \mathbf{0}. \quad (23e)$$

The generic matrix formulation (22) corresponds to the first-stage problem (1)-(8). \mathbf{x} and \mathbf{z} represent the first-stage decisions. Objective function (22a) corresponds to the first-stage objective function (1), where vector \mathbf{c}_z represents the corresponding coefficients of variable \mathbf{z} . Constraints (22b) contain first-stage constraints (2)-(7), where matrices \mathbf{A}_x and \mathbf{A}_z respectively denote the coefficients of variables \mathbf{x} and \mathbf{z} and vector \mathbf{b} contains the given constants. The generic matrix formulation (23) corresponds to the second-stage problem (9)-(20). Vectors $\mathbf{h}, \mathbf{p}, \mathbf{l}$ respectively subsume second-stage decisions h_{it}, p_{it} , and l_t for all $i \in \mathcal{I}$ and $t \in \mathcal{T}$. Objective function (23a) corresponds to the second-stage objective function (9), where \mathbf{c}_p and \mathbf{c}_h respectively denote the coefficients of variables \mathbf{p} and \mathbf{h} . Constraints (23b) contain constraints (10)-(15) and (18)-(19). Constraints (23c) and (23d) correspond to constraints (16) and (17), respectively. In constraints (23b), matrices $\mathbf{E}_p, \mathbf{E}_h, \mathbf{E}_l, \mathbf{D}_z$, and \mathbf{D}_x represent the coefficients of variables $\mathbf{p}, \mathbf{h}, \mathbf{l}, \mathbf{z}$, and \mathbf{x} , respectively. Vector \mathbf{q} contains the constants. Matrix \mathbf{G}_h denotes the coefficient of variable \mathbf{h} in constraints (23c). In constraints (23d), matrices $\mathbf{F}_p, \mathbf{F}_l$, and \mathbf{F}_h respectively correspond to the coefficients of variables \mathbf{p}, \mathbf{l} , and \mathbf{h} .

Since the designed algorithm adopts the framework of Benders' decomposition (Geoffrion, 1972), the two-stage robust model in formulation (22)-(23) is decomposed into a master problem and a subproblem in the algorithm. We first introduce the subproblem (24), which is expressed as follows:

$$\text{SP} : R(\mathbf{x}, \mathbf{z}) = \min_{\xi \in \Xi, \mathbf{d} \in \mathcal{D}, \mathbf{w} \in \mathcal{W}} R(\mathbf{x}, \mathbf{z}, \xi, \mathbf{d}, \mathbf{w}). \quad (24)$$

In the subproblem **SP**, $R(\mathbf{x}, \mathbf{z})$ equals the worst-case value of the second-stage problem $R(\mathbf{x}, \mathbf{z}, \boldsymbol{\xi}, \mathbf{d}, \mathbf{w})$ (23) over the uncertainty sets Ξ , \mathcal{D} , and \mathcal{W} given any \mathbf{x} and \mathbf{z} . However, the min-max optimization structure of the subproblem **SP** is very challenging to tackle. To deal with this issue, we first write out the dual of the inner maximization problem (23) and transform the subproblem **SP** into the following formulation (25):

$$\overline{\mathbf{SP}} : R(\mathbf{x}, \mathbf{z}) = \min_{\boldsymbol{\xi}, \mathbf{d}, \mathbf{w}, \boldsymbol{\alpha}_s, \boldsymbol{\beta}_s, \boldsymbol{\gamma}_s} (\mathbf{D}_x \mathbf{x} + \mathbf{D}_z \mathbf{z} + \mathbf{q})^\top \boldsymbol{\alpha}_s + \mathbf{d}^\top \boldsymbol{\beta}_s + \mathbf{w}^\top \boldsymbol{\gamma}_s \quad (25a)$$

$$\text{s.t. } \mathbf{E}_p^\top \boldsymbol{\alpha}_s + \mathbf{F}_p^\top \boldsymbol{\gamma}_s \geq \mathbf{c}_p + \boldsymbol{\xi} \quad (25b)$$

$$\mathbf{E}_h^\top \boldsymbol{\alpha}_s + \mathbf{G}_h^\top \boldsymbol{\beta}_s + \mathbf{F}_h^\top \boldsymbol{\gamma}_s \geq \mathbf{c}_h \quad (25c)$$

$$\mathbf{E}_l^\top \boldsymbol{\alpha}_s + \mathbf{F}_l^\top \boldsymbol{\gamma}_s \geq \mathbf{0} \quad (25d)$$

$$\boldsymbol{\alpha}_s, \boldsymbol{\beta}_s \leq \mathbf{0}, \boldsymbol{\xi} \in \Xi, \mathbf{d} \in \mathcal{D}, \mathbf{w} \in \mathcal{W} \quad (25e)$$

where $\boldsymbol{\alpha}_s$, $\boldsymbol{\beta}_s$, and $\boldsymbol{\gamma}_s$ are dual variables related to constraints (23b), (23c), and (23d), respectively. As shown in equation (25a), the objective function has a bilinear structure due to the existence of bilinear terms ($\mathbf{d}^\top \boldsymbol{\beta}_s$ and $\mathbf{w}^\top \boldsymbol{\gamma}_s$). These bilinear terms make formulation (25) a non-convex optimization problem. Fortunately, this issue can be addressed based on the linearization method introduced in Thiele et al. (2009) with the defined budget-constrained uncertainty sets Ξ , \mathcal{D} , and \mathcal{W} . Hence, subproblem $\overline{\mathbf{SP}}$ is able to be tackled after linearization.

Given any first-stage decisions \mathbf{x} and \mathbf{z} which satisfy constraints (22b)-(22c), the inner maximization problem (23) of subproblem **SP** can be infeasible with some outcomes of the uncertain parameters $\boldsymbol{\xi}$, \mathbf{d} , and \mathbf{w} . In this situation, $R(\mathbf{x}, \mathbf{z})$ cannot be evaluated and is conventionally set to be $-\infty$. Therefore, the first-stage decisions that will cause the second-stage problem infeasible have to be eliminated from the solution space using feasibility cuts. In the developed algorithm, we derive feasibility cuts via addressing a feasibility problem **FP** which is shown in the following formulation (26):

$$\mathbf{FP} : F(\mathbf{x}, \mathbf{z}) = \max_{\boldsymbol{\xi} \in \Xi, \mathbf{d} \in \mathcal{D}, \mathbf{w} \in \mathcal{W}} \min_{\mathbf{h}, \mathbf{p}, \mathbf{l}, \mathbf{u}, \mathbf{v}, \mathbf{e}^+, \mathbf{e}^-} \mathbf{1}^\top \mathbf{u} + \mathbf{1}^\top \mathbf{v} + \mathbf{1}^\top (\mathbf{e}^+ + \mathbf{e}^-) \quad (26a)$$

$$\text{s.t. } \mathbf{E}_p \mathbf{p} + \mathbf{E}_h \mathbf{h} + \mathbf{E}_l \mathbf{l} + \mathbf{u} \geq \mathbf{D}_x \mathbf{x} + \mathbf{D}_z \mathbf{z} + \mathbf{q} \quad (26b)$$

$$\mathbf{G}_h \mathbf{h} + \mathbf{v} \geq \mathbf{d} \quad (26c)$$

$$\mathbf{F}_l \mathbf{l} + \mathbf{F}_p \mathbf{p} + \mathbf{F}_h \mathbf{h} + \mathbf{e}^+ - \mathbf{e}^- = \mathbf{w} \quad (26d)$$

$$\mathbf{h}, \mathbf{p}, \mathbf{l}, \mathbf{u}, \mathbf{v}, \mathbf{e}^+, \mathbf{e}^- \geq \mathbf{0} \quad (26e)$$

where $\mathbf{u}, \mathbf{v}, \mathbf{e}^+, \mathbf{e}^-$ are the slack variables related to constraints (26b)-(26d), respectively. Note that the second-stage maximization problem (23) is feasible under all possible outcomes of the uncertain parameters in the defined uncertainty sets if and only if $F(\mathbf{x}, \mathbf{z}) = 0$. As shown in formulation (26), the optimization structure of the feasibility

problem **FP** is similar to that of the subproblem **SP**. Thus, we also dualize its inner optimization problem to transform it into the formulation (27):

$$\overline{\mathbf{FP}} : F(\mathbf{x}, \mathbf{z}) = \max_{\xi, \mathbf{d}, \mathbf{w}, \alpha_f, \beta_f, \gamma_f} (\mathbf{D}_x \mathbf{x} + \mathbf{D}_z \mathbf{z} + \mathbf{q})^\top \alpha_f + \mathbf{d}^\top \beta_f + \mathbf{w}^\top \gamma_f \quad (27a)$$

$$\text{s.t. } \mathbf{E}_p^\top \alpha_f + \mathbf{F}_p^\top \gamma_f \leq \mathbf{0} \quad (27b)$$

$$\mathbf{E}_h^\top \alpha_f + \mathbf{G}_h^\top \beta_f + \mathbf{F}_h^\top \gamma_f \leq \mathbf{0} \quad (27c)$$

$$\mathbf{E}_l^\top \alpha_f + \mathbf{F}_l^\top \gamma_f \leq \mathbf{0} \quad (27d)$$

$$\alpha_f \leq \mathbf{1} \quad (27e)$$

$$\beta_f \leq \mathbf{1} \quad (27f)$$

$$-1 \leq \gamma_f \leq 1 \quad (27g)$$

$$\alpha_f, \beta_f \geq \mathbf{0}, \xi \in \Xi, \mathbf{d} \in \mathcal{D}, \mathbf{w} \in \mathcal{W} \quad (27h)$$

where α_f , β_f , and γ_f are dual variables associated with constraints (26b), (26c), and (26d), respectively. Note that the bilinear terms in objective function (27a) also can be handled by the method proposed in Thiele et al. (2009).

Given the aforementioned subproblem $\overline{\mathbf{SP}}$ with formulation (25) and the feasibility problem $\overline{\mathbf{FP}}$ with formulation (27), we finally show the details of the designed algorithm for the two-stage robust model (1)-(20) in Algorithm 1. In the $(n+1)$ th iteration of the algorithm, the optimal values of the first-stage decisions $(\mathbf{x}_{n+1}^*, \mathbf{z}_{n+1}^*)$ and the auxiliary decisions $(\theta_{n+1}^*, \mathbf{h}^{1*}, \dots, \mathbf{h}^{n*}, \mathbf{p}^{1*}, \dots, \mathbf{p}^{n*}, \mathbf{l}^{1*}, \dots, \mathbf{l}^{n*})$ are first generated by solving the master problem **MP** in formulation (28) (lines 3-4). $(\mathbf{h}^1, \dots, \mathbf{h}^n, \mathbf{p}^1, \dots, \mathbf{p}^n, \mathbf{l}^1, \dots, \mathbf{l}^n)$ are decision variables of the master problem **MP**, which are created after solving the feasibility problem $\overline{\mathbf{FP}}$ and the subproblem $\overline{\mathbf{SP}}$ in the first n iterations. θ is an auxiliary decision variable which helps calculating the upper bound UB to the optimal value of the objective function (22a) (line 5). After solving the master problem, the feasibility problem $\overline{\mathbf{FP}}$ is addressed with the obtained first-stage decisions $(\mathbf{x}_{n+1}^*, \mathbf{z}_{n+1}^*)$ to generate feasibility cuts (line 6). If $F(\mathbf{x}_{n+1}^*, \mathbf{z}_{n+1}^*) > 0$, feasibility cuts (29a)-(29c) will be included in the master problem **MP** (lines 8-9). Note that \mathbf{d}_{n+1}^* and \mathbf{w}_{n+1}^* are the optimal realizations of the corresponding uncertain parameters derived by solving $F(\mathbf{x}_{n+1}^*, \mathbf{z}_{n+1}^*)$. Otherwise, the subproblem $\overline{\mathbf{SP}}$ in formulation (25) is solved (line 12). A lower bound LB to the optimal value of the objective function (22a) can be obtained based on the objective value of the subproblem $\overline{\mathbf{SP}}$ (line 13). If $|UB - LB|/LB$ is below the optimality gap threshold ϵ , the algorithm terminates and the optimal first-stage decisions $(\mathbf{x}_{n+1}^*, \mathbf{z}_{n+1}^*)$ are returned. Otherwise, optimality cuts (30a)-(30d) are generated and incorporated to the master problem **MP** (lines 18-19). Then, the algorithm continues. Note that ξ_{n+1}^* , \mathbf{d}_{n+1}^* , and \mathbf{w}_{n+1}^* are the optimal realizations of the uncertain parameters derived from solving $R(\mathbf{x}_{n+1}^*, \mathbf{z}_{n+1}^*)$.

Algorithm 1 A constraint generation algorithm

- 1: Set upper bound $UB = +\infty$, lower bound $LB = -\infty$, $n = 0$ and $\mathcal{M} = \emptyset$
- 2: **while** $|UB - LB|/LB > \epsilon$ **do**
- 3: Solve the master problem **MP** in formulation (28).

$$\text{MP} : \max_{\mathbf{x}, \mathbf{z}, \theta} \quad -\mathbf{c}_z^\top \mathbf{z} + \theta \quad (28a)$$

$$\text{s.t.} \quad \mathbf{A}_x \mathbf{x} + \mathbf{A}_z \mathbf{z} \leq \mathbf{b} \quad (28b)$$

$$\theta \leq \mathbf{c}_p^\top \mathbf{p}^m + \mathbf{c}_h^\top \mathbf{h}^m + \boldsymbol{\xi}_m^{*\top} \mathbf{p}^m, \quad \forall m \in \mathcal{M} \quad (28c)$$

$$\mathbf{E}_p \mathbf{p}^m + \mathbf{E}_h \mathbf{h}^m + \mathbf{E}_l \mathbf{l}^m \geq \mathbf{D}_x \mathbf{x} + \mathbf{D}_z \mathbf{z} + \mathbf{q}, \quad \forall m \leq n \quad (28d)$$

$$\mathbf{G}_h \mathbf{h}^m \geq \mathbf{d}_m^*, \quad \forall m \leq n \quad (28e)$$

$$\mathbf{F}_l \mathbf{l}^m + \mathbf{F}_p \mathbf{p}^m + \mathbf{F}_h \mathbf{h}^m = \mathbf{w}_m^*, \quad \forall m \leq n \quad (28f)$$

$$\mathbf{x}, \mathbf{z} \in \{0, 1\}^{|\mathcal{I}| \times |\mathcal{T}|} \quad (28g)$$

$$\mathbf{h}^m, \mathbf{p}^m, \mathbf{l}^m \geq \mathbf{0}, \quad \forall m \leq n \quad (28h)$$

- 4: Get an optimal solution $(\mathbf{x}_{n+1}^*, \mathbf{z}_{n+1}^*, \theta_{n+1}^*, \mathbf{h}^{1*}, \dots, \mathbf{h}^{n*}, \mathbf{p}^{1*}, \dots, \mathbf{p}^{n*}, \mathbf{l}^{1*}, \dots, \mathbf{l}^{n*})$.
- 5: Update $UB = \theta_{n+1}^* - \mathbf{c}_z^\top \mathbf{z}_{n+1}^*$.
- 6: Solve the feasibility problem $\overline{\text{FP}}$ in formulation (27) with $(\mathbf{x}_{n+1}^*, \mathbf{z}_{n+1}^*)$.
- 7: **if** $F(\mathbf{x}_{n+1}^*, \mathbf{z}_{n+1}^*) > 0$ **then**
- 8: Create second-stage decision variables $(\mathbf{h}^{n+1}, \mathbf{p}^{n+1}, \mathbf{l}^{n+1})$.
- 9: Add constraints (29a)-(29c) to the master problem **MP**.

$$\mathbf{E}_p \mathbf{p}^{n+1} + \mathbf{E}_h \mathbf{h}^{n+1} + \mathbf{E}_l \mathbf{l}^{n+1} \geq \mathbf{D}_x \mathbf{x} + \mathbf{D}_z \mathbf{z} + \mathbf{q} \quad (29a)$$

$$\mathbf{G}_h \mathbf{h}^{n+1} \geq \mathbf{d}_{n+1}^* \quad (29b)$$

$$\mathbf{F}_l \mathbf{l}^{n+1} + \mathbf{F}_p \mathbf{p}^{n+1} + \mathbf{F}_h \mathbf{h}^{n+1} = \mathbf{w}_{n+1}^* \quad (29c)$$

- 10: Update $n = n + 1$.
- 11: **else**
- 12: Solve the subproblem $\overline{\text{SP}}$ in formulation (25) with $(\mathbf{x}_{n+1}^*, \mathbf{z}_{n+1}^*)$.
- 13: Update $LB = \min\{LB, R(\mathbf{x}_{n+1}^*, \mathbf{z}_{n+1}^*) - \mathbf{c}_z^\top \mathbf{z}_{n+1}^*\}$.
- 14: **if** $|UB - LB|/LB \leq \epsilon$ **then**
- 15: Return the optimal first-stage decisions $(\mathbf{x}_{n+1}^*, \mathbf{z}_{n+1}^*)$.
- 16: **Break**
- 17: **else**
- 18: Create second-stage decision variables $(\mathbf{h}^{n+1}, \mathbf{p}^{n+1}, \mathbf{l}^{n+1})$.
- 19: Add constraints (30a)-(30d) to the master problem **MP**.

$$\theta \leq \mathbf{c}_p^\top \mathbf{p}^{n+1} + \mathbf{c}_h^\top \mathbf{h}^{n+1} + \boldsymbol{\xi}_{n+1}^{*\top} \mathbf{p}^{n+1} \quad (30a)$$

$$\mathbf{E}_p \mathbf{p}^{n+1} + \mathbf{E}_h \mathbf{h}^{n+1} + \mathbf{E}_l \mathbf{l}^{n+1} \geq \mathbf{D}_x \mathbf{x} + \mathbf{D}_z \mathbf{z} + \mathbf{q} \quad (30b)$$

$$\mathbf{G}_h \mathbf{h}^{n+1} \geq \mathbf{d}_{n+1}^* \quad (30c)$$

$$\mathbf{F}_l \mathbf{l}^{n+1} + \mathbf{F}_p \mathbf{p}^{n+1} + \mathbf{F}_h \mathbf{h}^{n+1} = \mathbf{w}_{n+1}^* \quad (30d)$$

- 20: Update $n = n + 1$ and $\mathcal{M} = \mathcal{M} \cup \{n + 1\}$.
 - 21: **end if**
 - 22: **end if**
 - 23: **end while**
-

5 Case study

5.1 Case statement

To show the effectiveness of the developed two-stage robust optimization model and the designed algorithm, a WTE CHP plant is chosen as a case study herein. The WTE CHP plant is operated by the private sector in the X municipality of Northern Europe. The municipality is covered by a well-developed district heating system. The WTE plant houses two extraction units to generate electricity and heat. The electricity generated is traded in the regional day-ahead power market and the heat produced is exported to the local district heating system. The structure of the WTE CHP plant and its operational scheme are similar to those shown in Fig. 1.

To ensure efficient operations and prevent unexpected failures, plant operators must schedule and perform necessary operational tasks for both extraction units of the WTE plant over a planning horizon. These tasks concern the shutting down of each extraction unit for fixed time periods to conduct maintenance actions. The planning horizon is set to be one month (30 days), over which each time period is considered to be one day. Of note, the maintenance tasks for both extraction units cannot be implemented synchronously. The technical parameters and the details of the maintenance tasks associated with the extraction units of the WTE plant are presented in Table 1. These parameters are generated based on the CHP units of the operating WTE plants as introduced in [Force Technology \(2019\)](#). The other important parameters related to MSW storage and treatment in the case study are presented in Table 2.

As uncertain electricity prices ξ_t , heat demand d_t , and MSW supply w_t can take any value in the corresponding uncertainty sets in equations (21a)-(21c), real-world data are employed to build these sets in the case study. We construct the uncertainty set Ξ for electricity prices based on data from the Elspot day-ahead market for Eastern Denmark in June 2016, which can be downloaded from [Energinet.dk \(2019\)](#). In this set, we assume that the nominal (forecast) values $\bar{\xi}_t$ of the uncertain electricity price ξ_t equal the calculated daily average prices in all planning periods. We also assume that the maximum deviation $\hat{\xi}_t$ of ξ_t is 20% of $\bar{\xi}_t$ for all $t \in \mathcal{T}$ in set Ξ . The nominal electricity prices over the scheduling horizon are shown in Fig. 3. We construct the uncertainty set \mathcal{D} for heat demand based on data of the total heat consumption in the west of Copenhagen from July 1995 to June 1996 ([Madsen, 2019](#)). Note that we rescale and modify the original data for heat demand in the case study. The nominal values \bar{d}_t of uncertain heat demand d_t over the scheduling horizon are also shown in Fig. 3. In set \mathcal{D} , \hat{d}_t is assumed to be $0.2\bar{d}_t$ for all $t \in \mathcal{T}$. We construct the uncertainty set \mathcal{W} for MSW supply by modifying the data provided in [Liu et al. \(2017\)](#). The nominal values \bar{w}_t of uncertain MSW supply w_t over the scheduling horizon are shown in Fig. 4. We assume

$\hat{w}_t = 0.2\bar{w}_t$ for all $t \in \mathcal{T}$ in set \mathcal{W} . Finally, the uncertainty budgets Γ_ξ , Γ_d , and Γ_w are assumed to be the same and equal to 7. The optimality gap threshold ϵ in the designed algorithm is set to be 0.01%.

Table 1. Parameters for the production units of the WTE CHP plant in the case study.

Parameters	Description	Extraction Unit 1	Extraction Unit 2
HD_i^{max}	Daily maximum heat production, (MWh)	336	444
HD_i^{min}	Daily minimum heat production, (MWh)	0	0
PD_i^{max}	Daily maximum power production, (MWh)	288	360
PD_i^{min}	Daily minimum power production, (MWh)	96	120
MD_i^{max}	Daily maximum MSW consumption, (tonne)	288	360
MD_i^{min}	Daily minimum MSW consumption, (tonne)	120	156
CV_i	Variable operating cost, (€/tonne)	53	50
CM_i	Daily maintenance cost, (€)	1500	1800
π_i^p	Marginal MSW consumption for power production, (tonne/MWh)	1	1
π_i^h	Marginal MSW consumption for heat production, (tonne/MWh)	0.19	0.20
τ_i	Heat-to-power ratio	0.65	0.60
ET_i	Earliest maintenance start time, (day)	5	1
LT_i	Latest maintenance start time, (day)	27	25
DT_i	Maintenance duration, (day)	4	5

Table 2. Parameters related to MSW storage and treatment in the case study.

Parameters	Description	Value
L^{max}	Maximum allowable amount of MSW in the waste bunker, (tonne)	6000
L^{min}	Minimum allowable amount of MSW in the waste bunker, (tonne)	2000
l_0	Amount of MSW in the waste bunker at the beginning of the scheduling horizon, (tonne)	3000
l_{end}	Minimum required amount of MSW in the waste bunker at the end of the scheduling horizon, (tonne)	4000
GF	Marginal gate fee for MSW treatment, (€/tonne)	75

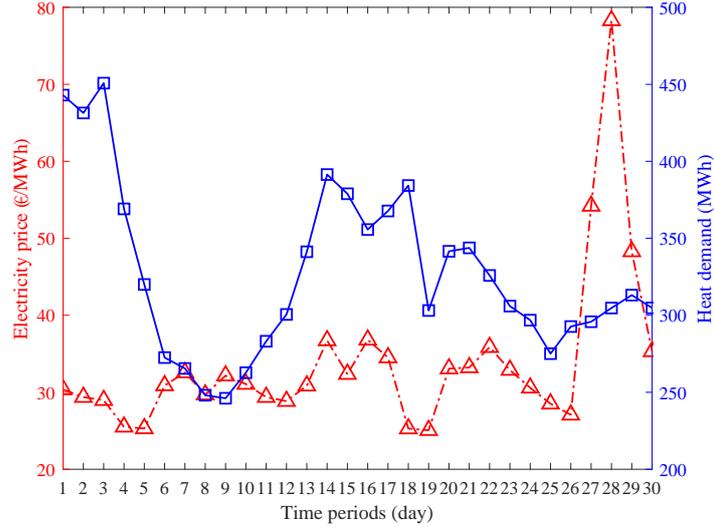


Fig. 3. Nominal values of uncertain electricity prices and heat demand over the scheduling horizon.

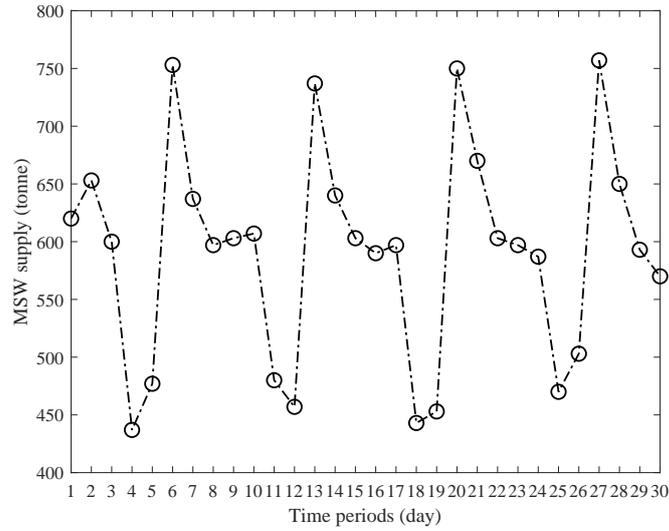


Fig. 4. Nominal values of uncertain MSW supply over the scheduling horizon.

5.2 Computational results

5.2.1 Robust schedule vs. Deterministic schedule

The two-stage robust model is implemented in the GAMS software and solved by the CPLEX 12.3 solver on a computer with a Xeon(R) 2.40GHz CPU and 32GB memory. With the aforementioned data, we first generate the robust schedule for the maintenance tasks for the extraction units of the WTE CHP plant, which is shown in Fig. 5(a). We

also obtain the deterministic schedule shown in Fig. 5(b) by solving the proposed model with only the nominal values of the uncertain parameters. Next, we use Monte Carlo simulation tests to evaluate the obtained robust and deterministic schedules. Monte Carlo simulation is a popular method to investigate the impact of uncertainty in many decision-making problems. It can help decision-makers see the performance of a decision under the possible realizations of uncertainty. In the simulation tests, we first randomly generate 1000 samples by assuming that each uncertain parameter ξ_t , d_t or w_t obeys a normal distribution with a mean equal to its corresponding nominal value and a standard deviation equal to 10% of its nominal value for all $t \in \mathcal{T}$. For each sampled outcome of all uncertain parameters, we solve the second-stage problem (9)-(20) with the schedule to be evaluated. Next, based on the 1000 generated samples, we calculate the average profit of the WTE CHP plant with the evaluated schedule. Note that penalties for failing to fulfill the heat demand and to treat the MSW are not considered when calculating the average profit of the plant. The robustness of a schedule is measured by a feasibility ratio: it is determined by calculating how many sampled outcomes can make the second-stage problem feasible with the evaluated schedule among the 1000 generated samples. The simulation results of the robust and deterministic schedules are shown in Table 3.

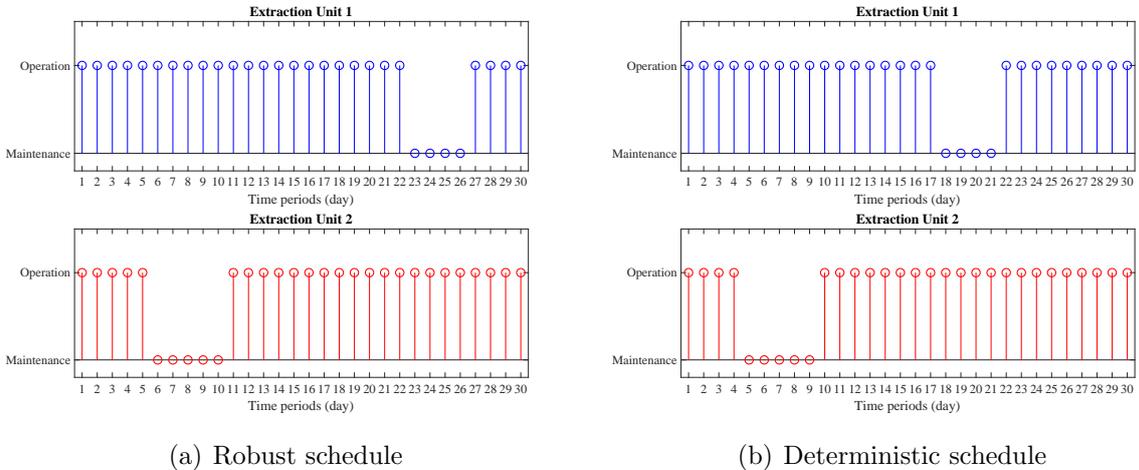


Fig. 5. Schedules for the production units of the WTE CHP plant: (a) robust schedule and (b) deterministic schedule.

Table 3. Simulation results for the robust and deterministic schedules.

Maintenance schedule	Average profit (€)	Feasibility ratio
Robust	877021.21	98.8%
Deterministic	879931.03	68.4%

Based on the deterministic schedule in Fig. 5(b), it is evident that Extraction Unit 2 is first shut down for maintenance during time periods 5-9, followed by Extraction Unit

1 during time periods 18-21. However, the robust schedule differs from the deterministic one. In Fig. 5(a), Extraction Unit 1 is scheduled to shut down for maintenance during time periods 23-26 whereas Extraction Unit 2 is scheduled to be instead during time periods 6-10. The underlying reason is that Extraction Unit 1 may fail to fulfill the allocated heat demand and the waste bunker may be unable to store the delivered MSW if Extraction Unit 2 is scheduled to shut down for maintenance during time periods 5-9, given the heat demand and MSW supply uncertainty. Based on the simulation results in Table 3, the average operating profit of the WTE CHP plant is 879931.03€ with the deterministic schedule, which declines slightly to 877021.21€ with the robust schedule. However, the feasibility ratio of the deterministic schedule is only 68.4%, which reflects its vulnerability. Moreover, the deterministic schedule may not enable the plant to cover the allocated heat demand in several time periods in real operations. Compared to the deterministic schedule, the robust schedule has a higher feasibility ratio of 98.8% which reflects not only its reliability, but also its lower likelihood of causing loss in heat demand or violating the upper storage limit of the waste bunker. Thus, we conclude that the robust schedule can achieve superior robustness at the (negligibly small) expense of the profit of the WTE CHP plant.

5.2.2 Effects of the uncertainty budgets

In this subsection, we analyze the effects of the uncertainty budgets Γ_ξ , Γ_d , and Γ_w of the corresponding uncertainty sets Ξ , \mathcal{D} , and \mathcal{W} on the robust schedule and the worst-case operating profit of the WTE CHP plant. Specifically, we assume $\Gamma = \Gamma_\xi = \Gamma_d = \Gamma_w$. Moreover, the maximum deviation of each uncertain parameter is assumed to equal 10% of its nominal value. The robust schedules with different uncertainty budgets are shown in Fig. 6. In addition, the corresponding worst-case operating profit of the plant and the CPU time are shown in Table 4.

Table 4. Results for the robust schedules with different uncertainty budgets.

Uncertainty budget Γ	7	14	21	28
Worst-case profit (€)	846933.83	810940.71	780533.49	734341.31
CPU time (s)	45.74	40.33	15.94	4.93

Concerning the uncertainty budget Γ , the robust schedules for the maintenance tasks are shown in Figs. 6(a) - 6(b) against the background of progressively larger budgets. A differential finding is evident: while it critically affects the scheduled shut-down time periods for Extraction Unit 1, the uncertainty budget does not affect those for Extraction Unit 2 (time periods 8-12). The underlying reason concerns the lower nominal values of uncertain electricity prices during time periods 8-12, during which it is judiciously

profitable to schedule the maintenance task for Extraction Unit 2.

Concerning the worst-case operating profit of the plant, when the values of Γ are elevated, such profit is noted to decrease as shown in Table 4. The underlying reason is that more possible outcomes of the uncertain parameters are considered in the defined uncertainty sets when Γ becomes larger. The worst-case profit of the plant diminishes since the operational task schedules have to be robust for more possible outcomes. From Table 4, we also find that all robust schedules with different uncertainty budgets are generated less than 60 seconds, attesting to the efficiency of the designed algorithm.

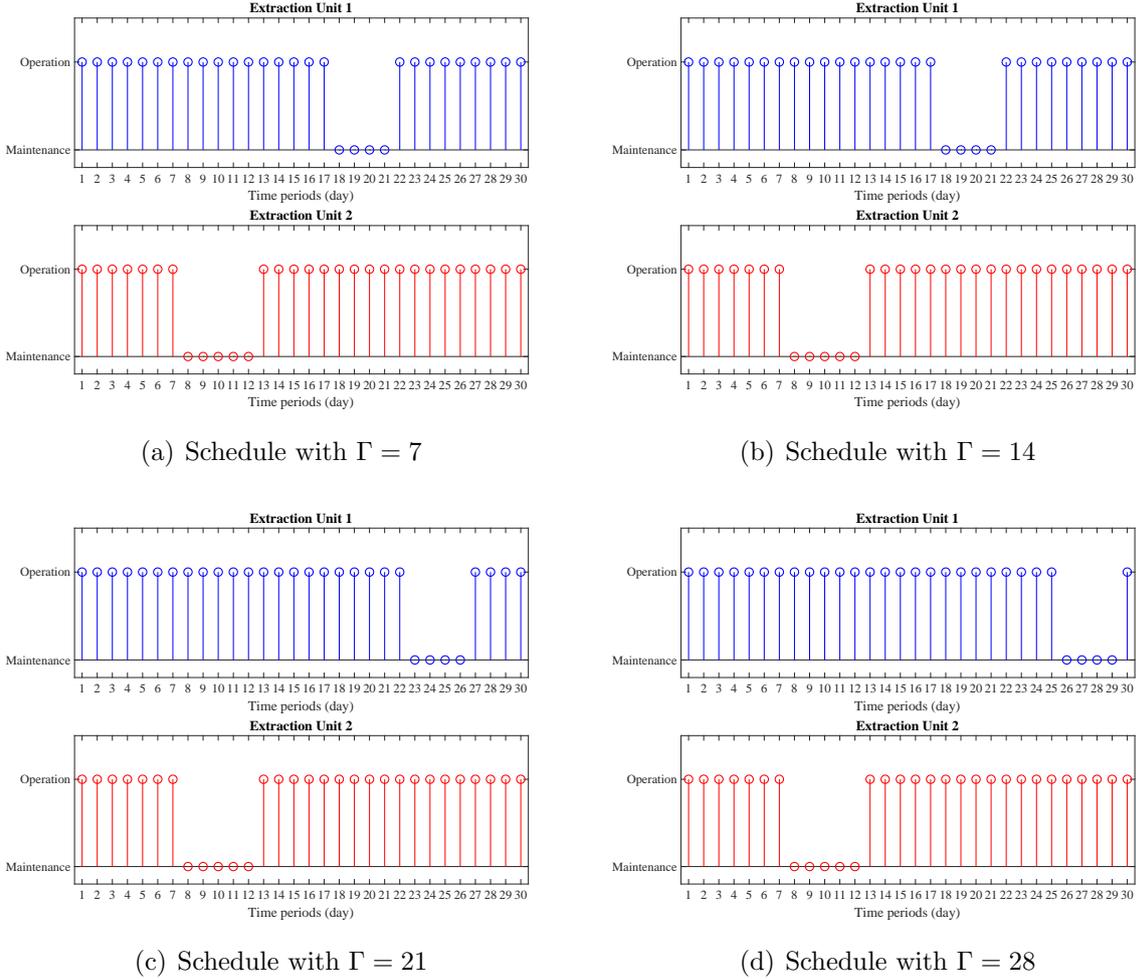


Fig. 6. Robust schedules with different uncertainty budgets: (a) schedule with $\Gamma = 7$, (b) schedule with $\Gamma = 14$, (c) schedule with $\Gamma = 21$ and (d) schedule with $\Gamma = 28$.

5.2.3 Effects of the uncertainty ranges

In this subsection, we investigate the impacts of the maximal deviations (uncertainty ranges) of the uncertain parameters on the robust schedule for the WTE CHP plant. Specifically, we assume that the maximum deviations $\hat{d}_t = U_r \bar{d}_t$, $\hat{\xi}_t = U_r \bar{\xi}_t$, and $\hat{w}_t = U_r \bar{w}_t$ for all $t \in \mathcal{T}$ in the uncertainty sets \mathcal{D} , Ξ , and \mathcal{W} , respectively. The parameter

U_r denotes the ratio between the maximum deviations of the uncertain parameters and their corresponding nominal values. It is introduced to reflect the uncertainty ranges of the uncertain parameters. In addition, all uncertainty budgets are still assumed to be the same and equal to 7. The robust schedules with different values of the maximum deviation ratio U_r are shown in Fig. 7. The corresponding worst-case operating profit of the plant and the CPU time are also shown in Table 5.

Table 5. Results for the robust schedules with different maximum deviation ratios.

Maximum deviation ratio U_r	5%	10%	15%	20%
Worst-case profit (€)	869481.59	846933.83	822983.77	799634.54
CPU time (s)	1399.74	45.74	405.60	1351.58

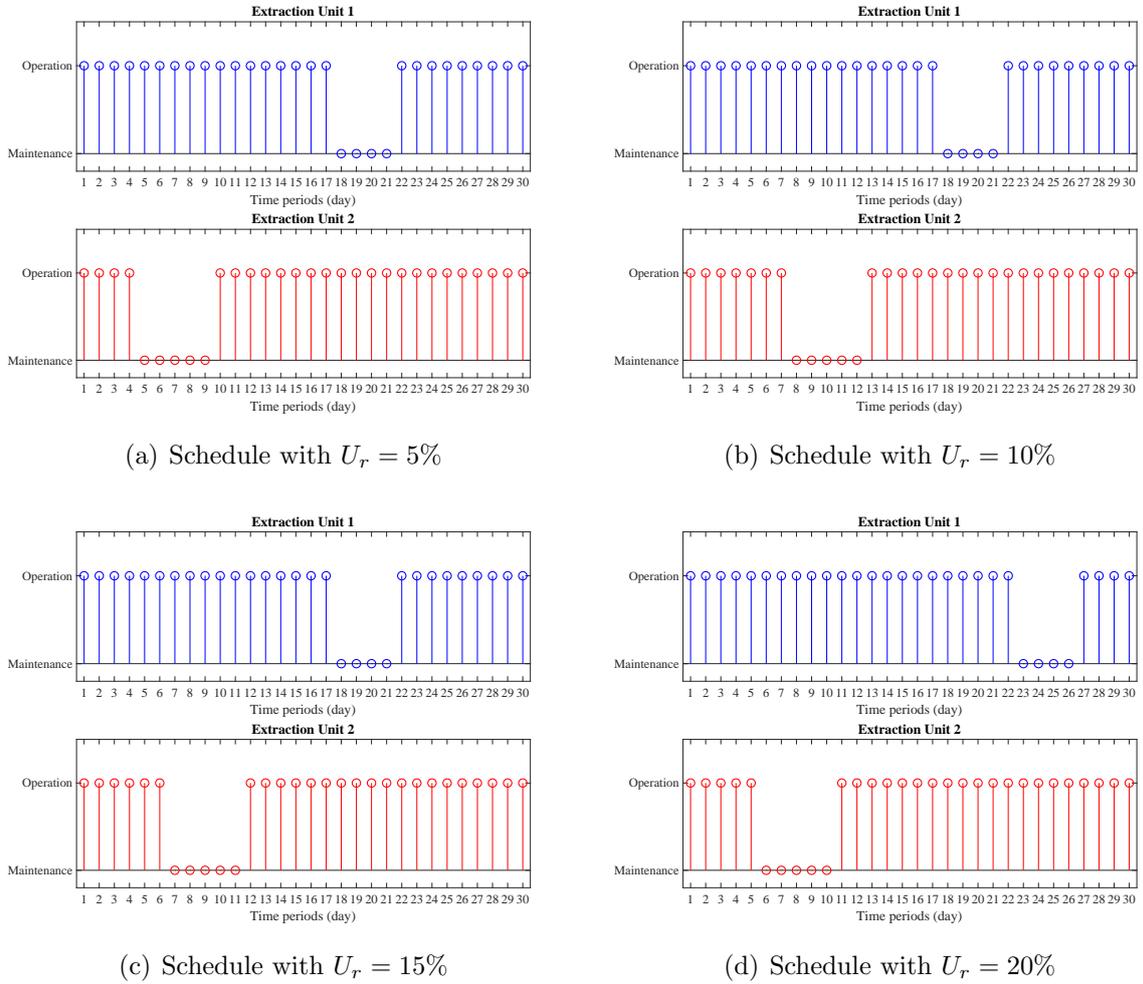


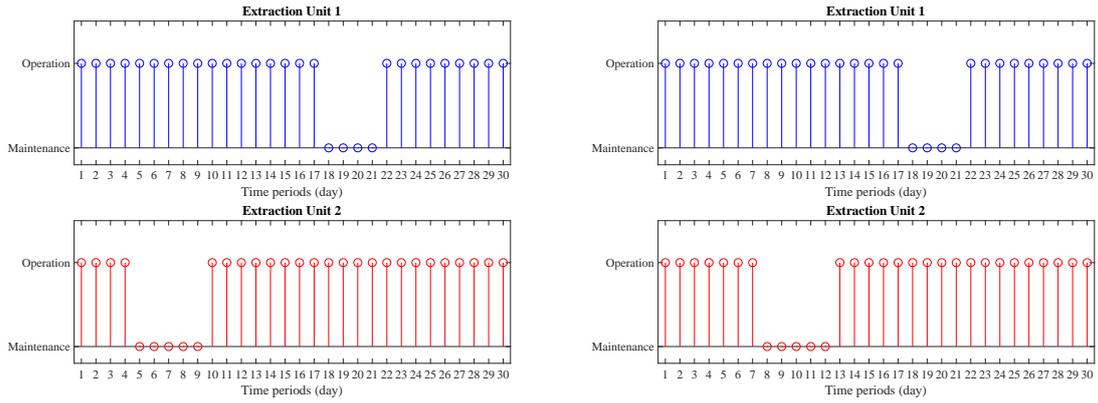
Fig. 7. Robust schedules with different maximum deviation ratios: (a) schedule with $U_r = 5\%$, (b) schedule with $U_r = 10\%$, (c) schedule with $U_r = 15\%$ and (d) schedule with $U_r = 20\%$.

Concerning the maximum deviation ratio U_r , it has an obvious impact on the derived

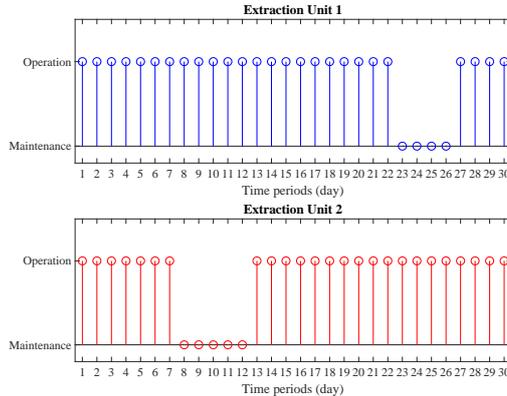
robust maintenance schedules for both extraction units as shown in Figs. 7(a)-7(d). From Table 5, we see that the worst-case operating profit of the WTE CHP plant over the scheduling horizon decreases as the maximum deviation ratio U_r increases. The underlying reason is that the uncertain parameters can have larger deviations when U_r elevates. The robust schedule has to compromise the worst-case operating profit of the plant since it needs to remain feasible when the maximum deviations of the uncertain parameters become larger.

5.2.4 Impacts of different types of uncertainty

In this subsection, we analyze the impacts of different types of uncertainty on the robust schedule and the worst-case profit of the WTE CHP plant. Specifically, we assume that the maximum deviation ratio U_r of all uncertain parameters is equal to 10%. Moreover, we assume $\Gamma_\xi = \Gamma_d = \Gamma_w = 14$. The robust schedules under different types of uncertainty are shown in Fig. 8.



(a) Schedule under electricity price uncertainty (b) Schedule under heat demand uncertainty



(c) Schedule under MSW supply uncertainty

Fig. 8. Robust schedules under different types of uncertainty: (a) schedule under electricity price uncertainty, (b) schedule under heat demand uncertainty and (c) schedule under MSW supply uncertainty.

Fig. 8(a) depicts the robust schedule generated by only considering the uncertainty in electricity prices. The worst-case operating profit of the plant is 848069.60€ with this schedule. Compared to the deterministic schedule in Fig. 5(b), it can be observed that uncertainty in electricity prices does not have an evident impact on the maintenance schedule but has a huge impact on the operating profit of the plant. The robust schedule obtained by only considering the uncertainty in heat demand is shown in Fig. 8(b). This schedule is different from the deterministic schedule. However, the operating profit of the plant with this schedule is 876197.91€, which is close to that with the deterministic one. Fig. 8(c) shows the robust schedule derived by only considering the uncertainty in MSW supply. This schedule is also different from the deterministic one. Moreover, the operating profit of the plant is 846796.50€ with this schedule, which is less than that with the deterministic schedule. Thus, uncertainty in MSW supply affects both the maintenance schedule and the operating profit of the plant.

6 Conclusion

In this paper, we have studied the problem of scheduling operational tasks for the production units of WTE CHP plants that participate in deregulated power markets and are connected to district heating networks. To address the problem, we have developed a two-stage robust optimization model that considers the uncertainty of electricity prices, heat demand, and MSW supply. A constraint generation algorithm has also been devised to solve the two-stage robust model to optimality. The proposed model and algorithm have been tested on a case study which aims to schedule operational tasks (preventive maintenance actions) for the production units of a WTE CHP plant.

The computational results suggest that the developed two-stage robust model can derive the optimal robust schedule for maintenance tasks for the production units of the WTE CHP plant. Furthermore, we have compared the robust schedule with the deterministic schedule generated without the consideration of uncertainty: the robust schedule outperforms its deterministic counterpart and yields superior robustness at the (negligibly small) expense of operating profit of the plant. In addition, we have analyzed the effects of the parameters defining the uncertainty sets on the optimal robust schedule for the WTE CHP plant. The results demonstrate that the uncertainty budgets and the maximal deviations of the uncertain parameters affect both the optimal robust schedule and the worst-case operating profit of the plant.

Our sensitivity analysis of the three different types of uncertainty further yields several interesting observations. Uncertainty in electricity prices exerts an evident impact on the operating profit of the WTE CHP plant. Conversely, uncertainty in heat demand tends to affect the robust schedule for maintenance tasks for the production units of the plant.

Uncertainty in MSW supply critically influences both the schedule and the operating profit of the plant. For future research, some directions deserve further pursuit. Firstly, it is worth developing more efficient algorithms for the developed two-stage robust model when the problem size increases. Secondly, it is interesting to extend the robust model to address potential operations scheduling problems of WTE plants in reality.

Acknowledgments

This research was funded by the National Research Foundation and the Prime Minister’s Office of Singapore under its Campus for Research Excellence and Technological Enterprise (CREATE) program.

References

- Abaecherli, M. L., Capón-García, E., Steinleitner, P., Weder, O., Szijjarto, A., and Hungerbühler, K. (2017a). Hierarchical integration of planning and scheduling for industrial waste incineration. *Industrial & Engineering Chemistry Research*, 56(27):7783–7798.
- Abaecherli, M. L., Capón-García, E., Szijjarto, A., and Hungerbühler, K. (2017b). Optimized energy use through systematic short-term management of industrial waste incineration. *Computers & Chemical Engineering*, 104:241–258.
- Avfall Svergie (2007). Towards a Greener Future with Swedish Waste-to-Energy: The World’s Best Example. URL < http://large.stanford.edu/courses/2016/ph240/ladow1/docs/forbranning_eng.pdf>.
- Ben-Tal, A., Goryashko, A., Guslitzer, E., and Nemirovski, A. (2004). Adjustable robust solutions of uncertain linear programs. *Mathematical Programming*, 99(2):351–376.
- Ben-Tal, A. and Nemirovski, A. (1998). Robust convex optimization. *Mathematics of Operations Research*, 23(4):769–805.
- Ben-Tal, A. and Nemirovski, A. (2000). Robust solutions of linear programming problems contaminated with uncertain data. *Mathematical Programming*, 88(3):411–424.
- Bertsimas, D. and Sim, M. (2004). The price of robustness. *Operations Research*, 52(1):35–53.
- Bhardwaj, A., Kamboj, V. K., Shukla, V. K., Singh, B., and Khurana, P. (2012). Unit commitment in electrical power system-a literature review. In *2012 IEEE International Power Engineering and Optimization Conference Melaka, Malaysia*, pages 275–280. IEEE.
- Cucchiella, F., DAdamo, I., and Gastaldi, M. (2014). Sustainable management of waste-to-energy facilities. *Renewable and Sustainable Energy Reviews*, 33:719–728.
- Energinet.dk (2019). Download of electricity market data. Website, URL < <https://en.energinet.dk>>.

- Force Technology (2019). European Biomass CHP in Practice. Website, URL < <https://bio-chp.force.dk/downloads/chp-plants-key-figures/>>.
- Froger, A., Gendreau, M., Mendoza, J. E., Pinson, É., and Rousseau, L.-M. (2016). Maintenance scheduling in the electricity industry: A literature review. *European Journal of Operational Research*, 251(3):695–706.
- Fruergaard, T., Christensen, T. H., and Astrup, T. (2010). Energy recovery from waste incineration: Assessing the importance of district heating networks. *Waste Management*, 30(7):1264–1272.
- Geoffrion, A. M. (1972). Generalized benders decomposition. *Journal of Optimization Theory and Applications*, 10(4):237–260.
- Guedes, L., Vieira, D., Lisboa, A., and Saldanha, R. (2015). A continuous compact model for cascaded hydro-power generation and preventive maintenance scheduling. *International Journal of Electrical Power & Energy Systems*, 73:702–710.
- Helseth, A., Fodstad, M., and Mo, B. (2018). Optimal hydropower maintenance scheduling in liberalized markets. *IEEE Transactions on Power Systems*, 33(6):6989–6998.
- Hongling, L., Chuanwen, J., and Yan, Z. (2008). A review on risk-constrained hydropower scheduling in deregulated power market. *Renewable and Sustainable Energy Reviews*, 12(5):1465–1475.
- Kelley, Jr, J. E. (1960). The cutting-plane method for solving convex programs. *Journal of the Society for Industrial and Applied Mathematics*, 8(4):703–712.
- Khalid, A. and Ioannis, K. (2012). A survey of generator maintenance scheduling techniques. *Global Journal of Researches in Engineering*, 12(1):10–17.
- Kralj, B. L. and Petrović, R. (1988). Optimal preventive maintenance scheduling of thermal generating units in power systemsa survey of problem formulations and solution methods. *European Journal of Operational Research*, 35(1):1–15.
- Kumar, A. and Samadder, S. (2017). A review on technological options of waste to energy for effective management of municipal solid waste. *Waste Management*, 69:407–422.
- Liu, Y., Shen, Z., Tang, X., Lian, H., and Li, J. (2017). Joint maintenance scheduling of the municipal solid waste incineration power plant and connected power system devices. *The Journal of Engineering*, 2017(13):1745–1749.
- Lombardi, L., Carnevale, E., and Corti, A. (2015). A review of technologies and performances of thermal treatment systems for energy recovery from waste. *Waste management*, 37:26–44.

- Madsen, H. (2019). Time series analysis. URL < <http://www.imm.dtu.dk/~hmad/time.series.analysis/assignments/>>.
- Mahor, A., Prasad, V., and Rangnekar, S. (2009). Economic dispatch using particle swarm optimization: A review. *Renewable and sustainable energy reviews*, 13(8):2134–2141.
- Nazari-Heris, M., Mohammadi-Ivatloo, B., and Gharehpetian, G. (2017). Short-term scheduling of hydro-based power plants considering application of heuristic algorithms: A comprehensive review. *Renewable and Sustainable Energy Reviews*, 74:116–129.
- Pan, S.-Y., Du, M. A., Huang, I.-T., Liu, I.-H., Chang, E., and Chiang, P.-C. (2015). Strategies on implementation of waste-to-energy (WTE) supply chain for circular economy system: A review. *Journal of Cleaner Production*, 108:409–421.
- Pant, D., Van Bogaert, G., Diels, L., and Vanbroekhoven, K. (2010). A review of the substrates used in microbial fuel cells (mfcs) for sustainable energy production. *Biore-source technology*, 101(6):1533–1543.
- Rogoff, M. J. and Screve, F. (2019). *Waste-to-energy: technologies and project implementation*. Academic Press.
- Ryu, C. and Shin, D. (2012). Combined heat and power from municipal solid waste: Current status and issues in South Korea. *Energies*, 6(1):45–57.
- Sepehri, A. and Sarrafzadeh, M.-H. (2018). Effect of nitrifiers community on fouling mitigation and nitrification efficiency in a membrane bioreactor. *Chemical Engineering and Processing-Process Intensification*, 128:10–18.
- Sepehri, A., Sarrafzadeh, M.-H., and Avateffazeli, M. (2019). Interaction between *Chlorella vulgaris* and nitrifying-enriched activated sludge in the treatment of wastewater with low C/N ratio. *Journal of Cleaner Production*.
- Shi, H., Mahinpey, N., Aqsha, A., and Silbermann, R. (2016). Characterization, thermochemical conversion studies, and heating value modeling of municipal solid waste. *waste Management*, 48:34–47.
- Thiele, A., Terry, T., and Epelman, M. (2009). Robust linear optimization with recourse. Technical report, Lehigh University, Bethlehem, PA, USA.
- Touš, M., Pavlas, M., Putna, O., Stehlík, P., and Crha, L. (2015). Combined heat and power production planning in a waste-to-energy plant on a short-term basis. *Energy*, 90:137–147.

- Yamin, H. Y. (2004). Review on methods of generation scheduling in electric power systems. *Electric Power Systems Research*, 69(2-3):227–248.
- Yanikoğlu, İ., Gorissen, B. L., and den Hertog, D. (2019). A survey of adjustable robust optimization. *European Journal of Operational Research*, 277(3):799–813.
- Yi, S., Jang, Y.-C., and An, A. K. (2018). Potential for energy recovery and greenhouse gas reduction through waste-to-energy technologies. *Journal of Cleaner Production*, 176:503–511.
- Zeng, B. and Zhao, L. (2013). Solving two-stage robust optimization problems using a column-and-constraint generation method. *Operations Research Letters*, 41(5):457–461.
- Zhen, J., Den Hertog, D., and Sim, M. (2018). Adjustable robust optimization via fourier–motzkin elimination. *Operations Research*, 66(4):1086–1100.